Contraception and Development: A Unified Growth Theory

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Abstract. This study investigates the interaction of the use of modern contraceptives, fertility, education, and long-run growth. It develops an economic model that takes into account that sexual intercourse is utility enhancing and that birth control by modern contraceptives is more efficient but more costly than traditional methods. The study shows how a traditional economy, in which modern contraceptives are not used and fertility is high, gradually converges towards a high growth regime, in which modern contraceptives are used. Lower prices or higher efficacy of contraceptives are conducive to an earlier onset of the fertility transition and a quicker take-off to modern growth. An extension of the model embeds the theory in the canonical unified growth model and assesses quantitatively how much the use of contraception has contributed to the fertility decline and the rise of education.

Keywords: fertility, sex, contraceptive use, education, economic development.

JEL: 040; I25; J10; N30.

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1. Introduction

There exists a vast literature on child demand and demographic-economic development built upon the idea that households spend part of their income to have children. The idea that households spend income in order to not have children, however, has received relatively little attention in the literature. This paper acknowledges that sexual intercourse (hereafter referred to as sex) is a utility enhancing activity and develops a theory of demand for children as well as for contraceptives. The use of modern contraceptives allows households to experience utility from sex without a proportional increase in child births. Consequently, family size becomes a function of the price and efficacy of contraceptives. A smaller family size requires less time dedicated to child rearing by the parents, and part of the released time is spent on children's education. Through this channel, the cost and efficacy of contraceptives have a significant impact on the onset of the fertility transition and on the speed of convergence towards a steady state of high economic growth.

This study contributes to the literature on unified growth theory by investigating the take-off from quasi-stagnation to modern growth and by emphasizing the importance of the fertility transition and the child quality-quantity trade-off for successful long-run development.[1] It extends the literature by exploring the role of sexual desire (besides the desire for fertility) and the impact of price and efficacy of contraceptives for the onset and speed of the fertility transition. The existence of contraceptives creates multiple equilibria of which the equilibrium of high growth is potentially latent. When households start using modern contraceptives, a threshold is crossed such that with rising income households gradually substitute child quantity with child quality (education). Crossing this threshold may be caused exogenously (by a sufficiently strong decline of the price for contraceptives) or endogenously (by gradually rising household income and education). The steady-state growth rates at the low-growth equilibrium and at the high-growth equilibrium are independent from the price and efficacy of contraception. Contraception only temporarily impacts on growth by initiating and accelerating the fertility transition. The fertility transition is accompanied by a rise of sexual intercourse (in marriage), i.e. sex increases while fertility declines.

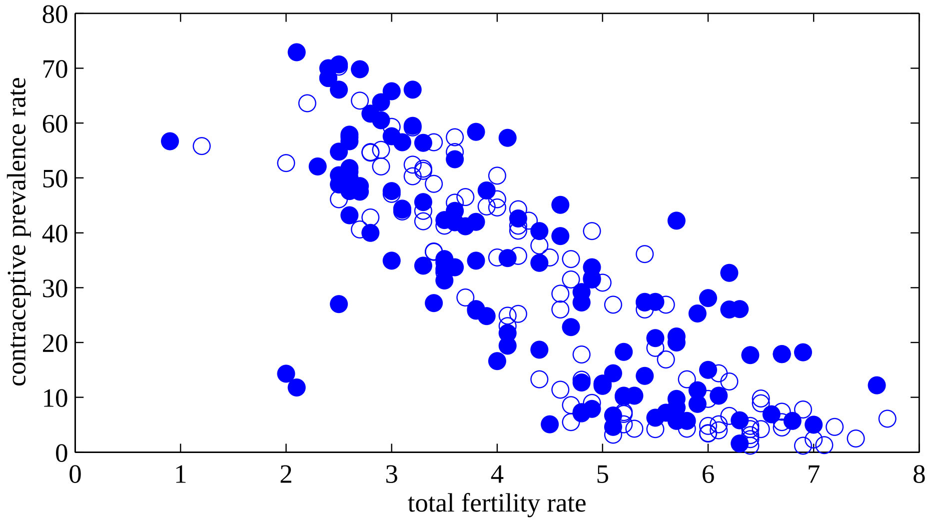
[1] For unified growth see Galor and Weil (2000), Kögel and Prskawetz (2001), Galor and Moav (2002, 2006), Doepke (2004), Strulik et al. (2013), and many others. See Galor (2011) for a survey.

The happiness literature has produced compelling evidence that sex is utility enhancing (Blanchflower and Oswald, 2004). This is true for both men and women. According to a study by Kahneman et al. (2004), sex is the activity that provides the single largest amount of happiness for a sample of women in the U.S. At the same time, there is surprisingly little support for the conventional Beckerian view that utility increases with family size (Margolis and Myrskylae, 2011; Deaton and Stone, 2014). These observations may appear puzzling from a conventional economic viewpoint but they are readily explained by evolutionary psychology. By Darwinian selection, the human brain evolved to experience joy from sex long before it was able to understand reproductive biology. During most of evolution humans had no clear notion of how sexual intercourse was related to fertility, which explains the desire for sex without the desire for (more) children and the use of contraceptives (Wright, 1994). According to the biologists' take on the demographic transition, summarized in Potts (1997), people always wanted to control family size but had no methods to achieve this goal efficiently for most of human history. The fertility transition begins after the discovery of effective methods of contraception. A negative relationship between income and fertility and between education and fertility then arises because the wealthy and well educated are more successful than the poor in obtaining costly contraception.

The two perhaps greatest innovations of contraceptive technology, the rubber condom (1855) and the contraceptive pill (1960), happened suspiciously close to the onset of the first demographic transition and the second demographic transition in many Western countries. Some economic historians are nevertheless reluctant to make a causal connection with respect to the first fertility transition because the price of condoms was still relatively high at the end of the 19th century (Guinnane, 2011). As shown below, this view is hard to support in a heterogenous society. Allowing for heterogeneity, there will always be some couples rich enough to find contraceptives attractive even at a high price. These pioneers of the fertility transition will have less children and initiate the demographic transition and the take-off to growth. With growing income levels, contraceptive use will be adopted by poorer individuals as well, which in turn further quickens the fertility transition and convergence to modern growth.[2]

[2]See Livi-Bacci (1986) and Haines (1989) for evidence that the fertility transition started among the rich. By taking the price of contraption into account, the present paper provides an alternative and possibly complementing mechanism to the more common view that contraception diffused through social learning from the rich to the poor (Seccombe, 1990).

Figure 1: Fertility vs. Contraceptive Prevalence Across Developing Countries 1985 - 2012



Data from available non-European DHS surveys (as of April 2014). White dots: 111 country surveys 1985-1999; blue dots: 127 country surveys 2000-2012. Fertility is the total fertility rate for the three years preceding the survey and the percentage of 15-49 years old women currently pregnant. Contraceptive prevalence is the current use of modern contraception methods among currently married women. Source: ICF (2012).

The argument that prices could be too high for modern contraceptives to play a role for the onset of the fertility transition, is actually supportive of the mechanism suggested in this paper. It shows that prices were decisive for the decision to use modern contraceptives during the historical fertility transition of the West. With respect to contemporaneous developing countries it is a far less contested issue that contraceptive use is a leading proximate cause of the fertility decline (e.g. Bongaarts and Potter, 1983; Westoff and Bankole, 2011; Lule et al., 2007; Darroch and Sing, 2013). Figure 1 shows the strong negative association between the total fertility rate and the prevalence rate of modern contraceptives in developing countries. The data is taken from the available DHS surveys (ICF, 2012). White dots represent data from surveys taken from 1985–1999, and blue dots represent surveys from 2000–2012. The association seems to be almost time-invariant since the 1980s. The fertility transition moves in sync with intensified use of modern contraceptives.

While the positive association of the fertility transition with the rise in education and its impact on the take-off to modern growth is well-established in the literature (e.g. Galor, 2011; Dalgaard and Strulik, 2014), the role of contraceptives for these developments is perhaps less intensively researched. With respect to oral contraceptives, Bailey (2010, 2013) used legal differences across US states (the Comstock laws) and health policy differences across US counties and showed that increased contraception in the 1960s and 1970s had not only a causal impact on fertility but also a positive impact on education and income of subsequent generations. In earlier times, legal as well as cultural constraints may have slowed down the uptake of modern contraceptives but they were not able to choke off the general trend. In the U.S., for example, notwithstanding the Comstock laws, the fifteen largest producers of condoms were producing a million and a half per day in the mid 1930s (D'Emilio and Freedman, 1988, Ch. 11). Now, close to the end of the fertility transition, the "contraceptive transition" is also close to completion. In the US in 2002, for example, 98 percent of all women who ever had intercourse used at least one contraceptive method and about 62 percent of the 15-44 years old women were using contraceptives at the time of the interview. Those not using contraceptives were mostly pregnant, or trying to become pregnant, or were unable to conceive (Mosher et al., 2004).

Economic theory only recently began to take into account that sex is a utility enhancing activity. Greenwood and Guner (2010), Kennes and Knowles (2013), and Fernandez-Villaverde et al. (2014) developed models of contraceptive use, pre-marital sex, and fertility and investigated how preventive technology affected the sexual revolution and the intergenerational transmission of sexual norms. However, the relevance of contraceptive use for the fertility transition and the take-off to modern growth were not investigated in these studies.

Bhattacharya and Chakroborty (2014) discuss the role of contraception for the fertility decline in Victorian England. According to their model, sex is not utility enhancing but (traditional or modern) contraceptives are needed to push fertility below a natural maximum. The use of contraceptives, in turn, is not costly in monetary terms but utility reducing. Economic growth is absent or constant and the use of modern contraceptives is triggered by a decline of child mortality. In contrast to the present study, it is argued that contraception by itself could not have triggered the decline in fertility and the fertility transition is not considered to be conducive to the take-off to modern growth.

