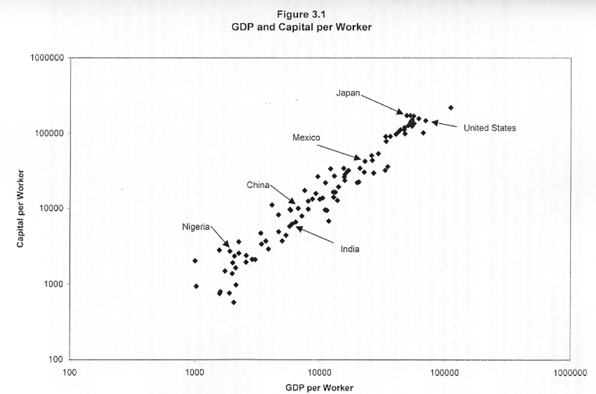
2. Capital Accumulation.

2.1. The Nature of Capital



- Note the linear path of expansion.
- Stylized (Kaldor-) Fact: the capital output ratio is constant (in the long-run).
- Yet, why is K so much higher in the U.S. than in Nigeria?
- And what is capital? (a difficult question)

Physical capital: the "tools", machines, equipment used by workers.

- It is productive (its use raises output).
- It has been produced (i.e. it is costly) $\rightarrow$ investment.
- It is rival in its use. Compare
  - private vs. public capital (congestion).
  - capital vs. ideas (e.g. blueprint for a machine)
- It can be accumulated (multiplied) without bound. Compare
  - capital vs. land
  - capital vs. people (workers)
- It depreciates (using it wears it down, it has to be renewed).

Compare: human capital, social capital, cultural capital,....

(heroic) assumption: It can be aggregated

- the capital stock of an economy, K
- the production function of an economy, $Y = AF(K, L, \cdot)$.

The Role of capital in explaining growth and development is focus of Neoclassical Growth Theory.

2.2. The Solow Model

Robert, M. Solow, 1956, A Contribution to the Theory of Economic Growth, *Quarterly Journal of Economics* 70, 65-94.
Barro and Sala-i-Martin, Chapter 1 (Weil Chapter 3.3 in discrete time).

Main ideas:

- capital accumulation as the engine of growth ($\leftrightarrow$ classical growth theory, Malthus, Ricardo, $\leftrightarrow$ New Growth Theory)
- Economic growth is an equilibrium process, i.e. balanced growth ($\leftrightarrow$ other growth theories: growth on a knife-edge: Harrod, Domar, Marx).

Elements of the Model

- one-good-economy (i.e. aggregation)
- closed economy ($\rightarrow$ later)
- no specific role for government ($\rightarrow$ later)

$$Y = C + I, \quad Y = C + S, \quad I = S \tag{1}$$

C: consumption, I: investment, S: savings.

The savings ratio s is a given constant.

$$S = s \cdot Y \tag{2}$$

Why? A behavioral assumption. Next chapter: resolution.

Equation of motion for the capital stock:

$$\dot{K} = I - \delta K \tag{3}$$

I: gross investment, δ: depreciation rate, i.e. $\dot{K}$: net investment.

Neoclassical production function $Y = AF(K, L)$, where F

- is constant returns to scale

$$\lambda F(K, L) = F(\lambda K, \lambda L)$$

  - no gains of specialization
  - no external effects
  - no limiting factors (land)
- has positive, decreasing marginal returns:

$$\frac{\partial F}{\partial K} > 0, \qquad \frac{\partial^2 F}{\partial K^2}, < 0$$
$$\frac{\partial F}{\partial L} > 0, \qquad \frac{\partial^2 F}{\partial L^2}. < 0$$

- fulfils the Inada conditions, in particular

$$\lim_{K \to 0} = \frac{\partial F}{\partial K} = \infty, \quad \lim_{K \to \infty} \frac{\partial F}{\partial K} = 0$$

  and likewise for L.

From c.r.s. follows

$$y \equiv \frac{Y}{L} = F\left(\frac{K}{L}, 1\right) = f(k)$$

with $k \equiv K/L$.
with full employment:

- y: per capita production, per capita income
- k machines per workplace, per capita capital stock
- f per capita production function.

