4. Population and Economic Growth

Reference: Weil, Chapter 4.1-4.2

3 important aspects:

- population size L
- population growth $\dot{L}/L \equiv n$
- given a fixed quantity of land, population density L/X

Compare: between 1960 and 2000

- Japan's population grew at rate 0.8% p.a.,
- Kenya's population grew at at rate 3.3. % p.a.

Yet, population density in 2000 was

- 131 people per km^2 in Japan
- 20 people per km^2 in Kenya.

Humans *seem* to grow different than other species. Growth of animals

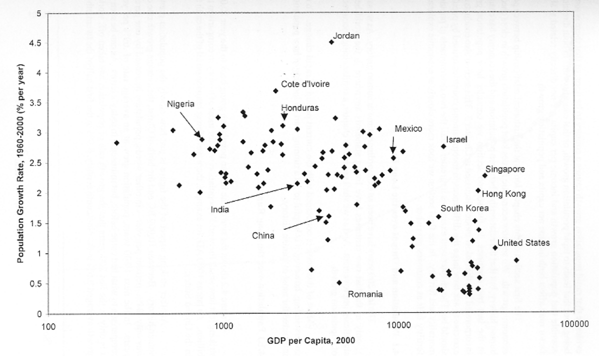
- logistic growth (carrying capacity)
- predator prey dynamics

Growth of mankind: exponential ?

- why?
- sustainable?

Population growth seems to be bad for the standard of living

Per capita income and population growth across countries:



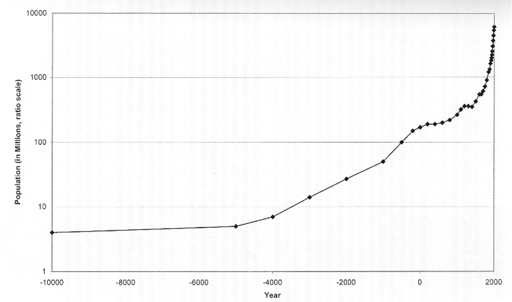
Causality:

- Does high population growth trigger poverty
- Does low income lead to high population growth?

As with income growth the last 200 years were different.

Consider 20000 BC - 1860 AD:

- Almost no income growth.
- Yet technological progress at slow pace.
- Population growth at slow pace.
  - 0.04 % p.a. between 10.000 BC and 1 AD
  - 0.09 % p.a. between 1 and 1800
  - 0.6 % p.a. in 1800 - 1900 , 0.9 % p.a. in 1900 - 1950
  - 1.8 % p.a. in 1950 - 2000
- Productivity gains have been exclusively used to increase L/X.



The Malthusian Model

Malthus (1798): Principle of Population:

- Ceteris paribus, the less people there are on a given piece of land the better off they are.
- The better off people are the more they multiply.

- The positive check: as for other animals there exists a given carrying capacity. Mortality balances fertility.
- Preventive check (unique to humans): anticipating this, humans (somewhat) control fertility.

Indeed there was birth control everywhere at all times:

- condoms
- late or no marriage (Europe)
- extended breastfeeding (Africa)
- child exposure (Ancient Greece, China).

The Malthusian reasoning can be easily diagrammatically analyzed...

[Insert: Malthusian Model: Diagrammatic Analysis]

Note: stable steady-state at: y^*, L^* and $n^* = 0$.

Consider technological progress:

- people become sedentary
- cultivation of a new crops
- import of the potato.

- Instantaneous effect: people are richer: $L(y)$ curve shifts to the right.
- Then, move towards a new Malthusian equilibrium
- y^* as before, $L^{**} > L^*$.

Thus, in a Malthusian world countries with higher productivity are not richer but more densely populated (consistent with history).

Yet, nowadays capital has predominantly replaced land in production.

- Interaction of population growth and capital accumulation in Neoclassical Growth Models:
- The capital dilution effect.
- Take, for example, the Solow model.

Basic idea:

- Ceteris paribus, population growth lowers the number of machines per worker (K/L).
- Income per capita will be lower.
- population growth operates like depreciation of k.

Re-differentiate the capital labor ratio $k = K/L$ w.r.t. time:

$$\dot{k} = \frac{\dot{K}}{L} - \frac{\dot{L}}{L}\frac{K}{L} \qquad \Rightarrow \qquad \dot{k} = \frac{\dot{K}}{L} - nk.$$

Insert the rest of the Solow model:

$$\dot{k} = sAk^{\alpha} - (\delta + n)k. \tag{1}$$

Comparative steady-state analysis: consider $n \uparrow$:

[Insert: population growth in the Solow Model]

Conclude:

- Countries with higher population growth are poorer.
- Population growth dilutes the capital stock per capita
- And thus drives down productivity or workers and income.

Algebraically Solve (1) at the steady-state

$$0 = \dot{k} = sAk^{\alpha} - (\delta + n)k \qquad \Rightarrow \qquad k^* = \left(\frac{sA}{n+\delta}\right)^{1/(1-\alpha)} \tag{2}$$

Implying income per capita:

$$y^* = Ak^{*\alpha} = A\left(\frac{sA}{n+\delta}\right)^{\alpha/(1-\alpha)} \tag{3}$$

Now consider 2 countries:

- $n_1 = 0$ (typical rich OECD)
- $n_2 = 0.04$ (typical very poor SSA)

Assume everything else is the same. Relative income

$$\frac{y_1}{y_2} = \left(\frac{n_2 + \delta}{n_1 + \delta}\right)^{\alpha/(1-\alpha)}.$$

Suppose $\alpha = 1/3$, $\delta = 0.05$:

$$\frac{y_1}{y_2} = \left(\frac{0.04 + 0.05}{0 + 0.05}\right)^{1/2} \approx 1.34.$$

Conclude:

- The country with high population growth is poorer.
- Yet, much less than the data suggests (inspect Figure 1).
- Assume: $\alpha = 2/3$
  - Why? $\rightarrow$ later
  - expect larger consequences from capital dilution.

$$\frac{y_1}{y_2} = \left(\frac{0.04 + 0.05}{0 + 0.05}\right)^{2} \approx 3.2.$$

Expect similar results for Ramsey-, Diamond-model ($\rightarrow$ why?).

Finally reconsider the poverty trap version of the Diamond model.

- Assume $n_t = n(w_t - \bar{c})$, $n(0) = 0$
- $n' > 0$ in neighborhood of $\bar{c}$
- $L_{t+1} = (1 - n_t)L_t$

At a long-run equilibrium where $n = 0$:

$$\frac{L}{X} = \left[\frac{(1-\alpha)A}{\bar{c}}\right]^{1/\alpha} \qquad \Rightarrow \qquad A \uparrow \; \rightarrow \; \frac{L}{X} \uparrow .$$

[Insert: Diamond model with population trap]

Observe:

- A different definition of subsistence: population stays constant.
- The poverty trap becomes a *population trap*.
- Technological progress triggers population growth.
- The higher A the lower the exogenous shock of k needed to escape from the trap.