The present paper also contributes to the general discussion of the income–fertility nexus. That there exists a strong negative association between income and fertility across countries, within countries over time, and within countries across households is relatively well-known (e.g. Herzer et al., 2013). What is perhaps less well-known is that it is surprisingly difficult to explain the negative association at the household level. A negative income-fertility nexus usually requires non-standard manipulations of the utility function or the introduction of special assumptions like the presence of exogenous non-labor income. As shown below, a negative association between fertility and income emerges "naturally" after acknowledging that sex is a utility enhancing activity and that contraception is costly.[3]

This paper is organized as follows. Section 2 sets up the model, computes the threshold for contraceptive use and its comparative statics, and compares behavior of households with and without use of contraceptives. Section 3 investigates the implied macro-economic performance at the steady states. Section 4 calibrates the model and assesses the quantitative impact of contraceptive use for the transition to modern growth. It extends the representative household model towards a stratified society and shows that it is sufficient that initially only the rich take up modern contraception in order to set the fertility transition in motion. Section 5 embeds the theory into the canonical unified growth model (Galor and Weil, 2000) such that the fertility transition and the take-off to growth are propelled by two mechanisms, contraceptive use and the well-known feedback mechanism between technological progress and education. It will be shown that the main results from the simple model hold in the extended framework as well. A calibrated version of the model is used in order to assess quantitatively how much the two mechanisms contributed to the historical fertility transition. Finally, price and efficacy of contraceptives will be considered time-variant and explained as a function of aggregate technological progress. Section 6 concludes.

2. The Basic Model

Consider an economy populated by a large number of households and competitive firms. At any given time, firms produce output according to the production function $y_t = h_t \ell_t$, in which ℓ_t is employment and h_t is human capital of the workforce. The wage per unit of human capital is thus unity and potential income of households is given by their endowment of human capital.

Households consist of couples who cooperatively maximize utility from consumption, from having surviving children, from future income of their children, and from having sex. We measure all variables in units per parent, such that n_t is the number of births per parent, and c_t is consumption expenditure per parent, etc. In order to derive an analytical solution, the

[3]This is not the first paper generating a negative income fertility nexus without non-standard assumptions, another example is Moav (2005).

household utility function is assumed to be separable and logarithmic. It reads

$$U = \log c_t + \alpha \log(\pi_t n_t) + \gamma \log h_{t+1} + \sigma \log s_t, \tag{1}$$

in which h_{t+1} is the human capital (future income) per child, π_t is the child survival rate, and s_t is the time devoted to sex. Besides its last element, the utility function is standard in unified growth theory. As shown in the Introduction, there is strong support from the happiness literature as well as from evolutionary psychology that sex is utility enhancing. The parameter σ is determined by the desire for sex.

For simplicity, we assume that fertility, without the use of modern contraceptives, is proportional to sexual activity s_t and normalize the factor of proportionality to one. The number n_t may be thought of as already taking into account costless traditional methods of contraception, like postponing marriage, breastfeeding, or withdrawal. For completeness, we note the existence of an upper limit of fertility, given by female reproductive capacity, $\bar{n}$. In the analysis below, however, fertility will be assumed to always lie below its biological maximum, in line with the historical evidence.[4] Using a unit of modern contraceptives prevents the birth of μ children. The parameter μ thus controls the effectiveness of modern contraceptives. Taking the corner solution into account, the number of births is

$$n_t = \min \left\{ s_t - \mu u_t, \, \bar{n} \right\}, \tag{2}$$

in which u_t is the use of modern contraceptives and μu_t are births prevented (additionally to births prevented by use of traditional methods).

Human capital is produced according to the production function

$$h_{t+1} = A e_{t+1} h_t, \tag{3}$$

in which e_{t+1} is the time spent on education per child, i.e. the education received by an adult of generation $t + 1$. Next period's human capital is thus a positive function of education and the human capital endowment of adults. The linearity in education is innocuous and could be avoided without loss of generality. The linearity in human capital is essential for the simple model to create perpetual growth at the long-run steady state but is inessential for the results

[4] See e.g. Wrigley and Schofield (1987) on postponing marriage, see e.g. Cinnirella et al. (2012) on birth control within marriage.

on contraceptive use and their implications for the fertility transition. In Section 5 we abolish the linearity assumption and consider an extended model embedding contraception into the standard unified growth model (Galor and Weil, 2000) with endogenous technological progress. This adds more complexity but preserves the main results from the simple model.

We assume that the probability that a child survives to adulthood π_t is taken as given at the household level and not partly controlled through expenditure on child health and nutrition (as in Strulik, 2008 and Strulik and Weisdorf, 2014). We also treat children as a continuous number in order to avoid problems of precautionary child-bearing (as in Kalemli-Ozcan, 2002, 2003). These simplifying assumptions are justified by the different focus of the present paper. In fact, it will be shown below that the uptake of modern contraceptives operates as a stand-alone mechanism taking child survival as constant. Adding a feedback effect between parental human capital (income) and child survival adds more realism to the quantitative experiments but it is not essentially driving any of the results. As many related studies we assume that child rearing costs as well as education costs are incurred only for surviving children. Child rearing costs ϕ and education investment e_{t+1} are measured in terms of income lost due to the time spent on rearing and education per child.

Households are endowed with one unit of time per adult and face the budget constraint

$$w_t h_t (1 - \phi \pi_t n_t - e_{t+1} \pi_t n_t - \tau s_t) = p u_t + c_t. \tag{4}$$

in which p is the price of modern contraceptives and $w_t = 1$ for the simple model. The price of contraceptives is taken as given by households but it may occasionally change. Here we treat the price and efficacy of contraceptives parametrically. In Section 5 we consider them as time-variant and driven by endogenous technological change. Notice that the budget constraint includes also a cost in terms of time for sex, denoted by $\tau > 0$. This assumption prevents sex from increasing without limit in a growing economy. An arbitrarily small cost in time is sufficient to achieve asymptotically constant sex. Alternatively, we could use a satiation level or a physical upper limit for sex without changing the results.[5] We refer to human capital h_t also as potential income and to the expression on the left-hand side of (4) as actual income, i.e. human capital multiplied by labor supply.

[5]Obviously the time cost of sex does not affect the comparative statics, which hold for all τ, including $\tau = 0$. As shown below, the steady state is also independent from τ. Replacing the time constraint for sex by a satiation level raises somewhat the predicted speed of increasing sexual activity in the 20th century. It leaves all other quantitative results, including the speed of the fertility transition and the take-off to growth, virtually unaffected.

Households maximize (1) subject to (2)–(4), given non-negativity constraints on all variables. Additionally, we assume $\gamma < \alpha$. The assumption ensures that parents prefer to have children even if they could be avoided without cost (i.e. for $p = 0$), see below. This plausible restriction prevents unnecessary case differentiation. The interior solutions for consumption, fertility, education, and contraceptive use, and sex are:

$$c_t = c_t^M \equiv \frac{h_t}{1+\alpha+\sigma} \tag{5}$$

$$n_t = n_t^M \equiv \frac{(\alpha-\gamma)\mu h_t}{(1+\alpha+\sigma)\left[(\phi\pi_t\mu h_t - p\right]} \tag{6}$$

$$e_{t+1} = e_{t+1}^M \equiv \frac{\gamma\left(\phi\pi_t\mu h_t - p\right)}{(\alpha-\gamma)\pi_t\mu h_t} \tag{7}$$

$$u_t = u_t^M \equiv \left(\frac{\sigma}{\mu\tau h_t + p} - \frac{\alpha-\gamma}{\phi\pi_t\mu h_t - p}\right)\frac{h_t}{1+\alpha+\sigma} \tag{8}$$

$$s_t = s_t^M \equiv \frac{\sigma\mu h_t}{(1+\alpha+\sigma)(\tau\mu h_t + p)}. \tag{9}$$

A superscript M identifies the optimal choice of a variable at an equilibrium where modern contraceptives are used.

Inspecting the relevant first-order derivatives of (6)–(9) proves the following proposition.

PROPOSITION 1. *At the interior solution of the households' problem we observe the following comparative statics:*

- *An increase in human capital (income) h_t reduces fertility and increases consumption, education, contraceptive use, and sex.*
- *An increase in child survival π_t leads to lower fertility, lower net fertility $\pi_t n_t$, higher education and more use of contraceptives. It leaves sex unaffected.*
- *A reduction in the price of contraceptives p reduces fertility and increases education, contraceptive use, and sex.*
- *An increase in the desire for sex σ increases contraceptive use and decreases fertility. It leaves education unaffected.*
- *An increase in the efficacy of contraception μ increases contraceptive use, sex, and education and it decreases fertility.*

Most of these comparative statics are expected. The impact of child survival on education is a non-standard result. The standard result, obtained when non-surviving children incur no costs, is that child mortality affects neither fertility nor education (e.g. Galor, 2011, Ch. 4). Here, the standard result would be obtained by setting the price of contraception $p = 0$ in (6) and (7). The standard model implicitly assumes that couples adjust their sexual activity without any (utility-) cost in order to produce the desired number of surviving children. Here, however, couples enjoy sex and do not adjust their sexual behavior (since the price p of having sex did not change). Instead they spend more on contraceptives and u_t increases as a response to improving child survival. This reduces net income (after contraceptive expenditure) and increases implicitly the marginal cost of having a child. Consequently household reduce fertility by more than in the standard model, implying that net fertility $\pi_t n_t$ declines as a response to increasing π_t.[6]

The negative association between income and fertility arises because better educated and richer adults can afford costly contraceptives and prefer a smaller family. A smaller number of children requires less child rearing time, and the saved time is invested in the children's education. The most surprising and certainly "non-Darwinian" result is that couples who like sex a lot, have less offspring, $\partial n/\partial\sigma < 0$. This is the case because sex is expensive when modern contraceptives are used such that couples partly substitute child costs when they increase their sexual activity.

