School Attendance and Child Labor – A Model of Collective Behavior

Appendix B and C

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Appendix B: Child Wage Work

If children are supplying labor on the market it is reasonable to assume that a child's wage is independent from its parent's wage. In order to deviate minimally from the analysis in the main text, let A continue to be a measure of aggregate productivity, let $\epsilon(i)$ be a household-specific index of adult income, $\epsilon(i) \in (0,1)$, $w(i) = \epsilon(i)A$, and let γA be the market wage for a unit of child labor, $0 < \gamma \leq 1$. For a child supplying $\ell_t(i)$ units of labor, the family budget constraint is thus given by $c_t(i) = [\epsilon(i) + \gamma\ell_t(i)]A$. Inserting this information into the utility function (5) from the main text and taking the first order condition with respect to child labor provides (A.1)

$$\ell_t(i) = \max\left\{0,\ (1-\alpha)(1-\tau h_t(i)) - \frac{\alpha}{\gamma}\cdot\epsilon(i)\right\}. \tag{A.1}$$

Child labor is a negative function of parental income. Children of richer families supply less wage work. This means that the three cases distinguished in the main text become dependent on the idiosyncratic variable $\epsilon(i)$:

- case WW: for $\gamma(1-\tau) > \alpha\epsilon(i)/(1-\alpha)$ school attending children continue to work.
- case NW: for $\gamma \geq \alpha\epsilon(i)/(1-\alpha) \geq \gamma(1-\tau)$ school attending children stop working.
- case NN: for $\alpha\epsilon(i)/(1-\alpha) > \gamma$ children are never working regardless of the schooling decision.

Comparing utilities and differentiating between the three cases we conclude that the child family i is attending school if

$$E_j(i)\cdot A^{1-\alpha} \geq \sigma(i)\cdot(\phi - S_t), \quad j \in \{WW, NW, NN\}, \quad \text{where} \tag{A.2}$$

$$E_{WW}(i) = \{[\epsilon(i) + \gamma(1-\tau)]\, a - \gamma\tau\}\,(1-\alpha)^{1-\alpha}\left(\frac{\alpha}{\gamma}\right)^{\alpha}$$

$$E_{NW}(i) = \epsilon(i)^{1-\alpha}(1-\tau)^{\alpha}(1+a) - (1-\alpha)^{1-\alpha}(\epsilon(i)+\gamma)\left(\frac{\alpha}{\gamma}\right)^{\alpha}$$

$$E_{NN}(i) = \epsilon(i)^{1-\alpha}(1-\tau)^{\alpha}(1+a) - 1.$$

(A.2) replaces (8) of the main text.

We assume that the schooling feasibility condition from the main text continues to hold, i.e.

$$a > \alpha\tau/(1-\tau). \tag{A.3}$$

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In order to verify a positive association between the schooling propensity and parental income take the derivative

$$\frac{\partial E_{NW}(i)}{\partial \epsilon(i)} = \epsilon^{-\alpha}(1-\alpha)(1-\tau)^{\alpha}(1+a) - (1-\alpha)^{1-\alpha}\left(\frac{\alpha}{\gamma}\right)^{\alpha}. \tag{A.4}$$

Since the second derivative is negative, it is sufficient to show that $\partial E_{NW}(i)/\partial\epsilon(i) > 0$ for the largest feasible $\epsilon(i)$ in the NW domain which is

$$\epsilon(i) = \gamma(1-\alpha)/\alpha.$$

Substitute $\epsilon(i)$ into (A.4) and conclude that the derivative is positive if

$$(1-\alpha)\gamma^{-\alpha}(1-\alpha)^{-\alpha}\alpha^{\alpha}(1-\tau)^{\alpha}(1+a) - (1-\alpha)^{1-\alpha}\alpha^{\alpha}\gamma^{-\alpha} > 0 \qquad \Rightarrow \qquad (1-\tau)^{\alpha}(1+a) > 1.$$

For this to hold it is sufficient that

$$(1-\tau) + a(1-\tau) > 1 \qquad \Rightarrow \qquad a > \tau/(1-\tau),$$

which is true because of (A.3). Thus $\partial E_{NW}(i)/\partial\epsilon(i) > 0$ for all i. From inspection of (A.2) it is immediately obvious that $\partial E_{WW}(i)/\partial\epsilon(i) > 0$ and $\partial E_{NN}(i)/\partial\epsilon(i) > 0$. Thus it remains to be true that children of richer parents are more likely attending school.