[Insert: shape of per capita production function]

An easy to handle production function fulfilling all that: Cobb-Douglas PF.:

$$Y = AK^{\alpha}L^{1-\alpha} \qquad \Rightarrow \qquad y = AK^{\alpha}L^{-\alpha} = Ak^{\alpha}. \tag{4}$$

Factor payment according to marginal product, i.e. interest rate

$$r = \frac{\partial Y}{\partial K} = \alpha A K^{\alpha-1} L^{1-\alpha}$$

Implying that the capital share of income

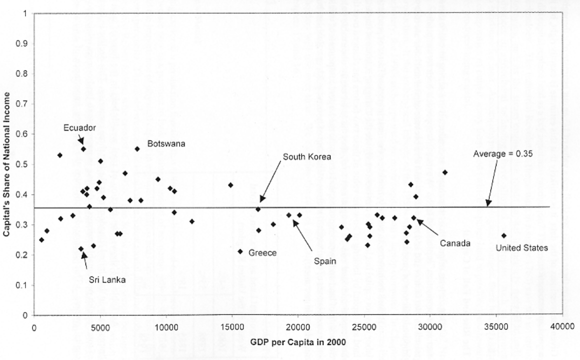
- equals the production elasticity of capital, i.e. the exponent of capital in the production function
- is constant over time.

$$\frac{rK}{Y} = \frac{\partial Y}{\partial K} \cdot \frac{K}{Y} = \frac{\alpha A K^{\alpha-1} L^{1-\alpha} \cdot K}{AK^{\alpha}L^{1-\alpha}} = \alpha$$

(likewise for labor).

benchmark: the capital share (α) is roughly 1/3.

Capital share of income in a cross-section of countries



Next step: summary of the model in one equation.

Differentiation of $k = K/L$ w.r.t. time (L constant):

$$\dot{k} = \dot{K}/L \tag{5}$$

Insert (1) – (3):

$$\dot{k} = sY/L - \delta K/L \qquad \Rightarrow \qquad \dot{k} = s \cdot f(k) - \delta k.$$

And for Cobb-Douglas: $\dot{k} = s \cdot Ak^{\alpha} - \delta k$.

Graphical Analysis:

[Insert 2 diagrams]

Thus:

- There exists a unique non-trivial dynamic equilibrium at $k = k^*$
- It is globally stable:

$$\dot{k} > 0 \quad \text{for } k < k^*$$
$$\dot{k} < 0 \quad \text{for } k > k^*.$$

We say: at k^* the economy is in a steady-state.

Def. steady-state: a situation in which the variables of a model are

- either constant.
- or growing at constant rate.

Explicit solution: at k^*:

$$\dot{k} = k^* \left[sA{k^*}^{\alpha-1} - \delta \right] = 0 \qquad \Rightarrow \qquad k^* = \left(\frac{\delta}{sA} \right)^{1/(\alpha-1)} = \left(\frac{sA}{\delta} \right)^{1/(1-\alpha)} . \quad (6)$$

and thus income

$$y^* = A{k^*}^{\alpha} = A^{1/(1-\alpha)} \left(\frac{s}{\delta} \right)^{\alpha/(1-\alpha)} . \quad (7)$$

2 methods of further analysis:

- comparative statics (i.e. of steady-states)
- comparative dynamics (of adjustment behavior towards the s.s.)

Comparative statics: consider 2 countries

- savings rate in country 1: s_1
- savings rate in country 2: s_2, $s_2 < s_1$.

Graphical analysis

[Insert: comparative statics: *s*]

Conclude: countries with higher savings rate (investment rate) have higher income per capita.

Algebraically

$$\frac{y_1}{y_2} = \left(\frac{s_1}{s_2}\right)^{\alpha/(1-\alpha)}$$

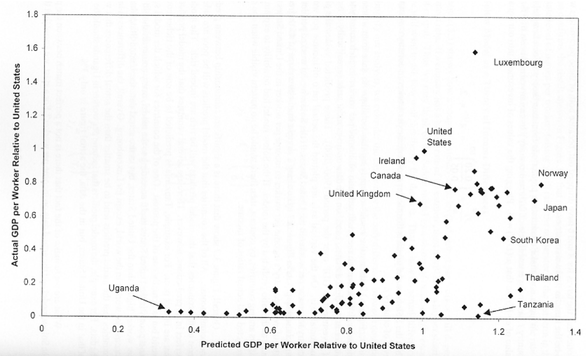
For example, $\alpha = 1/3$, $s_1 = 0.2$ (Korea), $s_2 = 0.05$ (Uganda):

$$\frac{y_1}{y_2} = \left(\frac{0.2}{0.05}\right)^{1/2} = 4^{1/2} = 2.$$

The Solow model predicts that Korea is twice as rich as Uganda.