Proposition 2. *Modern contraceptives are not used ($u_t = 0$) if human capital (income) is sufficiently low compared to the price of modern contraceptives that is if*

$$h_t \leq \bar{h} \equiv \frac{p(\alpha - \gamma + \sigma)}{\mu[\sigma\phi\pi_t - (\alpha - \gamma)\tau]}. \tag{10}$$

Ceteris paribus, the threshold $\bar{h}$ is

- *increasing in the price of contraceptives p and the desire for children α*
- *declining in the child survival rate π_t, the efficacy of contraceptives μ, the desire for sex σ, the time cost of child rearing ϕ, and the desire for education γ.*

The proof evaluates (8) for $u_t \leq 0$, i.e. when the non-negativity constraint on contraceptive use is binding. In line with the literature, the proposition confirms both low income (human capital) and high cost of contraceptives (approximating as well difficult access) as causes of the lack of demand for contraceptives (Ainsworth et al., 1996; Gakidou and Vayena, 2007). Yet

[6]There are, however, other channels, not considered here, motivating a *positive* association between child mortality and net fertility like, for example, costs of non-surviving children (Doepke, 2005) and partly endogenous survival by costly nutrition (Strulik and Weisdorf, 2014).

it is also true that non-users of contraceptives have more children than users with the same preferences (see Proposition 3 below). In a questionnaire, non-users may thus state that they do not use contraceptives because they want more children (Pritchett, 1994; World Bank, 2007; Guenther and Harrtgen, 2013). The ultimate reason, however, is not found in preferences, but in constraints. Given higher education or income, or lower costs of contraceptives, the same person would prefer to use contraceptives and desire a smaller family size. The proposition also identifies, like Bhattacharya and Chakraborty (2014), the prevalence of high child mortality as an obstacle to contraceptive use. The reason here is plainly that in a high mortality environment, less contraception is needed to achieve a given family size.[7]

The corner solution without use of modern contraceptives is obtained as:

$$n_t = n_t^T \equiv \frac{\alpha - \gamma + \sigma}{(1 + \alpha + \sigma)(\phi\pi_t + \tau)} \tag{11}$$

$$e_{t+1} = e_{t+1}^T \equiv \frac{\gamma(\phi\pi_t + \tau)}{\pi_t(\alpha - \gamma + \sigma)} \tag{12}$$

$$s_t = s_t^T \equiv \frac{\alpha - \gamma + \sigma}{(1 + \alpha + \sigma)(\phi\pi_t + \tau)}. \tag{13}$$

and $c_t = c_t^T = c_t^M$. A superscript T indicates an equilibrium at which only traditional methods of contraception are used. Notice that parents invest into education at the traditional equilibrium but education does not rise with income. Education at the corner can best be conceptualized as children learning the basic techniques of a trade or of subsistence agriculture. Comparing the traditional and the modern equilibrium, we find:

Proposition 3. *At the traditional equilibrium, fertility is higher and education and sex are lower than at the modern equilibrium.*

The proof begins with noting from Proposition 2 that the modern equilibrium fulfills $p(\alpha - \gamma + \sigma) < [\sigma\phi\pi_t - (\alpha - \gamma)\tau]\,\mu h_t$, that is $(\alpha - \gamma)(\phi\pi_t + \tau)\mu h_t < (\alpha - \gamma + \sigma)(\mu\phi\pi_t h_t - p)$, that is $n_t^M < n_t^T$. The proofs for education and sex are analogous.

Interestingly, people enjoy less sex at the traditional equilibrium. The economic transition from a traditional society to a modern one is also accompanied by "sexual liberation" such that individuals enjoy more sex (in marriage). We assume that $n_t^T < \bar{n}$, which avoids unnecessary

[7]Appendix A shows how these results generalize with respect to extensions of the model by variable elasticity of intertemporal substitution and non-market income.

case differentiation. Inspecting the respective derivatives of (11)–(13) verifies the following proposition.

PROPOSITION 4. *At the traditional equilibrium we observe the following comparative statics:*

- *An increase in human capital (income) h_t leads to more consumption and leaves fertility, education, and sex unaffected.*
- *An increase in child survival π_t leads to lower fertility, higher net fertility $\pi_t n_t$, lower education per child e_{t+1}, and less sex s_t.*
- *An increase in the desire for sex σ increases fertility and reduces education.*

Notice that the model predicts very different comparative statics for the traditional society compared to the modern society. In particular, a stronger desire for sex has different consequences at the traditional and modern equilibria. At the traditional equilibrium, we observe the "Darwinian" result that a greater desire for sex increases fertility. Furthermore, improving child survival reduces education at the traditional equilibrium. To see why, observe from (11) and (12) that that the model would provide the standard result of no effect of π on net fertility $\pi_t n_t$ and education e_{t+1} if there were no time cost of sex. Here, without the use of modern contraceptives, couples adjust to higher child survival by having less sex. This frees parental time and creates an income effect. It reduces the child costs per unit of income ($\tau s_t h_t$ in the budget constraint) such that parents reduce fertility by less than in the standard model, implying higher net fertility and lower education as a response to improving child survival. However, since the time cost of sex is small, we expect this effect to be small as well.

The results on the differentiated impact of child survival at the traditional and modern equilibrium shed new light on the debate as to whether the historical decline in child mortality could have been the causal mechanism for the demographic transition (see e.g. Cleland, 2001; Guinnane, 2011; Kalemli-Ozcan, 2003; Doepke, 2005; Galor, 2005). The solution suggested by the present model is that mortality may have contributed to the fertility decline only *after* the introduction of modern contraceptives. Net fertility and education are independent from child mortality if contraceptives are for free (or income approaches infinity), $\lim_{h_t \to \infty} \pi_t n_t^M = (\alpha - \gamma)/[(1 + \alpha + \sigma)\phi]$. In the limit, the model thus confirms the view that child mortality is irrelevant for (net-) fertility (Galor, 2011, Ch. 4). As long as the price of contraceptives is small compared to income, we expect a small effect of child mortality during the transition. This hypothesis is confirmed by the numerical experiments below. The crucial driver of the fertility transition is the availability and affordability of modern contraceptives. Before the uptake of modern contraceptives, net fertility rises when child survival improves.

Inspection of (7) seems to suggest that another corner solution where the non-negativity constraint on e_{t+1} binds may exist. This, however, is not the case.

Lemma 1. *There exists no other corner solution besides* $u_t = 0$ *and* $n_t = \bar{n}$*.*

For the proof, let $\underline{h}$ denote the potential income threshold below which $e_{t+1} = 0$, i.e. $\underline{h} = p/(\phi\mu)$. Since $0 < (\alpha - \gamma)(\phi\pi_t + \tau)$, we have $\sigma\phi\pi_t - (\alpha - \gamma)\tau < \phi\pi_t(\alpha - \gamma + \sigma)$ and thus, $\underline{h} < \bar{h}$. But for $h < \bar{h}$ the corner solution (11)–(13) holds, at which education is positive. Thus, there is no solution with $e_{t+1} = 0$. This outcome could be easily avoided by assuming that some minimum education is picked up without incurring costs. Here we focus on the decision to use modern contraceptives and thus abstain from introducing another corner solution.[8]

3. Long-Run Economic Development

Inserting (12) into (3) we obtain the gross growth rate of the traditional economy

$$\frac{h_{t+1}}{h_t} = g_t^T \equiv \frac{\gamma(\phi\pi_t + \tau)A}{\pi_t(\alpha - \gamma + \sigma)}. \tag{14}$$

Assume that productivity in education A is large enough such that the modern society is capable of long-run growth and that child survival π_t converges to unity in a perpetually growing economy. For $h_t \to \infty$ we obtain

$$n_\infty^T = \frac{\alpha - \gamma}{(1 + \gamma + \sigma)\phi}, \qquad e_\infty^T = \frac{\gamma\phi}{\alpha - \gamma}, \qquad s_\infty^T = \frac{\sigma}{(1 + \alpha + \sigma)\tau}. \tag{15}$$

Inserting the solution for education in (3) we obtain the steady-state growth rate of the modern economy

$$\frac{h_{t+1}}{h_t} = g^M \equiv \frac{\gamma\phi A}{\alpha - \gamma}. \tag{16}$$

Sufficiently large productivity in education, namely $A > (\alpha - \gamma)/(\gamma\phi)$, ensures that the gross growth rate exceeds unity, i.e. that there exists positive long-run growth. It is a necessary and sufficient condition for the modern economy to converge towards (15) and is assumed to hold henceforth.

[8]A corner solution without education will naturally arise in the context of the canonical unified growth model and will be discussed in Section 5.

Proposition 5. *The modern economy grows at a higher rate than the traditional economy.*

This is verified by comparing (16) and (14). Inspection of the first order derivatives of the growth equations (14) and (16) verifies the following proposition.

Proposition 6.

- *Growth at the traditional and modern equilibrium is increasing in the desire for education γ, the productivity of education A, the time cost of child rearing ϕ; and is declining in the desire for children α.*
- *Both growth rates are independent from the price and efficacy of contraceptives.*
- *Growth of the traditional society g^T is declining in the desire for sex σ and increasing in the time cost of sex τ.*
- *Growth of the modern society g^M is independent from the time cost of sex and the desire for it.*

Interestingly, while a high level of sexual desire does not harm growth of the modern economy, it does affect growth of the traditional economy.

The cost and efficacy of contraceptives are irrelevant for growth at the steady state for a given equilibrium of the economy. But cost and efficacy are decisive for whether an economy is situated at the traditional equilibrium or at the modern equilibrium. If Proposition 2 is fulfilled, the economy is situated at the traditional equilibrium. A sufficiently strong decline of the price of contraceptives or a sufficiently high increase of its efficacy would move the economy onto the modern growth path.

The transition towards the modern economy, however, does not necessarily require an exogenous impulse. In order to make the problem interesting and to build a unified growth theory, we assume in the following that A is large enough such that the traditional economy is growing as well, albeit at a (much) smaller rate than the modern economy. This means that eventually education becomes large enough such that the threshold is crossed and the economy switches to the modern regime. The price and efficacy of contraceptives are decisive for how fast an economy transits from the traditional regime to the modern regime.

