5. A Model of R&D-driven Growth

Reference: Jones, Chapter 5

- A complete model of endogenous growth clarifying the points
  - how R&D incentives operate
  - how specialization leads to i.r.s.
  - how market power is integrated into general equilibrium.
- This will be almost Paul Romer's (1990) famous model.
- Non-crucial simplification: a constant savings rate.

Consider an economy consisting of 3 sectors:

1. final products
2. specialized inputs (capital goods)
3. R&D

- R&D firms develop blueprints for new input goods.
- for each input good they sell a patent (right to produce) to a single firm.
- input goods producers rent their goods to final good producers.
- production of final goods is sold to consumers.

Recall: Knowledge spillovers:

- nobody besides the patent holder is allowed to use a blueprint to produce a certain good.
- However, everybody may use a blueprint to invent *new* goods.
- knowledge is a non-rivalrous, partially excludable good.

The final goods sector
produces with c.r.s. (i.e. competitively)

$$Y = L_Y^{1-\alpha} \sum_{i=1}^{A} x_i^{\alpha} \tag{1}$$

L_Y: labor (human capital) employed in manufacturing.
x_i: differentiated intermediate inputs, currently there exist A different ones.

Profits are:

$$L_Y^{1-\alpha} \sum_{i=1}^{A} x_i^{\alpha} - wL_Y - \sum_{i=1}^{A} p_i x_i$$

w_L: wage
p_i: price of intermediate good i.

The FOC's for maximum profits are :

$$(1-\alpha)\frac{Y}{L_Y} - w = 0 \tag{2}$$

$$\alpha L_Y^{1-\alpha} x_i^{\alpha-1} - p_i = 0 \quad \text{for all } i. \tag{3}$$

$\rightarrow$ indirect demand functions for workers $L_Y(w, Y)$ and intermediate goods $x_i(p_i, Y)$.

Intermediate Goods Sector.

Simple technology:

- a firm who has the right to produce good i can transform x_i units of raw capital into x_i units of specialized capital
- a unit of raw capital ($=$ forgone consumption) costs r, i.e. marginal costs are $1 \cdot r$.

The producers face demand functions $p_i(x_i)$ according to (3) for their goods. Profits:

$$\pi_i = p_i(x_i) \cdot x_i - r x_i \tag{4}$$

FOC:

$$p_i' x_i + p_i - r = 0 \qquad \Rightarrow \qquad \underbrace{p_i' \frac{x_i}{p_i}}_{\text{price elasticity of demand}} + 1 = \frac{r}{p_i} \tag{5}$$

From (3) $p_i' x_i / p_i = \alpha - 1$, i.e.

$$\alpha = \frac{r}{p_i} \qquad \Rightarrow \qquad p_i = p = \frac{1}{\alpha} \cdot r \tag{6}$$

for all i.

- all capital goods sell at the same price r/α.
- all capital goods are demanded in equal quantities $x_i = x$
- we can drop the good index i henceforth.
- Observe: prices for intermediate goods (r/α) are above marginal costs (r).

With $x_i = x$ for all i aggregate output can be rewritten as

$$Y = L_Y^{1-\alpha} \cdot A \cdot x^{\alpha}, \tag{7}$$

i.e. c.r.s in physical factors L, x; but i.r.s when the number of available products, A, is taken into account.

The aggregate capital stock (forgone consumption) is

$$K = \sum_{i=1}^{A} x = Ax \qquad \Rightarrow \qquad x = \frac{K}{A} \tag{8}$$

Insert (8) into (7):

$$Y = L_Y^{1-\alpha} \cdot A \cdot K^\alpha A^{-\alpha} \qquad \Rightarrow \qquad Y = K^\alpha (A L_Y)^{1-\alpha}. \tag{9}$$

Observe:

- Aggregate production has the same structure as in the (augmented) Solow model.
- Knowledge (A) increases productivity of labor.
- Yet, A is explained endogenously (a view into the black box).

The R&D Sector.

