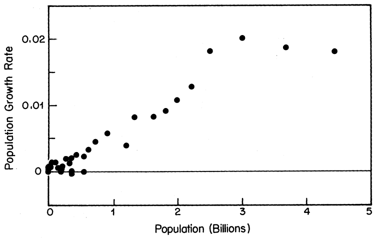
Reference: Michael Kremer, 1993, Population Growth and Technological Change: One Million B.C. to 1990, *Quarterly Journal of Economics* 108, 681-716.
Weil Chapter 4.1., 9.1.

Empirical observation: in world history:

- $\hat{L} \equiv n \neq$ const.
- $n \approx$ proportional to L

$\rightarrow$ L grows more than exponentially (hyper-exponentially) for most of world history.
$\rightarrow$ human kind grows different to all other species in the world.



[World population over the long run]

A theoretical model that combines

- endogenous technological progress (last lecture)
- endogenous population growth (Part I of the course)

can explain this stylized fact.

- The criticized scale effect is now useful.
- It explains why among societies with no contact to each other those with higher initial L have faster growth

The Model

$$Y = AL^{\alpha} T^{1-\alpha} \tag{1}$$

T: arable land, not-accumulable, normalized to 1.

Observe:

- no capital; land is important.
- decreasing returns to scale w.r.t. to accumulable physical factors ($0 < \alpha < 1$).

Income per capita:

$$y \equiv \frac{Y}{L} = AL^{\alpha-1}$$

Recap from last course: Malthus (1798): Principle of Population:

- Ceteris paribus, the less people there are on a given piece of land the better off they are.
- The better off people are the more they multiply.

- The positive check: as for other animals there exists a given carrying capacity. Mortality balances fertility.
- Preventive check (unique to humans): anticipating this, humans (somewhat) control fertility.

[Insert: Malthusian Model: Diagrammatic Analysis]

Observe: There exists a unique equilibrium income $y = \bar{y}$

- above which $\dot{y} < 0$ because $\dot{L} > 0$ (drive for reproduction)
- below which $\dot{y} > 0$ because $\dot{L} < 0$ (premature death) Thus a unique stable equilibrium L at $\bar{y}$

$$\bar{y} = AL^{\alpha-1} \quad \Rightarrow \quad L = \left(\frac{\bar{y}}{A}\right)^{1/(\alpha-1)} \tag{2}$$

Yet this was last course.

- Malthus: A is constant $\rightarrow$ replaced by
- Kremer/Jones/Romer: A grows permanently.

We get an explanation for the puzzling observation

- that there were new ideas/productivity growth all time.
- yet there was no (pronounced) improvement of income per capita.

The technological advances were "eaten up" by population growth.
Economic development meant population growth: $A \uparrow \rightarrow L \uparrow \rightarrow y$ const.

This notion is correct until ≈ 1800 (when Malthus wrote his essay).

TABLE 9.1
Growth Accounting for Europe, A.D. 500–1700

Period	Annual Growth Rate of Income per Capita, $\hat{y}$	Annual Growth Rate of Population, $\hat{L}$	Annual Growth Rate of Productivity, $\hat{A}$
500–1500	0.0%	0.1%	0.033%
1500–1700	0.1%	0.2%	0.166% i

Observe

- $y_0 \approx y(1800) \approx 250\$ \approx y|$ poorest countries today.
- Economic growth *must* be recent phenomenon:

$$250 \cdot \mathrm{e}^{0.025 \cdot 205} = 42044\$ \quad (\approx \text{ the USA today}).$$

- We call $\bar{y}$ subsistence income.

Formally with our simplest approach:

$$\dot{A} = A \cdot B \cdot L \quad \Rightarrow \quad \hat{A} = B \cdot L \tag{3}$$

- B is the probability that you or I invent the wheel / the clock / the compass.
- (apply law of large numbers:) B is average research productivity.

Note the difference to our R&D model:

- Innovation is costless (the same L's are also engaged in production of goods).
- Notion of a society / economy: set of people who
  - are in contact which each other
  - have institutions to access knowledge created elsewhere
- Germany $\leftrightarrow$ Europe $\leftrightarrow$ Asia $\not\leftrightarrow$ America.