4. Transition to Modern Growth

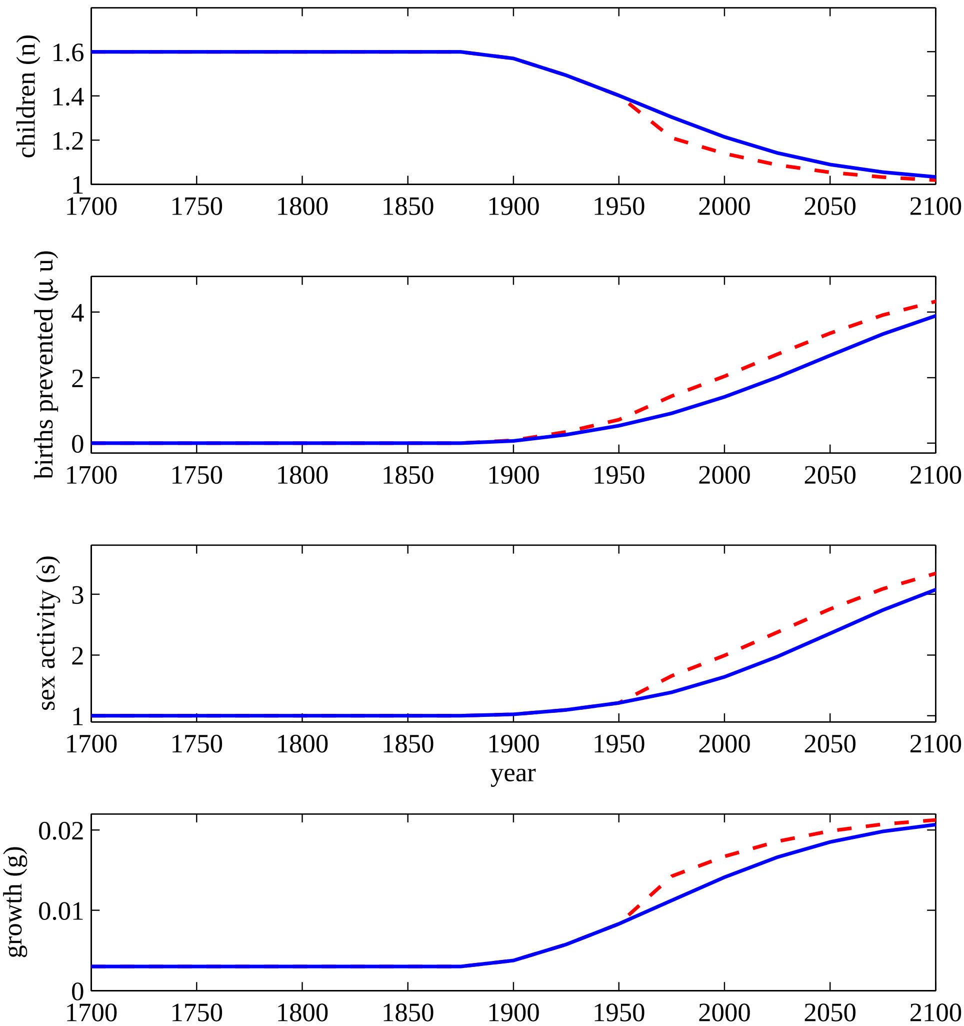
We next explore transitional dynamics with a series of numerical experiments. For this purpose we specify a typical Western European country undergoing a fertility transition. Naturally, this attempt provides only a rough approximation of any particular country, given the simple structure of the model and the lack of extensive historical data on contraceptive use and sexual activity. However the numerical experiments are sufficient for exploring the basic mechanisms and to roughly assess the quantitative importance of contraception. First, I set child rearing costs ϕ to 0.15, according to Haveman and Wolfe (1995) and set α arbitrarily equal to 0.5. I then calibrate the remaining parameters such that the modern economy grows at an annual rate of between 1.5 and 2 percent in the late 20th century (steady state 2.2 percent), the traditional economy grows at a rate of 0.3 percent, fertility per couple approaches unity in the modern economy (convergence towards a stationary population), and modern couples at the end of the fertility transition have about three times as much sex as in pre-modern times.[9] This provides the estimates $\gamma = 0.24$, $\sigma = 0.21$, $A = 12$, and $\tau = 0.02$.

As shown above, price and efficacy of contraceptives do not affect the steady state. I use the data in Table 2 of Greenwood and Guner (2010) to obtain a first estimate of μ. For this purpose, I assume that the traditional method consists of an average of no contraception at all (failure rate 0.85 percent) and withdrawal (failure rate 0.225), providing a failure rate of the traditional method of 0.53. For the effectiveness of condoms, I use an average between rubber condoms (failure rate 0.45) and latex condoms, which became available in the 1920s (failure rate 0.175). This provides a failure rate of 0.31, and an estimate of $\mu = (1 - 0.31)/(1 - 0.53) = 1.46$. Finally I set the initial time to the year 1400 and the initial endowment $h(0)$ to 0.1. I then determine p such that modern contraceptives are used for the first time in 1900, i.e. with a delay of two generations after the invention of vulcanized rubber (patented in 1844) and the introduction of the rubber condom. This provides the estimate $p = 0.06$.[10] After running the numerical

[9]Sensitivity analysis confirms that results do not respond to the scale of α, as long as the other parameters are adjusted to fulfil the conditions on steady-state growth and fertility. Likewise, assuming a different size of σ leads to a readjustment of the estimated parameters and leaves the time paths for all variables but u_t and s_t unaffected.

[10]Notice that this procedure is consistent with the idea that modern contraceptives were not available before the mid-19th century. When the threshold (10) is binding for $\mu = 1.46$ and $p = 0.06$, it is of course also binding for higher prices and for any lower values of the efficiency parameter; capturing (much) less price-efficient contraceptives than condoms. In an extension of the model in Section 5 we consider the price-efficacy ratio of contraception to be reduced by endogenous technological progress.

Figure 2: Long-Run Adjustment Dynamics



Dashed lines: μ rises from 1.46 to 1.75 in the year 1960; n is births per adult; sexual activity is measured relative to its initial value. Growth is the implied income growth rate per year, assuming a generation takes 25 years.

experiments, I convert the measure of growth rates from per-generation to per-year assuming that a generation takes 25 years.

For the first experiment I counterfactually assume that the child survival rate stays constant at unity. This experiment is useful in order to establish contraceptive use as a stand-alone mechanism that operates independently from child mortality. Blue (solid) lines in Figure 2 show the implied evolution of fertility, contraceptive use, annual growth of potential income (of human capital), and sex. The uptake of contraceptives in the late 19th century sets in motion a virtuous circle of development. It leads to less fertility, more education, and higher growth. Annual economic growth rises from 0.3 percent before the transition to about 1.5 percent in the year 2000. The model predicts that households in 1925 spend about 2.5 percent of their income on modern contraceptives.[11] This figure somewhat underestimates the real cost when compared with the observation that "in the early twentieth century, a year's supply of condoms cost a Berlin worker the equivalent of ten to fifteen days of wages" (Brown, 2009, cited according to Guinnane, 2011). Given 250 workdays per year, this estimate implies an expenditure share of $10/250 = 4$ percent.

The second and third panel of Figure 2 show the evolution of the two functions of contraceptive use. It motivates the fertility transition and it allows couples to enjoy more sex. Over the 20th century modern contraception is predicted to have prevented about 2 births per adult (on top of births prevented using traditional methods) and to have allowed a doubling of sexual intercourse. The released resources of child care are partly spent on increasing education such that economic growth takes off to a rate of about 1.5 percent at the end of the century.

While the positive impact of the fertility decline on education and the onset of modern growth is a well-known feature (e.g. Galor, 2011), the positive association of the fertility decline and the take-off to growth with the rise of sexual intercourse is less well established but there exists some supporting evidence. According to Kinsey (1948) the average frequency of sexual intercourse of married couples was twice a week in the late 1940s. According to Hunt (1974) it increased to about 3.25 times weekly by the mid 1970s. These earliest quantitative studies of human sexual behavior were not obtained from random samples and are somewhat speculative. According to the National Fertility Studies, coital frequency increased by 25 percent between 1965 and 1975, albeit from a lower base level than suggested by the Kinsey and Hunt studies, see Trusell and Westoff (1980) who also documented a positive association of coital frequency with the use of effective contraceptive methods. For the 1990s, Janus and Janus (1993) report that 85 percent of married people in their sample enjoy sexual activity at least weekly (14 percent daily and 44 percent a few times weekly). Michael et al. (1996) report that 41 percent of U.S. American couples have sex twice a week or more. For the time before the mid 20th century we have to rely on historical narratives. Here, scholars agree that the arrival of new methods of birth control led to the demise of the 19th century Victorian prescriptions about continence and self-control. In the early 20th century, the writings of Sigmund Freund and Havelock Ellis pioneered the strife for sexual gratification rather than procreation in marriage and they were soon followed by many other writers. Inspired by these new ideas, marriage was increasingly approached with

[11] The expenditure share of contraceptives is computed as pu_t/y_t, in which y is income.

