Should Continued Family Firms Face Lower Taxes Than Other Estates?

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Abstract. Taxes on estates and inheritances may induce heirs to discontinue family firms. Because firm dissolution incurs transaction costs, a preferential tax treatment of transferred family businesses seems to be desirable from a macroeconomic viewpoint. The support of dynastic succession, however, entails also a cost on the economy if firm continuation by less able heirs prevents entry into entrepreneurship. Here, we investigate analytically and quantitatively the trade-off between transaction costs saved and creative destruction prevented. We find that a unique general equilibrium exists at which, depending on the institutional setup, low-ability heirs either abandon (Type 1) or continue (Type 2) a family business. A calibration of the model with German data suggests that preferential tax treatment of family firms has severe negative consequences on macroeconomic performance if it causes a threshold crossing from Type 1 to Type 2 equilibrium. It also reveals that the descendants of less able entrepreneurs who were caused by continuation-friendly tax policy to keep a family business always lose relative to their status in an economy without such a policy.

Keywords: Bequest Taxation, Creative Destruction, Entrepreneurship, Family Firms, Preferential Tax Treatment.

JEL: H25, L26, J24.

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1. Introduction

Taxation of inheritances and estates is under steady debate in many industrialized countries.¹ On the one hand, proponents argue that taxing inheritances is an effective mean to "level the playing field", i.e. to mitigate wealth inequality and improve equality of opportunity. On the other hand, supporters of tax reduction or repeal argue that it provides disincentives to accumulate capital and retards work effort.²

Here we focus on one point that is always stressed in the debate, namely that taxes on inherited family firms impose a burden on the heirs that may induce them to discontinue the business. Business closures and the start up of new firms entail transaction costs, i.e. real efficiency losses without any gains elsewhere in the economy. It has been argued that, therefore, continuation of family firms is desirable from a macroeconomic viewpoint and should not be punished by the tax law.³

In many industrialized countries, like the US and many EU member states, the tax law treats inherited firms preferentially or reforms in this direction are planned for the future. Already in 1994 the European Commission (1994) published its recommendations on the transfer of small and medium size enterprises where it reads "We want to encourage the Member States to adopt concrete and specific measures to prevent SME closures, which have an adverse effect on attempts to maintain and increase employment. [...] The Commission requests the Member States to ensure that family law, inheritance law and the payment of financial compensation cannot jeopardize the survival of the business [and to] reduce taxation on assets in the event of transfer by succession or by gift, provided that the heirs continue to operate the business." In 2006 the European Commission (2006) reviewed the implementation of its recommendations and concluded that 21 out of 25 states had either implemented the recommendation of reduced taxation of inheritances or were planning an implementation for the future.

¹We do not distinguish in our analysis between inheritance taxation (which refers to individual taxation of beneficiaries) and estate taxation (where the tax payment solely depends on the total estate left, rather than on the distribution of inherited wealth among heirs). For instance, the US and UK have estate taxation at the federal level, whereas Germany, France and some US states have inheritance taxation.

²These issues are discussed by, among many others, Holtz-Eakin et al. (1994), Weil (1994), Holtz-Eakin (1999), Kopczuk and Slemrod (2001), Gale and Perozek (2001) Holtz-Eakin and Marples (2001), and in a general equilibrium context by Laitner (2001) and Cagetti and de Nardi (2007). Gale and Slemrod (2001) provide a short survey over "rhetoric and economics in the estate tax debate". In a recent normative approach Farhi and Werning (2005) find that the optimal inheritance tax is progressive if future generations are directly valued in welfare maximization.

³This reasoning is somehow weakened by the fact that the current owner could (and in many cases does) take care of expected tax payments through life insurance so that there is no sudden liquidity constraint to bear for offsprings when they take over the business and face the tax burden. But the general argument that taxes on inheritances may prevent firm continuation and cause transaction costs remains, of course, valid.

The high importance attributed to taxation of inherited family firms stems from the recognition that this institution is a quantitatively important determinant of employment, income per capita, and many other macroeconomic aggregates. In Germany, for example, about 85 percent of all firms in the manufacturing sector are family-owned and managed (BDI, 2006). The small and medium enterprises (SMEs) of the so called German "Mittelstand" encompass 99 percent of all German companies and employ about 70 percent of the labor force. Many European countries (but not the U.S.) show similar characteristics. 99 percent of the European enterprises are SMEs. The average European SME employs 6 people and 66% of the European labor force are employed in SMEs (see European Commission, 2003, and Deutsche Bank, 2007).

The business continuation argument shifts the discussion from the general pros and cons of inheritance taxation towards the specific tax treatment of one particular item, the family firm. Strictly speaking, the continuation argument cannot be used in order to generally justify inheritance tax cuts or the preferential tax treatment of inherited businesses. Instead, tax alleviation should be contingent on an action of the heirs, namely to continue the inherited firm.

Preferential tax treatment of continued family firms, however, may also incur a cost on society. While the founder of a family business is almost by definition endowed with high entrepreneurial skills this is not necessarily true for his or her heirs. Unlike financial wealth, management skills cannot be inherited perfectly. Given a possibly small but inevitably positive probability that heirs do not inherit the entrepreneurial spirit and the management skills of their parents, management abilities, like other personal characteristics, regress towards the mean (Galton, 1877, Mulligan, 1999). On average and over the long-run, heirs of family firms will have just average skills to run the business and probably underperform vis-à-vis new entrants who are – by self selection into entrepreneurship – more likely to be endowed with high entrepreneurial skills.