Log-diff. (2)

$$n = \frac{1}{1-\alpha}\hat{A} \tag{4}$$

and insert (3)...

$$n = \frac{1}{1-\alpha} B \cdot L \tag{5}$$

Result: population growth is proportional to its level.

A 1st Generalization

Considering general effects of

- standing on shoulders
- stepping on toes

→ (3) is replaced by

$$\dot{A} = A^{\phi} B L^{\lambda} \tag{6}$$

i.e.

$$\hat{A} = A^{\phi-1} B L^{\lambda} \tag{7}$$

And thus along a balanced growth path (steady-state) where $\hat{A}$ is constant

$$\hat{A} = \frac{\lambda}{1-\phi} n$$

Compare with semi-endogenous growth theory (last lecture)

Now insert (7) into (4):

$$n = \frac{1}{1-\alpha} A^{\phi-1} B L^{\lambda} \tag{8}$$

and substitute A from (2)

$$n = \frac{1}{1-\alpha} B L^{\lambda} \left(\bar{y} L^{1-\alpha}\right)^{\phi-1} \quad \Rightarrow \quad n = \frac{1}{1-\alpha} B L^{\lambda-(1-\phi)(1-\alpha)} \bar{y}^{\phi-1} \tag{9}$$

Example

- $\lambda = 1$ (normal case)
- $\phi = 0$ (extreme case: no knowledge spillovers)
- $\alpha = 2/3$ (labor share in production, normal case)

i.e.

$$n = \frac{1}{1-\alpha} B L^{2/3}$$

$\Rightarrow$ n is "almost" proportional to L

Henceforth we assume: $\lambda - (1-\phi)(1-\alpha) > 0$ (not very restrictive)

A 2nd Generalization

- so far: infinite speed of adjustment towards subsistence.
- now: finite adjustment speed.

$$n = n(y) \qquad \text{with } n(\bar{y}) = 0$$
$$n'(\bar{y}) > 0$$

How does n adjust to income ?

- fertility
- mortality

not modelled here $\rightarrow$ Part I of the course.

Instead we postulate a $n(y)$ function consistent with demographic history (of fully developed countries):

[Insert Figure 2: Population Growth vs. Income]

As before:

$$y = AL^{\alpha-1} \quad \Rightarrow \quad \hat{y} = \hat{A} - (1-\alpha)n(y) \tag{11}$$

Observe: with positive $\hat{A}$ there exists no equilibrium $\bar{y}$

- Suppose an equilibrium $\bar{y}$ with $n(\bar{y}) = 0$ (constant population) exists.
- Then $\hat{y}(\bar{y}) = 0$ (def. of an equilibrium).
- Thus, $n = \frac{1}{1-\alpha}\hat{A} > 0 \Rightarrow$ a contradiction.

Intuition: $n' = |\infty|$ (in the simple model) has been replaced by finite n'.

There can't be a steady-state because more people invent more.

Insert (7) in (11):

$$\hat{y} = BA^{\phi-1}L^{\lambda} - (1-\alpha)n(y) \tag{12}$$

Insert from (1):

$$y = AL^{\alpha-1} \quad \Rightarrow \quad A = yL^{1-\alpha}$$

$$\hat{y} = By^{\phi-1}L^{\lambda-(1-\phi)(1-\alpha)} - (1-\alpha)n(y) \tag{13}$$

$$n = n(y)$$

$\rightarrow$ 2-dim system of differential equations.