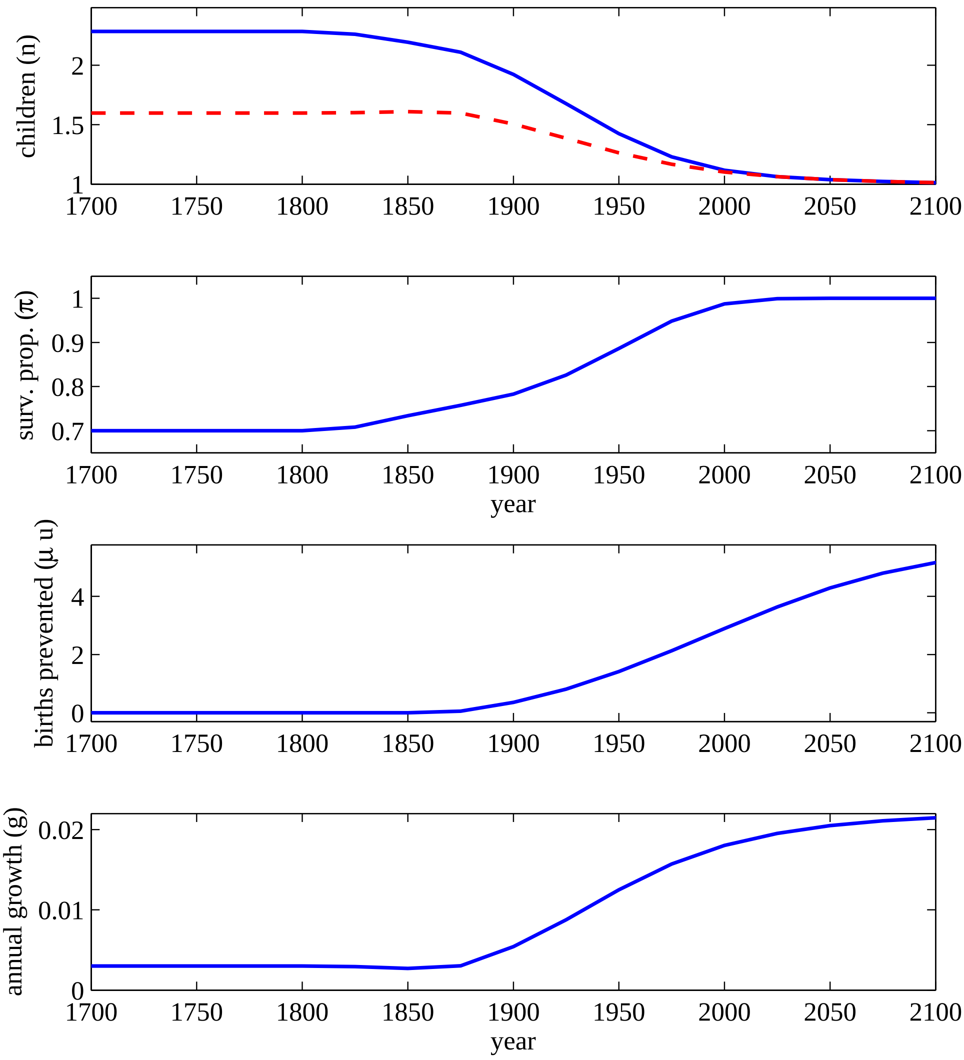
the expectation of erotic enjoyment (D'Emilio and Freedman, 1988, Ch. 10). The overarching impression from the literature is thus that sexual intercourse increased significantly during the 20th century, along with the decline in fertility. That sexual intercourse continues to rise in the 21st century could admittedly be considered as a questionable prediction of the model but notice that at this time the fertility transition is largely completed such that the mechanism could be choked off by, for example, a physiological cap on sexual activity with little quantitative impact on fertility, education, and growth.

The next experiment concerns the introduction of oral contraceptives in 1960. Using the same methodology as above and the data from Table 2 in Greenwood and Guner (2010) provides a new estimate for $\mu = (1-0.075)/(1-0.53) = 1.96$ from 1960 onwards. The implied results, assuming that p remains constant, are shown by dashed lines in Figure 2. The first generation that takes up oral contraception are the adults of 1975. The fertility transition accelerates somewhat and the economy adjusts somewhat faster towards the steady-state. Annual economic growth is predicted to be 1.7 percent in the year 2000.

We next consider the results under evolving child mortality. For this purpose, I assume that π_t rises from 0.7 in the first half of the 19th century, to almost unity at the end of the 20th century. A parsimonious way to do this is to relate child survival to income of the parent such that $\pi_t = \max\{0.7, 1 - \exp(-bh_t)\}$. The parameter b is set such that the mortality transition begins in 1825 and almost ends in 2000 (child survival rate 0.985 in 2000). This provides the estimate $b = 2.2$. I take all parameters from the benchmark model and re-estimate p such that modern contraceptives are first used in 1900. This provides a re-adjustment of p from to 0.06 to 0.04. The break-even price of contraceptives has to be lower than in the benchmark run because households invest less in education when child survival is uncertain. Consequently, economic growth is lower and the level of income is lower at any given time than for the benchmark run. The model now predicts that individuals spend about 4 percent of the income on contraceptives in the 1920, close to the Brown (2009) estimate from above.

Results are shown in Figure 3. The trajectories look very similar to the ones of Figure 2. The introduction of endogenous mortality improves the prediction of fertility, which is now 2.4 per adult (4.8 per woman), a figure closer to actual fertility in most Western countries at the dawn of the fertility transition. The main takeaway, however, is that the mortality decline does not initiate the fertility transition. In the years after the onset of the mortality decline and before

Figure 3: Long-Run Adjustment Dynamics: Evolving Child Mortality



Top panel: born children (solid line) and surviving children (dashed line) per adult. Parameter values as for Figure 2 and $b = 2.2$, $p = 0.04$.

the take up of modern contraceptives, net fertility $\pi_t n_t$ actually rises (see Proposition 4). Only after the take up of contraceptives does fertility start to decline, and education and economic growth take off.

Finally we consider a stratified society. This numerical experiment is useful to invalidate the argument that contraceptives could not have had an impact on the historical fertility decline and economic development because the initial price was too high to make them affordable to the *average* citizen. The argument is invalid because it is sufficient that *some rich* citizens take up modern contraception in order to set the virtuous cycle of declining fertility, rising education, and rising economic growth into motion.

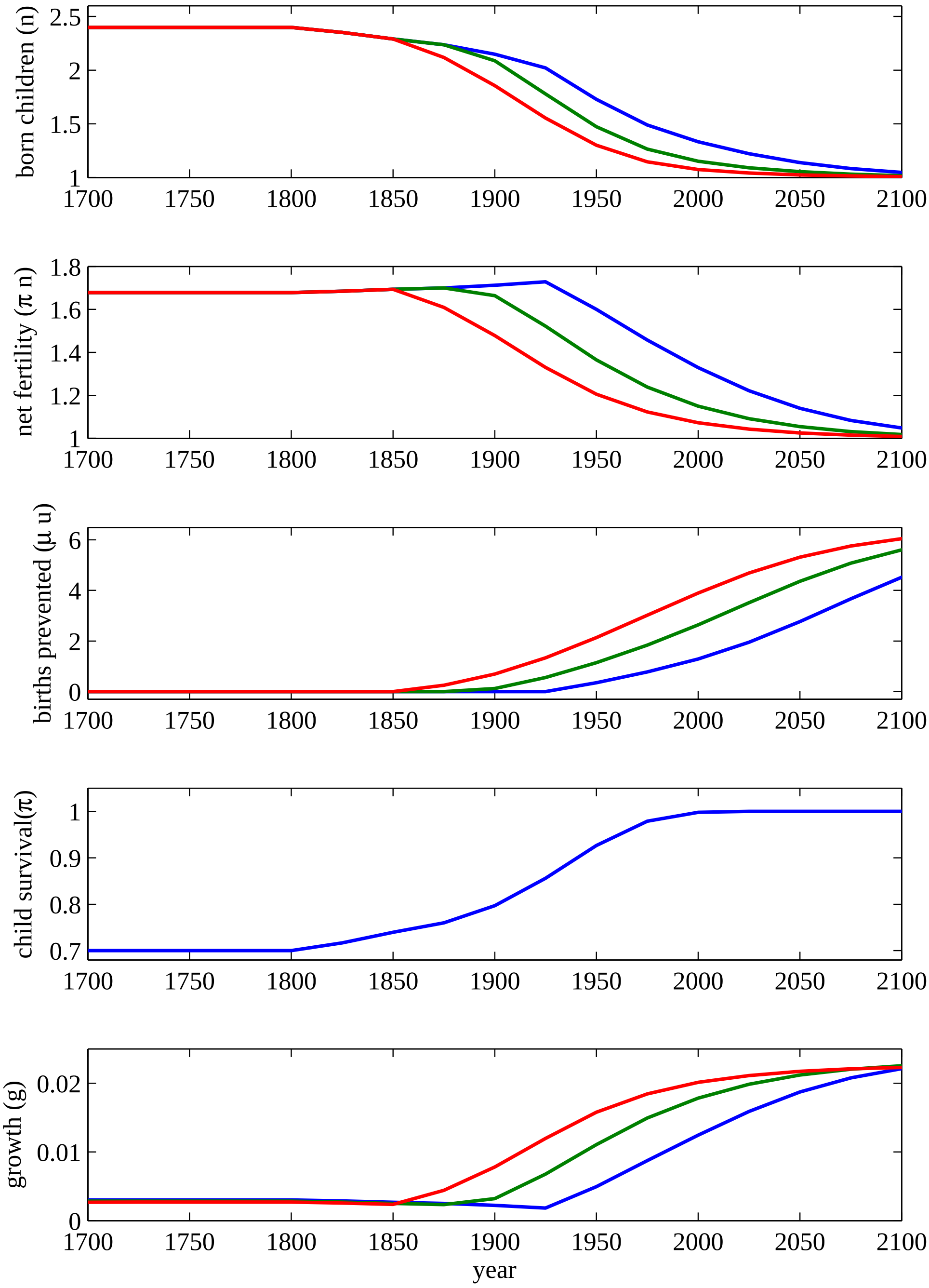
In order to establish this result, I assume, for simplicity, that society is subdivided into three classes: the rich, the middle class, and the poor, which differ in their initial endowments of h. I allow for spillovers in education such that (3) is replaced by

$$h_{t+1,j} = Ae_{t,j}h_{t,j}^{\lambda}\bar{h}_t^{1-\lambda}, \tag{17}$$

in which $e_{t,j}$ and $h_{t,j}$ are education and human capital of class j at time t, and $\bar{h}_t$ is average education at time t, $\bar{h}_t = 1/L_t \sum_{j=1}^{3} h_{t,j}\ell_{t,j}$. The size of the classes evolves according to $\ell_{t+1,j} = n_{t,j}\ell_{t,j}$ and where $L_t = \sum_{j=1}^{3} \ell_{t,j}$ is aggregate population size. I keep all parameter values from the previous experiment, but assume that child survival depends on income of the rich (proxying human capital of doctors) and I adjust b to 0.6 and p to 0.15. I set the initial endowment and the initial population sizes such that the rich take up modern contraceptives in 1875, shortly after they become first available. I set λ such that the middle class (the poor) start using contraceptives with one (three) generations delay. This provides the estimates $\lambda = 0.97$ and $h_0 = (1, 1.25, 1.5)$ for initial population shares $\ell_0 = (10, 1, 0.1)$.