Production of ideas (as in last lecture):

$$dA = \bar{\delta} L_A dt, \quad \bar{\delta} = A^\phi / L_A^{1-\lambda} \tag{10}$$

L_A engineers employed over a time interval of dt create new knowledge (ideas) dA.

Profits in R&D sector:

$$\underbrace{P_A}_{\text{price of a blueprint}} \bar{\delta} L_A dt - \underbrace{w L_A dt}_{\text{labor costs}} \tag{11}$$

Free entry into R&D => positive profits cannot be an equilibrium solution. => zero profits in an equilibrium with R&D. Hence:

$$P_A\bar{\delta} = w \tag{12}$$

Consider the decision to buy a blueprint (a patent of infinite duration) and establish a new firm. In an equilibrium with R&D a no-arbitrage rule applies:

$$P_A(t) = \int_t^{\infty} \pi e^{-\int_t^v r(s)ds} dv \tag{13}$$

i.e. price of blueprint = present value of all future earnings

Diff. w.r.t. time [use Leibnitz rule]:

$$\dot{P}_A = -\pi + rP_A \tag{14}$$

i.e. current profits + possible increase (or loss) of market value of patent $(\pi + \dot{P}_A)$ = return if an amount P_A would have been invested alternatively on the market for consumption loans (rP_A).

Market clearing condition:

$$L = L_Y + L_A \tag{15}$$

(1) - (15): complete model. Solution:

We focus on steady-state:

- $\hat{A}$ and $\hat{y}$ grow at equal rates
- constant labor allocations, L_A/L, L_Y/L.

First Step.
Consider labor demand L_Y from (2):

$$L_Y = (1-\alpha)\frac{Y}{w}$$

Insert free-entry-condition (12):

$$L_Y = (1-\alpha)\frac{Y}{P_A\bar{\delta}}. \tag{16}$$

Second Step.
Insert prices (6) into profits (4):

$$\pi = px - rx = px(1-\alpha).$$

Insert p from (3):

$$p = \alpha L_Y^{1-\alpha} x^{\alpha-1} = \alpha \frac{Y}{A}\frac{1}{x} \qquad \Rightarrow \qquad px = \alpha \frac{Y}{A}.$$

Thus

$$\pi = (1-\alpha)\alpha \frac{Y}{A} \tag{17}$$

Conclude: profits grow at rate:

$$\hat{\pi} = \hat{Y} - \hat{A} = \hat{y} + \hat{L} - \hat{A} = \hat{L} \equiv n.$$

3rd Step.

Conclude from no-arbitrage

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A} \tag{18}$$

at the steady-state:

- r is constant
- $(\dot{P}_A/P_A)$ is constant
- thus, π and P_A grow at equal rates, i.e. at rate n.

i.e.

$$r = \frac{\pi}{P_A} + n$$

Insert profits from (17):

$$r - n = (1-\alpha)\alpha \frac{Y}{A}\frac{1}{P_A}$$

And insert this into L_Y

$$L_Y = \frac{r-n}{\alpha\bar{\delta}} \cdot A. \tag{19}$$

Final Step.

$$\hat{A} = \bar{\delta}\frac{L_A}{A} \qquad \Rightarrow \qquad \frac{\hat{A}}{L_A} = \frac{\bar{\delta}}{A} \tag{20}$$

Insert this into (19):

$$L_Y = \frac{r-n}{\alpha}\frac{L_A}{\hat{A}} \qquad \Rightarrow \qquad \frac{L_A}{L_Y} = \frac{\alpha \hat{A}}{r-n}$$

and expressed in market shares:

$$\frac{L_A/L}{L_Y/L} = \frac{s_R}{1-s_R} = \frac{\alpha \hat{A}}{r-n} \qquad \Rightarrow \qquad s_R = \frac{1}{1+\frac{r-n}{\alpha \hat{A}}} \tag{21}$$

Conclude:

- The faster the economy grows (the higher $\hat{A}$) the higher the fraction of workers in R&D.
- The higher the interest rate r (i.e. the lower the capital stock) the lower the fraction of workers in R&D.
- Why? The higher r the higher are the opportunity costs of R&D i.e. the lower is the present value of profits.