Next, establish schooling feasibility. For $E_{NW}(i) > 0$,

$$\epsilon(i)^{1-\alpha}(1-\tau)^{\alpha}(1+a) > (1-\alpha)^{1-\alpha}(\alpha/\gamma)^{\alpha}(\epsilon(i)+\gamma) > 0. \tag{A.5}$$

Since $\partial E_{NW}(i)/\partial\epsilon(i) > 0$ it is sufficient to show that (A.5) holds for the smallest feasible $\epsilon(i)$ in the NW domain, i.e. for

$$\epsilon(i) = \gamma(1-\tau)(1-\alpha)/\alpha.$$

Inserting $\epsilon(i)$ into (A.5),

$$\gamma^{1-\alpha}(1-\tau)^{1-\alpha}(1-\alpha)^{1-\alpha}\alpha^{\alpha-1}(1-\tau)^{\alpha}(1+a) > (1-\alpha)^{1-\alpha}\alpha^{\alpha}\gamma^{-\alpha}\left[\gamma(1-\tau)\frac{(1-\alpha)}{\alpha} + \gamma\right],$$

the condition simplifies to

$$(1+a) > (1-\alpha) + \frac{\alpha}{1-\tau} \qquad \Rightarrow \qquad a > \frac{\alpha\tau}{1-\tau},$$

which is true because of (A.3). For $E_{WW}(i) > 0$,

$$[\epsilon(i) + \gamma(1-\tau)]\, a - \gamma\tau > 0. \tag{A.6}$$

Since $\partial E_{WW}(i)/\partial\epsilon(i) > 0$ it is sufficient to show that (A.6) holds for smallest feasible $\epsilon(i)$ in the WW domain, i.e. for $\epsilon(i) = 0$. Inserting this information, the condition becomes

$$\gamma(1-\tau)a > \gamma\tau \qquad \Rightarrow \qquad a > \tau/(1-\tau),$$

which is true because of (A.3). Thus $E_{WW}(i) > 0$ for all i. Finally note that (A.3) implies $(1-\tau)^{\alpha}(1+a) > 1$ which verifies that $E_{NN}(i) > 0$ for all i. Summarizing, all school propensities are positive and increasing in income.

From inspection of (A.2) it is obvious that $\partial E_j(i)/\partial a > 0$ and $\partial E_j(i)/\partial\tau < 0$ for all j and i, verifying that Lemma 1 continues to hold. Furthermore we have

$$\frac{\partial E_{NW}(i)}{\partial\gamma} = -(1-\alpha)^{1-\alpha}\alpha^{\alpha}\left[-\gamma^{-\alpha-1}(\epsilon(i)+\gamma) + \gamma^{-\alpha}\right] = (1-\alpha)^{1-\alpha}\frac{\alpha^{\alpha}}{\gamma^{1+\alpha}}\left[\epsilon(i)+\gamma-\gamma\right] > 0.$$

Finally, observe that the sign of $\partial E_{WW}(i)/\partial\gamma$ equals the sign of