Results are shown in Figure 4. The upper middle class (red line) starts using modern contraceptives in 1875, implying that at that time, 1 percent of the population were using contraceptives. This sets in motion a virtuous cycle of declining fertility, rising education, and income growth. Because of rising income and improving child survival gross fertility declines for all classes. However, net fertility of the lower middle class and the poor continues to rise (mildly) because of declining mortality (see the second panel from above). Only the upper middle class experiences declining net fertility already before the 20th century. In 1900 the middle class (green line) starts using contraceptives such that at that time, 11% of the population are using them. Two generations later, in 1950, the poor (blue line) start using them (implying an acceptance rate of 100%). At that time the poor spend about one percent of their income on modern contraceptives. The predicted population shares using modern contraceptives coincide roughly with Woods' (2000) estimates for England and the values imputed by Bhattacharya and Chakraborty (2013). Notice that the stratified society provides the prediction that contraceptive use is strongly associated with education not only over time, as in the previous experiments, but also across social strata at a given time, in line with the empirical evidence for developed and developing countries (e.g. Martin, 1995; Mosher et al., 2004).

Figure 4: Long-Run Adjustment Dynamics: Stratified Society



Parameter values as for Figure 3; except $b = 0.6$, $p = 0.15$, 3 population classes; initial values $h_0 = (1, 1.25, 1.5)$ and $\ell_0 = (10, 1, 0.1)$.

5. Contraception in the Galor–Weil (2000) Framework

One potential drawback of the simple model is that growth is solely driven by human capital, which disconnects it from standard unified growth theory emphasizing the interaction between technological progress and education. In this section I thus embed the theory in the Galor-Weil (2000) framework according to which human development is driven by the mutual enforcement of technological progress and education. This makes the model less simple and elegant and not all results from the simple model can be re-established analytically. The main results, however, will be shown to continue to hold. Moreover, the extended model can be used to assess quantitatively how much of the fertility transition and the take-off to growth can be contributed to the Galor-Weil mechanism vs. the contraception mechanism. Specifically, we consider the parametrization of the Galor-Weil model suggested by Lagerlof (2006). For simplicity, we neglect land in production and focus on the main mechanism of the Galor-Weil model, the interaction between education and technological progress. This means that the production function is modified to include time-varying total factor productivity $\tilde{A}_t$, $y_t = \tilde{A}_t h_t \ell_t$ such that the wage per unit of human capital supplied grows at the rate of technological progress, $g_{t+1} \equiv (\tilde{A}_{t+1} - \tilde{A}_t)/\tilde{A}_t$. The evolution of human capital is given by

$$h_{t+1} = \frac{e_{t+1} + \rho\phi}{e_{t+1} + \rho\phi + g_{t+1}}. \tag{18}$$

It is a positive function of eduction and a negative function of technological progress. The parameter ρ controls the concavity of the function (and thus the speed of adjustment) for given g_{t+1}. The growth rate of technological progress, is determined as a positive function of population size and education whereby the positive impact of population is limited from above:

$$g_{t+1} = g(e_t, L_t) = (e_t + \rho\phi) \min\{\theta L_t \;\;, a\}. \tag{19}$$

As Galor and Weil (2000), I assume that the growth rate of technological progress is correctly foreseen by parents and that child survival is certain, i.e. $\pi_t = 1$ for all t. Parents maximize (1) subject to (2), (4), (18) and (19). The first order conditions for n_t, e_t, and u_t can be summarized as follows.

$$f(e_{t+1}, g_{t+1}) \equiv \frac{\gamma g_{t+1}}{(1 + \alpha + \sigma)(e_{t+1} + \rho\phi + g_{t+1})(e_{t+1} + \rho\phi)} \leq n_t(e_{t+1}) \tag{20}$$

$$n_t(e_{t+1}) = \begin{cases} n_t^T \equiv \frac{\alpha+\sigma}{(1+\alpha+\sigma)(e_{t+1}+\phi+\tau)} & \text{for } u_t = 0 \\ n_t^M \equiv \frac{\alpha\mu w_t h_t}{(1+\alpha+\sigma)[(\phi+e_{t+1})\mu w_t h_t - p]} & \text{for } u_t > 0 \end{cases} \tag{21}$$

$$u_t = \max\left\{0,\ \left(\frac{\sigma}{\mu\tau w_t h_t + p} - \frac{\alpha}{(\phi + e_{t+1})\mu w_t h_t - p}\right)\frac{w_t h_t}{1+\alpha+\sigma}\right\} \tag{22}$$

in which (20) holds with equality if $e_{t+1} > 0$. As for the original Galor-Weil setup, the solution for optimal education and fertility is only implicitly given and it potentially assumes a corner solution with no education, see (20) and (21). The difference here is that fertility (and thus education at the interior solution) can assume two possible values depending on whether modern contraceptives are used or not. As for the basic model, there exists a threshold below which modern contraceptive are not used.

Proposition 7. *Modern contraceptives are not used ($u_t = 0$) if potential income $z_t \equiv w_t h_t$ is sufficiently low that is if*

$$z_t \leq \bar{z}_t \equiv \frac{p(\alpha+\sigma)}{\mu[\sigma(\phi + e_{t+1}) - \alpha\tau]}. \tag{23}$$

Ceteris paribus, the threshold $\bar{z}$ is

- *increasing in the price of contraceptives p and the desire for children α*
- *declining in the efficacy of contraceptives μ, the desire for sex σ, the time cost of child rearing ϕ, and education expenditure e_{t+1}.*

The proof evaluates (22) for $u_t \leq 0$, i.e. when the non-negativity constraint on contraceptive use is binding. Proposition 7 resembles Proposition 2 of the simple model. Here, we observe additionally that the threshold can be crossed even for constant human capital, namely by technological progress through its positive impact on wages per unit of human capital w_t. Moreover, the threshold is only implicitly defined; more education e_{t+1} reduces the threshold. The positive association of education expenditure and contraceptive use was also present in the simple model but there the feedback effect was resolved in the closed-form representation of the threshold $\bar{h}$.

Proposition 8. *Fertility is lower when modern contraceptives are used than when they are not used ($n_t^M < n_t^T$).*

The result confirms Proposition 3 for the new setup. For the proof notice from (22) that when modern contraceptives are used $\sigma\left[(\phi + e_{t+1}\mu w_t h_t - p\right] > \alpha\left[\tau\mu w_t h_t + p\right]$ and thus $(\alpha +$

$\sigma)\,[(\phi + e_{t+1})\mu w_t h_t - p] > \alpha\mu w_t h_t\,[e_{t+1} + \phi + \tau]$, which proves together with (21) the claim of Proposition 8.

The fertility transition is now propelled by two forces, the interaction between technological progress and education and the increasing use of modern contraception. Depending on the price of contraceptives the evolving economy either proceeds successively through the stages

(1) $e_t = 0$, $n_t = n_t^T$ (pre-modern era)

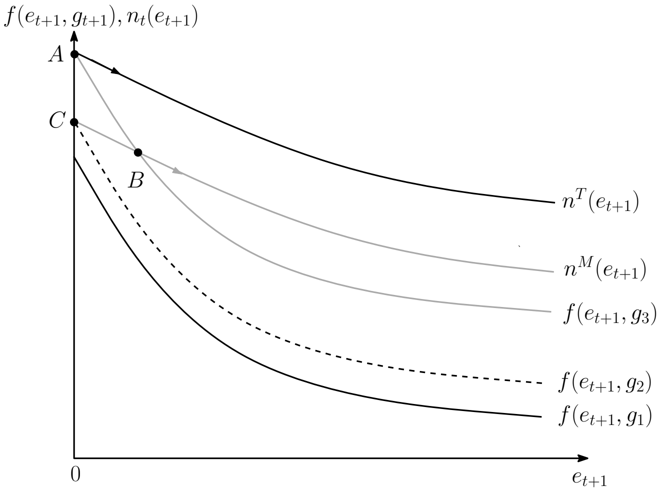
(2) $e_t > 0$, $n_t = n_t^T$ (schooling, trad. contraception)

(3) $e_t > 0$, $n_t = n_t^M$ (schooling, modern contraception),

or, alternatively, stage (2) is characterized by $e_t = 0$, $n_t = n_t^M$ (no schooling, modern contraception). Figure 5 illustrates these results. Suppose that the society is initially in a situation without education and contraceptive use. Diagrammatically it is situated at point A. The initial rate of technological progress g_1 is too low to elicit education. The downward sloping curve $f(e_{t+1}, g_1)$ lies below the downward sloping curve $n^T(e_{t+1})$ and, according to (20), the society is situated at the corner solution. Population growth propels technological progress and the f-curve shifts out gradually. Suppose education sets in before the use of modern contraceptives. This situation is reached when technological progress grows at rate $g_3 > g_1$. The fertility transition is initiated a la Galor-Weil: fertility declines with increasing education and further rising technological progress and society travels along the n^T-curve. Eventually, with further rising education and human capital, the society crosses the contraception threshold, the fertility curve shifts downward and the society travels along the n^M-curve. In the figure, this situation is reached at point B. The uptake of modern contraceptives amplifies the fertility decline and the incentive for education.