Phase Diagram Analysis

The $\dot{L} = 0$–locus

- from $n(y) = 0 \Rightarrow y = \bar{y}$
- $L \uparrow$ if $y > \bar{y}$
- $L \downarrow$ if $y < \bar{y}$

The $\dot{y} = 0$–locus from (13)

$$0 = By^{\phi-1}L^{\lambda-(1-\phi)(1-\alpha)} - (1-\alpha)n(y) = G(y, L)$$

Thus

$$\frac{\partial y}{\partial L} = -\frac{\partial G/\partial L}{\partial G/\partial y} = \frac{[\lambda - (1-\phi)(1-\alpha)]\,L^{\lambda-(1-\phi)(1-\alpha)-1}}{(1-\phi)By^{\phi-2}L^{\lambda-(1-\phi)(1-\alpha)} + (1-\alpha)n'} > 0$$

The $\dot{y} = 0$–locus

- has positive slope
- goes through $L = 0$, $y = \bar{y}$
- with $\hat{y} > 0$ if $L > L|\hat{y} = 0$ i.e. $y \uparrow$ to the right of $\dot{y} = 0$
- and $\hat{y} < 0$ if $L < L|\hat{y} = 0$ i.e. $y \downarrow$ to the left of $\dot{y} = 0$

Together:

[Insert Figure: Phase Diagram]

Note that it is impossible for any trajectory to cross $L = 0$ Slope of trajectories:

$$\frac{dy}{dL} = \frac{\frac{dy}{dt}}{\frac{dL}{dt}} = \frac{\dot{y}}{\dot{L}} = \frac{\hat{y} \cdot y}{n \cdot L}$$

which goes to ∞ for $L \to 0$.

Conclusion

- y is permanently growing
- n follows the path of demographic transition
- Eventually $\frac{\partial n}{\partial y} = 0$

$\Rightarrow \hat{y}$?

If a stable balanced growth path exists (not proven by Kremer)

$$g_{\hat{A}} = 0 \quad \Rightarrow \quad \hat{A} = \frac{\lambda}{1-\phi} n$$

and thus

$$\hat{y} = \underbrace{\left[\frac{\lambda}{1-\phi} - (1-\alpha)\right]}_{>0 \text{ since } \lambda-(1-\phi)(1-\alpha)>0} n$$

$\Rightarrow$ Long-run positive growth if $n > 0$ (the Jones-Result)

Empirical Results

1. Kremer estimates

$$n = \beta_0 + \beta_1 L$$

using historical data starting with the homo erectus one million years ago (from archeological and anthropological evidence) and finds support for his theory that suggests

- $\beta_1 > 0$ ($\beta_1 = \frac{B}{1-\alpha}$ in the simple model)
- β_0 insignificant

He estimates the general model

$$n = \beta_0 + \beta_1 L^{\beta_2} \quad \beta_2 \equiv \lambda - (1-\phi)(1-\alpha)$$

and finds estimates in the range $[0.9, 1.4]$

Thus, the simple model is a good approximation.

2. Evidence from technologically separate regions

Idea

- Land size is given
- Assumption: initial population L_0 is proportional to land size (same L/T everywhere)
- According to the theory $L_0 \uparrow \rightarrow \hat{A} \uparrow \rightarrow L \uparrow \rightarrow \hat{A} \uparrow$ etc
- Thus, larger land areas
  - create more tech. progress $\rightarrow$ their pop. grows at higher rate
  - end up with higher population density

Order 5 areas of the world by size

- Old World
- Americas
- Australia
- Tasmania
- Flinders Island

separated since end of last ice age (10.000 BC) until C. Columbus (1490 AC)

The chance that the order by population density is the same is 1/120 (or 1/24 without Flinders Island)

TABLE VII
POPULATION AND POPULATION DENSITY, C. 1500

	Land area (million km^2)	Population c. 1500 (millions)	Population/(km^2)
Old World[a]	83.98	407	4.85
Americas[b]	38.43	14	0.36
Australia[c]	7.69	0.2	0.026
Tasmania	0.068	0.0012–0.005	0.018–0.074
Flinders Island	0.0068	0.0	0.0

See also: Jared Diamond: Guns, Germs, and Steel.

Conclusions / Open Questions:

- The Kremer model explains the demo-economic history of the world "until recently".
  - but the demographic transition was not explained ($\rightarrow$ 1st course).
- But what about today's LDCs? Obviously, high population growth does not push technological progress in SSA.
- The model fails also to explain growth in today's fully developed countries
  - innovations a no longer a by-product of our existence.
  - R&D is a big business / a market activity.

  More on the last issue next.