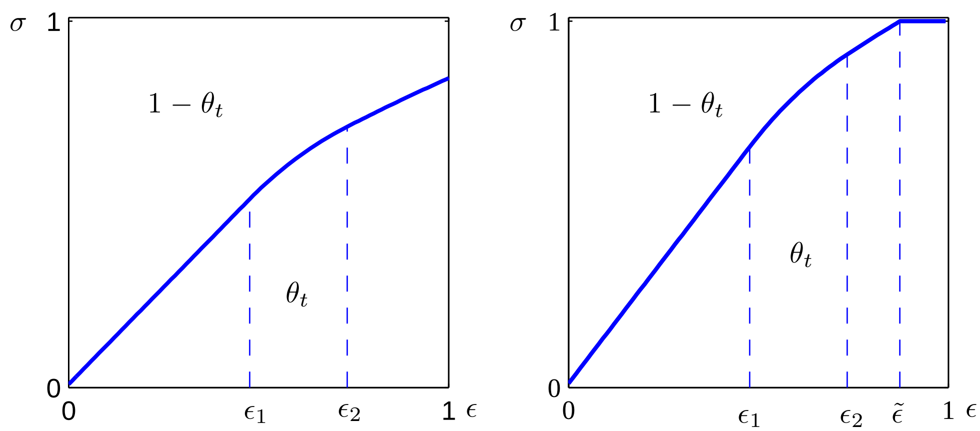
$$(1-\tau)a\gamma - \tau\gamma - \alpha\epsilon(i)a - \alpha(1-\tau)a\gamma + \alpha\gamma\tau = \left[(1-\tau)(1-\alpha)a - \tau(1-\alpha)\right]\gamma - \alpha\epsilon(i)a.$$

It is negative for

$$\gamma < \bar{\gamma} \equiv \frac{\alpha a\epsilon(i)}{(1-\tau)(1+a)-1}, \tag{A.7}$$

and positive for $\gamma > \bar{\gamma}$. Thus Lemma 2 holds in the modified context as well.

Figure A.1: School Attendance Threshold for Market Wage Work



In the income domain $\epsilon < \epsilon_1$ all children are working, in the income domain $\epsilon_1 \leq \epsilon \leq \epsilon_2$ children are either working or going to school, in the income domain $\epsilon > \epsilon_2$ children are never working. As S_t rises the threshold bends upward and an income domain $\epsilon > \tilde{\epsilon}$ emerges where all children are attending school. (right hand panel). As S_t continues to rise $\tilde{\epsilon}$ falls below ϵ_2 and eventually below ϵ_1.

The threshold for school attendance is given by

$$\sigma(\epsilon) = \frac{A^{1-\alpha}E_j}{\phi - S_t}\cdot\epsilon^{1-\alpha}, \quad \text{with } E_j = \begin{cases} E_{WW} \text{ for } \epsilon < \epsilon_1 \equiv (1-\tau)\gamma(1-\alpha)/\alpha \\ E_{NW} \text{ for } \epsilon_1 < \epsilon < \epsilon_2 \equiv \gamma(1-\alpha)/\alpha \\ E_{NN} \text{ for } \epsilon > \epsilon_2. \end{cases} \tag{A.8}$$

With the results proven above it remains to be true that the threshold is a positive, increasing function in the $\epsilon - \sigma$ space. However, at any point in time there may now be different modes of child activity observable at the same time. The most complex case is shown in Figure A.1 where we observe simultaneously the WW, NW, and NN case, i.e. children going to school and working (for $\epsilon < \epsilon_1$ and σ below the threshold), children going to school and not working, children not going to school and working, and children neither going to school nor working (for $\epsilon > \epsilon_2$ and σ above the threshold).

Since it remains to be true that $\partial\sigma(\epsilon)/\partial S_t > 0$, the threshold rotates upwards with rising S_t, establishing an increasing function $\theta(S_t)$. As the threshold rotates upwards the critical $\tilde{\epsilon}$ (above which $\sigma(\epsilon) = 1$ and all families send their children to school) moves to the left from the NN domain into the NW domain, and finally into the WW domain. As shown in the main text $S(\theta)$ is concave above $\tilde{\epsilon}$ and convex below. With contrast to the main text, $\tilde{\epsilon}$ can no longer be obtained analytically everywhere implying that the $S(\theta)$ function can no longer be obtained analytically. However, since the $S(\theta)$ function remains to be positive, increasing, and convex-concave, and since it has been shown that Lemma 1 and 2 continue to hold, the modified model supports qualitatively the same comparative statics of long-run social equilibria as established in the main text.