Alternatively, the use of contraception sets in before education. This would be the case when technological progress rises the unit wage w_t and thus potential income z_t sufficiently such that the contraception threshold $\bar{z}$ is crossed without education and with constant human capital. Diagrammatically, the society moves from A to C in Figure 6. The fertility transition is now initiated by the use of modern contraception. Declining fertility increases the incentive for education, which sets in when the rate of technological progress reaches g_2, $g_3 > g_2 > g_1$. Afterwards, the society moves from C along the n^M-curve. This scenario is the relevant one when the price for contraceptives p is sufficiently low compared to income. To see this, notice that for $p = 0$ there would be always use of modern contraceptives (but not always education). The

Figure 5: Stages of Development

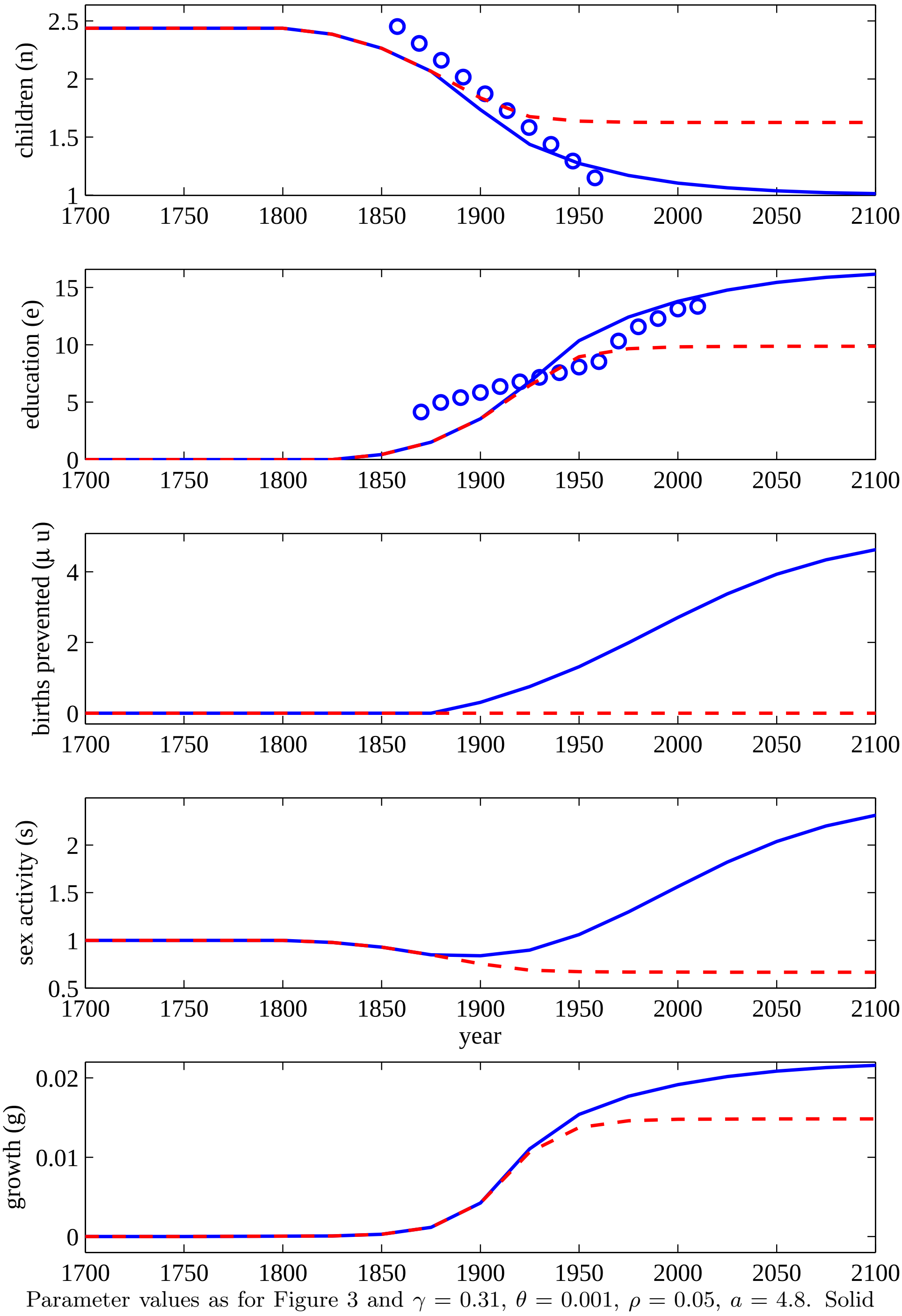


historical fertility transition of the West, however, is perhaps better described by a high relative price of modern contraceptives and the onset of education and the fertility transition before the use of modern contraceptives. In this case technological progress first causes education, which then causes contraceptive use, implying that contraception is not essential to initiate the fertility transition. Contraceptive use, however, amplifies the fertility transition and the question is how much it contributes to the fertility decline and the rise of education. This question will be addressed next with a calibration of the model.

Specifically, we consider the following numerical experiment of "counterfactual history" (Fogel, 1964). The model is calibrated such that the predicted evolution of fertility and education provides the best fit of the historical evolution of these series in England. We then counterfactually assume away the uptake of modern contraceptives (by assigning an infinite price) and investigate how much of the transition is explained by the reduced model, i.e. solely by the Galor-Weil mechanism.

For the calibration I keep all parameter values from the basic model, aside from γ, which is re-calibrated such that fertility converges to replacement level ($\gamma = 0.31$ instead of 0.24). I set $\theta = 0.001$ to approximate the low pre-industrial growth rates and calibrate a such that the economy grows at 2.2 percent annually at the modern steady state (such that it grows at 2 percent in the late 20th century). The best fit of fertility during the transition is obtained for $\rho = 0.05$.

Figure 6: Long-Run Adjustment Dynamics: Galor-Weil (2000) Setup



Parameter values as for Figure 3 and $\gamma = 0.31$, $\theta = 0.001$, $\rho = 0.05$, $a = 4.8$. Solid lines: basic run. Dashed lines: counterfactual removal of use of modern contraceptives. Dots: data for England; fertility from Guinnane (2011), here with linear interpolation between 1840 and 1940; years of education from Morrison and Murtin (2009).

Figure 7 shows the model's predictions. Solid lines show the basic scenario, according to which technological progress initiates the fertility transition in 1825 (the Galor-Weil mechanism) and modern contraceptives are used from 1900 onwards and amplify the fertility transition. According to the Galor-Weil model there is no formal education before the onset of the fertility transition. It is thus impossible to get the historical evolution exactly right for both education and the fertility transition. Figure 6 shows the compromise according to which the calibrated model minimizes the distance to the actually observed trends, visualized by dots. Data for years of education is from Morrison and Murtin (2009). Data for cohort fertility per woman is from Guinnane (2011). In order to compare data and model predictions, I divide fertility per woman by two to get fertility per parent. I assume that fertility is realized one model-generation after the birth year of the cohort. Moreover, I eliminate a trough in fertility for the 1890 to 1930 cohorts by computing the linear trend for fertility from the 1831 cohort to the 1936 cohort. In order to compare the trajectory of education expenditure predicted by the model with the observed years of education, I rescale the outcome for e_{t+1}. Specifically, I assume that years of education equal 0.4 times the investment in education. This implies that model and data are aligned in 1930, in the middle of the observation period from 1860 to 2010.

Before the onset of use of modern contraceptives the only way to bring down fertility is reduced sexual intercourse, for example by marrying late (Wrigley and Schofield, 1989, Chapter 10, Figure 10.9) or traditional methods of birth control including abstinence (Cinnirella et al., 2012; Clark and Cummins, 2015). This is also visible in Figure 7. The model predicts that the Galor-Weil-mechanism sets in 1825 and that fertility and sexual activity decline during the Victorian era, between years 1825 and 1900. In 1900, individuals start using modern contraceptives, which causes a further decline of fertility and increases sexual intercourse in marriage. As can be seen from the upper two panels, the model approximates the actual trends of fertility and education reasonably well.[12]

The evolution without the innovation of modern contraception is shown by dashed lines. The model now counterfactually predicts that individuals reduce their sexual activity further during the 20th century since this is the only way to bring fertility further down and to increase education per child. Individuals, however, like sex and thus they reduce sexual activity by less

[12] The model is calibrated such that the first small decline in fertility rates is predicted for 1825 while the big decline happens after 1875. Integrating a stratified society, as in Section 4, could allow fertility of a rich group to decline earlier, e.g. in 1780 and associate it with the first industrial revolution (Clark and Cummins, 2015).

than required in order to bring fertility down to replacement level. Fertility stabilizes at a level of about 1.6 children per adult (3.2 children per women) and education stabilizes at about 9 years of schooling. Inferior education affects growth, which is predicted to be about 0.7 percentage points lower than in the contraception case.

In order to assess the robustness of these findings we consider a brief sensitivity analysis. For that, we vary successively the values for the utility weight of children α, the utility weight of education γ, the time cost of children ϕ, the time cost of sex τ, and ρ, the curvature parameter of the education function, and compute the steady-state deviations of the Galor-Weil model with and without use of modern contraceptives. Parameter changes of p, μ, and θ are not considered because they leave the steady state unaffected. The parameter values of $\tilde{A}$ and σ are adjusted endogenously such that any version of the model with contraceptives produces the same long-run growth rate and fertility at the replacement level at the steady state.

The first data column of Table 1 (benchm.) reiterates the results just presented. Without the use of modern contraceptives, fertility according to the Galor-Weil model without contraception is predicted to be 0.6 children per adult (1.2 children per woman) higher, education 6.7 years lower, and growth 0.7 percentage points lower than in the full model. Naturally, the deviation gets larger when individuals place more weight on the number of offspring ($\alpha = 0.7$ instead of 0.5) or less weight on education ($\gamma = 0.2$ instead of 0.3), as shown in the next two columns. In these cases fertility without contraception is higher at all values of income such that the gap created by contraception widens. The same is true if child costs are lower ($\phi = 0.1$ instead of 0.15) as evidenced in the next column, or if the education function is less concave, as shown in the last column. If the time cost of sex is higher ($\tau = 0.05$ instead of 0.02), in contrast, the deviation is reduced because there is less sexual intercourse at any level of income and the power of the contraception channel is smaller.

In another counterfactual experiment we could assume that condoms (and other contraceptive innovations of the 19th and early 20th century) played no role for the historical fertility transition and that it is the innovation of the contraceptive pill in 1960 and the second demographic transition that brought fertility down to replacement level. In that case we would obtain the same deviations of the standard Galor-Weil model from the full model as shown in Table 1. To see this, recall that p and μ are irrelevant for steady states.[13]

Table 1: Sensitivity Analysis

	benchm.	$\alpha = 0.7$	$\gamma = 0.2$	$\phi = 0.1$	$\tau = 0.05$	$\rho = 0.1$
Δn	0.62	2.52	1.92	3.15	0.37	0.91
Δe	-6.71	-7.66	-6.12	-9.75	-4.73	-8.01
Δg	-0.71	-1.45	-1.31	-1.55	-0.49	-0.91

The table shows the steady-state deviation for fertility (Δn), education (Δe), and annual growth in percent (Δg) between the Galor-Weil model without modern contraceptives and the Galor-Weil model with contraceptives for alternative parameter values.