And for a bunch countries...

Actual vs. predicted GDP per worker



Conclude:

- The Solow model explains some variation of the world income distribution.
- E.g., it explains why Korea is twice as rich as Uganda (but it is actually 13 times as rich).
- 1 within-model explanation for lack of better fit: countries are not at their steady-state k^*.
- Yet we had'nt expected that savings rates explain already everything (then, what were we supposed to do for the rest of course?)

Conclusion (continued):

- Differences in A will explain a lot ($\rightarrow$ later)
- Other stuff will explain a lot.
- And, by the way, why do people in different countries show different savings behavior? ($\rightarrow$ deep determinants).

Comparative Dynamics:

Along the adjustment path growth is higher for small values of k (and thus y):

[Insert: phase diagram]

Yet, does this imply that poor countries grow faster than rich ones?

$\rightarrow$ No, not necessarily. Compare:

[Insert: 2 adjustment paths]

Conclude:

- The Solow model explains Conditional Convergence.
- countries with the *same fundamentals* (here s, A, δ, α) converge towards the same steady-state.
- But convergence does not apply for all countries (compare: twin peaks).

In Weil's words: Ceteris paribus

- if two countries have the same savings rate but different levels of income, the country with lower income grows faster.
- if two countries have the same level of income but different savings rates, the country with the higher savings rate grows faster.

Adjustment dynamics after an increase of the savings rate:

[Phase diagram]

[Impulse response functions for $\dot{k}/k$, y, and c]

Observe:

- It takes time to built up a capital stock.
- Necessarily there will be an initial drop in consumption.

But then, is an increasing savings rate desirable after all? The Solow model cannot answer this question $\rightarrow$ later.

Even at a steady-state there can be to much savings (theoretically). Recall

$$y^* = A^{1/(1-\alpha)} \left(\frac{s}{\delta}\right)^{\alpha/(1-\alpha)}.$$

Implied consumption:

$$c = (1-s)A^{1/(1-\alpha)} \left(\frac{s}{\delta}\right)^{\alpha/(1-\alpha)}.$$

First order condition for maximum consumption:

$$0 = \frac{\partial c}{\partial s} = \frac{A^{1/(1-\alpha)}}{\delta^{\alpha/1-\alpha}} \left[-s^{\frac{\alpha}{1-\alpha}} + (1-s)\frac{\alpha}{1-\alpha} s^{\frac{\alpha}{1-\alpha}-1} \right]$$

Implying

$$0 = -s + (1-s)\frac{\alpha}{1-\alpha} \qquad \Rightarrow \qquad \frac{s}{1-s} = \frac{\alpha}{1-\alpha}$$

and $s = s_{gold} = \alpha$.
This is the GOLDEN RULE OF SAVING. $s \uparrow$ beyond $s_{gold} \;\rightarrow\; c^* \downarrow$ (dynamic inefficiency).

Yet, there is hardly any country with savings rates above 1/3.

- The former USSR?
- Singapore?

How can the savings rate be risen (in democracies)?

- funded public pension programs
- less finance of public expenditure with government debt.

A final assessment:

- Capital accumulation and savings are an important determinant of growth and development.
- Yet, it can by far not explain fully the variance of income levels across countries.
- The neoclassical growth model can explain long-run differences in levels of y and short-run differences in growth but not long-run differences of growth rates (there is no long-run growth) $\rightarrow$ the role of A.
- Actually the savings rate is not a constant. It is generated through individual behavior given other fundamentals. Generally, richer individuals / countries save more ($\rightarrow$ note the problem of causality)
- For about 40 years after the WW II economists (neoclassical growth theory) and politicians (e.g. the IMF) have put too much emphasis on savings.
- Thereby other important stuff was neglected (e.g. the role of governance and institutions).

Yet, we will use the simple idea of capital accumulation to introduce some other ways of modelling growth. These models will be important vehicles of economic reasoning throughout the course.