Appendix C: Education – Productivity Interaction

If we conceptualize the model period no longer as a day but as a generation, it is reasonable to assume that the number of educated children this period affects next period's adult productivity. The simplest possible association is a linear one as in (A.9) where $\bar{A}$ is a constant productivity factor. With A_t being endogenous the share of educated children becomes a bivariate function $\theta_t = \theta(S_t, A_t)$ and the law of motion for community approval modifies to (A.10).

$$A_{t+1} = \bar{A}\theta_t \tag{A.9}$$

$$S_{t+1} = (1-\delta)\theta(S_t, A_t) + \delta S_t, \qquad \text{with } \theta(S_t, A_t) = \frac{E_j A_t^{1-\alpha}}{(2-\alpha)(\phi - S_t)}, \quad j \in \{WW, NW, NN\}\,. \tag{A.10}$$

The schooling propensities E_j are as defined in the main text and we continue to assume that schooling is feasible.

The long-run equilibrium is where $S_t = \theta_t$, as before. Insert this information and (A.9) into (A.10) to see that the long-run equilibrium θ fulfils

$$\frac{E_j \bar{A}^{1-\alpha}}{2-\alpha} = \theta^{\alpha}(\phi - \theta) \equiv f(\theta). \tag{A.11}$$

Inspect (A.11) to see that $f(0) = 0$, $f(1) = \phi - 1 < 0$, $f'(0) = \infty$, and $f''(\theta) < 0$. Thus f emerges from the origin with infinite slope and has the hump-shaped curvature shown in Figure A.2. Since schooling is feasible there are either no or two intersections with the constant $E_j\bar{A}^{1-\alpha}/(2-\alpha)$, identifying the equilibria θ_{low} and θ_{mid} (for simplicity we ignore the degenerate case where f is tangent to $E_j\bar{A}^{1-\alpha}/(2-\alpha)$).

Solve $f'(\theta) = 0$ to see that the maximum of f is where

$$\theta = \bar{\theta} \equiv \frac{\alpha\phi}{1+\alpha}, \qquad f(\bar{\theta}) = \alpha^{\alpha}\left(\frac{\phi}{1+\alpha}\right)^{1+\alpha}. \tag{A.12}$$

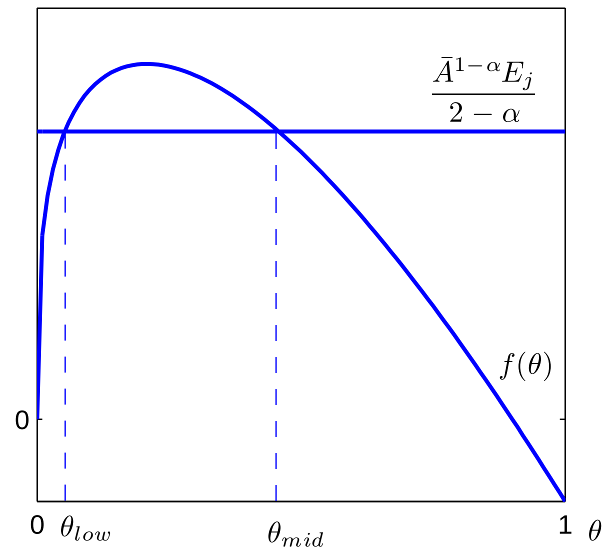
The domain of parameter values for $\bar{A}$ and E_j supporting an equilibrium is thus larger if ϕ is high. Since Lemma 1-3 from the main text remains to hold and since the constant gets larger (the $E_j\bar{A}^{1-\alpha}/(2-\alpha)$ line moves upwards) as a gets larger or as τ gets smaller and with a non-monotonous reaction on γ, we observe the same conditions for existence and the same comparative statics for θ_{low} as for the simple model of the main text. In general, however, there exists no analytical solution.