Finally, we endogenize the price and efficacy of modern contraception. In principle, technological progress can be conceptualized as quality-enhancing or price-reducing. In order to capture both aspects conveniently we define the price-efficacy ratio, $\omega_t \equiv p_t/q_t$, in which p_t and q_t are the now time-varying price and efficacy of modern contraception. With the new notation, equation (21) and (22) are rewritten as:

$$n_t(e_{t+1}) = \begin{cases} n_t^T \equiv \frac{\alpha+\sigma}{(1+\alpha+\sigma)(e_{t+1}+\phi+\tau)} & \text{for } \mu_t u_t = 0 \\ n_t^M \equiv \frac{\alpha w_t h_t}{(1+\alpha+\sigma)[(\phi+e_{t+1})w_t h_t - \omega_t]} & \text{for } \mu_t u_t > 0 \end{cases} \tag{24}$$

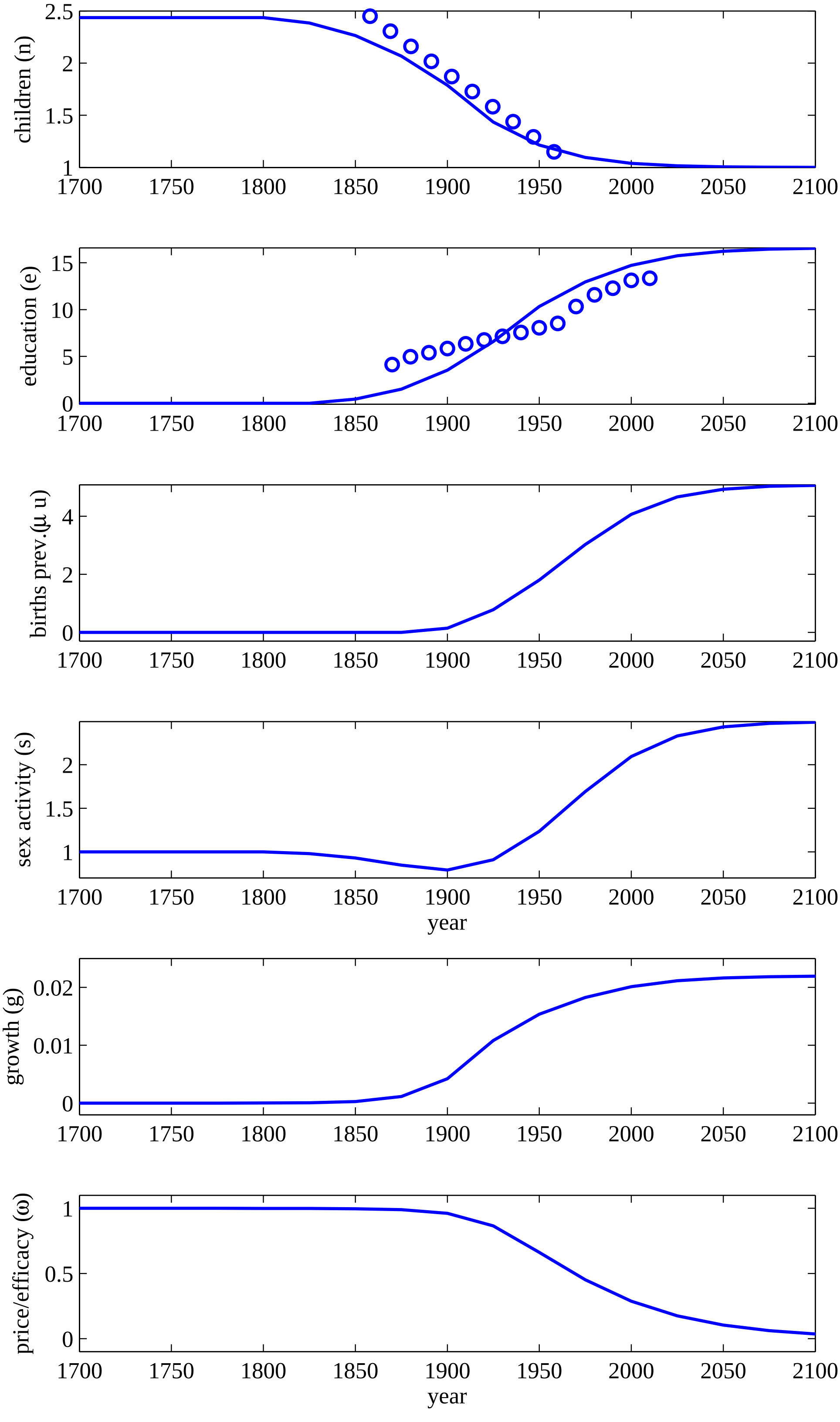
$$\mu_t u_t = \max\left\{0,\ \left(\frac{\sigma}{\tau w_t h_t + \omega_t} - \frac{\alpha}{(\phi + e_{t+1})w_t h_t - \omega_t}\right) \frac{w_t h_t}{1+\alpha+\sigma}\right\}. \tag{25}$$

Notice that n_t, e_{t+1}, and $\mu_t u_t$, i.e. the number of births prevented, converge towards constants for $\omega_t \to 0$ and $w_t \to \infty$, implying that sexual activity s_t also converges towards a constant. Assuming that technological progress in the contraceptive sector is proportional to aggregate technological progress, we have $\omega_t = \nu/\tilde{A}_t$. We calibrate ν such that modern contraceptives are used for the first time in 1900. This provides the estimate $\nu = 0.068$.

Figure 7 shows the results. The bottom panel shows the predicted price-efficacy ratio. The model predicts that in 1925, individuals spend 4 percent of the income on contraceptives. Compared to the previous model, the fertility transition proceeds somewhat faster. Fertility now reaches replacement level at the turn of the century. But otherwise the results are very similar.

[13]For the calibrations we always assume, in line with the UN population projections, that steady-state fertility is at replacement level (2 children per woman). Also, from a conceptual viewpoint, a steady state with a non-stationary population would make no sense. Since adjustment to the steady-state is monotonous this means that the phenomenon of fertility below replacement level cannot be captured by the model. See Strulik and Weisdorf (2008) for an approach to undershooting fertility along the transition.

Figure 7: Long-Run Adjustment Dynamics: Endogenous Price-Efficacy Ratio



Parameter values as for Figure 6 and $\nu = 0.07$. Sexual activity and the price-efficacy ratio are scaled relative to their initial values.

6. Conclusion

Inspired by insights from evolutionary psychology and from happiness economics, I have integrated sex in the utility function and suggested a theory of demand for children and for modern contraceptives. The approach provides a natural explanation for a negative income fertility nexus and motivates an income threshold below which modern contraceptives are not used. The fertility transition commences only after the threshold has been crossed. The fertility transition is accompanied by increasing investments in child education, which further amplifies income growth and the use of contraceptives. The economy develops towards a steady state at which couples experience more sex but have fewer children than couples at the traditional equilibrium, and at which the economy grows at a (much) higher rate.

While the simple model focusses on contraception as a stand-alone mechanism for the fertility transition and the take-off to growth, an extended model embeds the mechanism in the canonical unified growth theory (Galor and Weil, 2000) and confirms all main results from the basic model. According to the extended model, societies may move from a pre-modern state without education in one of two ways. If the initial price of modern contraceptives is high, education due to the Galor-Weil mechanism sets in first and then triggers the uptake of modern contraceptives. If the initial price is low, contraceptive use is initiated first such that fertility declines before the onset of education. Assuming that the first way is the relevant one for the historical fertility transition of the West, a calibrated model has been used for a counterfactual historical experiment. It showed that when contraception is taken away from the full model, the fertility transition stops halfway to replacement level, implying about 6 years less of education, and sexual intercourse at a frequency below the level during the Victorian era.

This paper provides a first attempt to integrate sexual desire and contraceptive use into unified growth theory. Several extensions of the theory are conceivable. Social interaction and cultural evolution have been identified as important determinant of fertility and the demand for contraceptives (Palivos, 1995; Kohler et al., 2001; Munshi, 2006; Iyer, 2003; Heaton, 2011). Prettner and Strulik (2014) use the present framework and extend it with special focus on the role of traditional religious beliefs. Such demand-driven mechanisms are useful to motivate the local stability of a traditional equilibrium at which contraceptives are not used although prices are low. As demonstrated above, however, these elements are not essential to explain the role of contraceptives for the historical fertility transition of the West.

Appendix A

The main text assumed an iso-elastic utility function for consumption. Alternatively, we could replace $\log c_t$ in (1) by $\log(c_t - \bar{c})$, for $c_t > \bar{c}$. For this Stone-Geary-type utility function the elasticity of intertemporal substitution is not constant during the process of economic development. It assumes a value of zero at subsistence level and converges towards one as income and consumption go to infinity. It is a convenient way to add more realism to the model and to capture elasticities of intertemporal substitution different from unity. In this case, the interior solution for fertility is obtained as

$$n_t = \frac{(\alpha - \gamma)\mu_t(h_t - \bar{c})}{(1 + \alpha + \sigma)(\phi\pi\mu h_t - p)}$$

and the derivative with respect to potential income (h_t) is obtained as

$$\frac{\partial n_t}{\partial h_t} = -\frac{(\alpha - \gamma)\mu\,[p - \bar{c}\mu\phi\pi]}{(1 + \alpha + \sigma)(\phi\pi\mu h_t - p)^2}.$$

This means that increasing income induces lower fertility as long as the efficiency-adjusted price of contraceptives is sufficiently large compared to $\bar{c}$, that is if $p/(\mu\phi\pi) > \bar{c}$. The same result is obtained under the utility function of the main text when household expenditure contains a non utility enhancing part $\bar{c} > 0$, i.e. when budget constraint (4) is replaced by

$$h_t(1 - \phi\pi_t n_t - e_{t+1}\pi_t n_t - \tau s_t) = pu_t + c_t + \bar{c}.$$

All other result of the main text continue to hold without qualification.

Alternatively, we could allow households to receive a non-market income $b > 0$. In that case fertility at the interior solution is obtained as

$$n_t = \frac{(\alpha - \gamma)\mu_t(h_t + b)}{(1 + \alpha + \sigma)(\phi\pi\mu h_t - p)}$$

and it is straightforward to see that all results of the main text continue to hold without qualification.

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