Yet r is itself endogenous...

From (3):

$$p = \alpha \frac{Y}{xA}$$

and from (8) $K = Ax$. Hence:

$$\frac{Y}{K} = \frac{p}{\alpha}$$

insert from (6) $p = r/\alpha$:

$$\frac{Y}{K} = \frac{r}{\alpha^2} \qquad \Rightarrow \qquad r = \alpha^2 \frac{Y}{K}$$

Recall from the Solow model: $r = \alpha Y/K$. Conclude:

- Physical capital is identified as the factor earning *less* than its marginal product.
- The differential income is used to compensate R&D costs.

From capital accumulation (with depreciation rate d):

$$\dot{K} = sY - dK \qquad \Rightarrow \qquad \hat{K} = s\frac{Y}{K} - d$$

and at a steady state with r constant: $\hat{K} = \hat{Y} = \hat{y} + n = \hat{A} + n$. Thus...

$$\hat{A} + n + d = s\frac{Y}{K} \qquad \Rightarrow \qquad r = \frac{\alpha^2(\hat{A} + n + d)}{s}.$$

and

$$s_R = \frac{1}{1 + \frac{\alpha^2(\hat{A}+n+d)-ns}{s\alpha\hat{A}}} = \frac{s\alpha\hat{A}}{s\alpha\hat{A} + \alpha^2(\hat{A} + n + d) - ns} \tag{22}$$

Finally: case differentiation

1. Endogenous growth:

$$\bar{\delta} = \delta \cdot A \qquad \Rightarrow \qquad \hat{A} = \delta \cdot L_A$$

Thus

$$s_R = \frac{L_A}{L} = \frac{s\alpha\delta L_A}{s\alpha\delta L_A + \alpha^2(\delta L_A + n + d) - ns}$$

Solving for L_A yields the endogenous growth rate:

$$\hat{y} = \hat{A} = \delta L_A \quad \text{with} \quad L_A = \frac{L \cdot s\alpha\delta + ns - \alpha^2(d+n)}{\alpha\delta(\alpha+s)} \tag{23}$$

Observe:

- Growth depends on the savings rate.
- Expect: impact of R&D policy.
- Scale effect: growth depends on population size.

2. Semi-endogenous growth:

$$\bar{\delta} = \delta A^{\phi}/L_A^{1-\lambda} \qquad \Rightarrow \qquad \hat{A} = \delta A^{\phi-1} L_A^{\lambda}$$

At a steady-state where $\hat{A}$ and L_A/L are constant:

$$0 = (\phi - 1)\hat{A} + \lambda n \qquad \Rightarrow \qquad \hat{y} = \hat{A} = \frac{\lambda n}{1-\phi}. \tag{24}$$

Observe:

- Long-run growth depends only on population growth and parameters about externalities.
- It is independent from the share of workers allocated to R&D.
- The R&D share s_R adjusts to given $\hat{A}$.
- Expect: no effect of R&D policy.
- Growth is semi-endogenous
  - fully endogenously explained
  - yet exogenous to / uncontrollable by (standard) fiscal policy.

Conclusion

- At the aggregate the Romer model looks like the Solow model.
- Yet economic growth if fully endogenously explained.
- Economic growth is understood as efficiency gain from increasing division of labor / increasing specialization (Adam Smith, 1776).

- Growth is induced by market incentives (profits in the intermediate goods sector).
- External effects: firms do not consider/internalize the positive impact of their R&D on total knowledge and further developments.
- Market imperfection: too few intermediate goods are supplied because of monopoly profits.

- A benevolent social planner can take these effects into account and can thereby achieve through reasonable policy (subsidies for R&D, subsidies for purchase of intermediate products)
  - higher economic growth in the original model with scale effect.
  - higher levels of income in the semi-endogenous growth model (with exogenously determined long-run growth rate).

- Missing obsolescence: new products (MS Word) complement existing ones (typewriter). The Schumpeterian "Process of creative destruction" is not displayed.
  $\rightarrow$ next lecture.