To assess stability of the equilibria take the partial derivatives of (A.9) and (A.10).

$$\frac{\partial A_{t+1}}{\partial A_t} = 0$$

$$\frac{\partial A_{t+1}}{\partial S_t} = \bar{A}\frac{\partial \theta_t}{\partial S_t} = \bar{A}\frac{E_j}{2-\alpha}\frac{A_t^{1-\alpha}}{(\phi - S_t)^2}$$

Figure A.2: Equilibria of the Two-Dimensional System



Numerical example for $\alpha = 0.3$, $\phi = 0.9$ and $\bar{A}^{1-\alpha}E_j/(2-\alpha) = 0.35$.

$$\frac{\partial S_{t+1}}{\partial A_t} = (1-\alpha)\frac{E_j}{2-\alpha}\frac{A_t^{-\alpha}}{\phi - S_t}$$
$$\frac{\partial S_{t+1}}{\partial S_t} = (1-\delta)\frac{E_j}{2-\alpha}\frac{A_t^{1-\alpha}}{(\phi - S_t)^2} + \delta.$$

At the steady-state where $\theta_t = \theta$ for all t we have $S_t = \theta$, $A_t = \bar{A}\theta$ and $\theta^{\alpha}(\phi - S_t) = \bar{A}^{1-\alpha}E_j/(2-\alpha)$. Inserting this information into the above derivatives provides the elements of the Jacobian evaluated at the steady-state

$$J_{11} = 0$$
$$J_{12} = \frac{\bar{A}^{\alpha}\theta^{1+\alpha}(2-\alpha)}{E_j}$$
$$J_{21} = (1-\alpha)/\bar{A}$$
$$J_{22} = (1-\delta)\frac{\bar{A}^{\alpha-1}\theta^{1+\alpha}(2-\alpha)}{E_j} + \delta.$$

For stability (1) $|\det J| < 1$ and (2) $|\text{trace } J| < 1 + \det J$. For (1) to hold, $\theta < \phi/(2-\alpha)$ For (2) to hold

$$(1-\delta)\frac{\theta}{\phi-\theta} + \delta < 1 - (1-\alpha)\frac{\theta}{\phi-\theta} \qquad \Rightarrow \qquad \theta < \frac{\phi}{2 + \frac{1-\alpha}{1-\delta}}$$

and thus necessarily $\theta < \phi/(3-\alpha) < \phi/2 < 1/2$. Thus, the equilibrium at $\theta_{low} < 1/2$ is locally stable.

The equilibrium at θ_{mid} is unstable if $\theta_{mid} \geq \phi/(2-\alpha)$. Since $\theta_{mid} > \bar{\theta}$, a sufficient condition for θ_{mid} being unstable is that $\bar{\theta} > \phi/(2-\alpha)$, i.e.

$$\frac{\phi}{2-\alpha} > \frac{\alpha\phi}{1+\alpha} \qquad \Rightarrow \qquad \alpha^2 - \alpha + 1 > 0 \qquad \Rightarrow \qquad \alpha^2 - 2\alpha + 1 = (1-\alpha)^2 > -\alpha,$$

which is always true. The following Proposition summarizes the results.

PROPOSITION 1. *For the intergenerational model of school attendance and child labor, augmented with endogenous productivity, there exists at most one locally stable equilibrium θ_{low} which displays, qualitatively, the same comparative statics as the simple model of the main text. If θ_{low} does not exist or if the community is outside its domain of attraction, then the community converges to $\theta = \theta_{high} = 1$.*

Thus all characteristics of the simple model are present for the augmented model as well. The two model-economies, however, are not observationally equivalent since the augmented model provides additionally income dynamics. Inspect (A.9) to verify the following result.

PROPOSITION 2. *As the economy converges towards θ_{high} aggregate productivity and average income per capita rises.*

The positive income effect on schooling speeds up the convergence process such that the augmented economy, ceteris paribus, converges faster towards θ_{high} than the economy of the main text.