While there are also good reasons to believe that heirs of family firms are endowed with particular management traits (tacit firm-specific knowledge and longer planning horizons, for example), the recent empirical evidence suggests that the negative regression-to-the-mean effect dominates. Comparing publicly traded businesses it has been found that heir-controlled firms underperform relative to those managed by unrelated CEOs. This is shown by Morck et al. (1998), Perez-Gonzalez (2006), Villalonga and Amit (2006), Bloom and van Reenen (2007), and Bennedsen et al. (2007) for US American, British, Canadian, Danish, French, and German firms.

A continuation-friendly tax policy that causes low-ability heirs to continue a family business has

direct and indirect repercussion on the macroeconomy. If managing ability complements factor input, which seems to be a natural characteristic of managing qualities, low-ability heirs invest less and employ less workers than their high-ability counterparts. In short, they run inefficiently small businesses. A second, indirect effect on efficiency occurs if the presence of low-ability descendants of firm owners blocks entry into entrepreneurship of high-ability descendants of workers. In this case, a continuation-friendly tax policy reduces aggregate total factor productivity and through this channel probably not only current GDP per capita but also economic growth. It slows down the Schumpeterian process of creative destruction. Finally, firm continuation may have a negative "third generation" effect on welfare that occurs when low-ability heirs invest little or nothing, live off the capital stock of the inherited firm instead, and transfer small wealth to their own offspring.

It is our impression that these negative repercussions of firm continuation are largely overlooked in the inheritance tax debate. This does, of course, not necessarily imply that fostering firm continuation is a bad idea. Firm continuation may still be worthwhile because of the saved transaction costs through prevented exit and entry. A trade-off exists and the question is which effect dominates. The purpose of the present paper is thus to investigate whether lower inheritance taxes for family firms are efficiency-enhancing or whether they are reflecting family values, which are harmful for aggregate economic performance and welfare.

In order to solve this problem we propose a simple general equilibrium model with endogenous exit and entry of heterogeneous family-owned firms and inheritance taxation. We model the decision of descendants of firm owners whether to operate the inherited firm or to sell it and become a worker. Descendants of workers decide whether to become entrepreneur or worker. Entrepreneurs choose the amount of investment and bequeath the firm to their offspring. With this dynastic business transfer we intend to capture the revealed preferences of many firm owners, i.e. the desire to ensure survival and family control of their firm. Workers, by contrast, choose the amount of their bequests through foregone life-time consumption. The fact that individuals differ by provenance, i.e. origin from worker- or entrepreneur-households, by inherited wealth, and by entrepreneurial talent drives the heterogeneity of firms and the performance of the macroeconomy.

In the analytical part of the paper we show that there exists a unique general equilibrium of the model economy, which assumes one of two possible types. In a Type 1 equilibrium low-ability heirs of family firms sell the business and exit the market immediately, in a Type 2 equilibrium low-ability heirs continue the business unless they have inherited it from a parent who was also of low ability. We show how the threshold separating Type 1 from Type 2 depends, among other things, on inheritance tax arrangements and we investigate performance of the economy at the two types of equilibria.

We then continue by calibrating the model and investigate numerically how introducing preferential tax treatment of inherited businesses contingent on continuation affect the performance of the economy, aggregate welfare, and utility of the different groups in society. We also examine whether tax deterrence of continued family firms, rather than preferential treatment, could be socially desirable.

Our model shares some elements with Caselli and Gennaioli (2006) who also investigate firms where ownership and control are passed from one generation to the other. They show that dynastic management reduces total factor productivity if the heirs have little talent and use this result to explain cross-country differences in total factor productivity. The incidence of family firms is explained by weak institutions and underdeveloped financial markets. As a result, family firms are predicted to be more prevalent in less developed countries from which the productivity difference across countries derives. With contrast, we investigate family firms in fully industrialized countries with strong institutions and developed financial markets. Family values motivate entrepreneurs to pass on their firm to their offspring and the interaction of tax legislation, transactions cost, and wealth inequality determines whether the heirs continue the business.⁴

2. The Model

2.1. **The Population.** We consider an economy populated by a unit mass of families, indexed by $i \in [0,1]$. Families are intergenerationally linked and conceptualized as dynasties whereby we assume for simplicity that each parent generation has one child. In each period t = 0, 1, 2, ... there is one generation of each dynasty economically active, either as a worker or as an entrepreneur. The number of entrepreneurs (n_t) and the number of workers $(1 - n_t)$ in the economy is generally endogenous and predetermined only for the initial period.

From period one onwards economic agents have to make a career decision depending on kind and magnitude of their inherited wealth and their endowment with entrepreneurial skills (in short

⁴Caselli and Gennaioli investigate a much richer wealth distribution than the present paper. Since our main arguments are based on efficiency considerations we impose a very stylized distribution of wealth in order to obtain our main results analytically hoping that distributional issues are of second order for efficiency outcomes. How bequests affect the wealth distribution is an interesting aspect in itself which is addressed by Laitner (2001), Heer (2001), Cagetti and de Nardi (2006, 2007), and Bossmann, Kleiber and Wälde (2007). The link between entrepreneurship, savings, and wealth distribution is also investigated by Quadrini (1999) and Gentry and Hubbard (2004).

"ability"). Specifically, we assume that the ability to manage a firm is either high or low, i.e. ability of the member of family i who is economically active in period t is given by $a_t(i) \in \{a^L, a^H\}$, $a^L < a^H$. Like wealth, managerial ability may be inherited. Although the recent empirical literature provides little support for an intergenerational transfer of talent (see Introduction) it is nevertheless useful to control for this possibility. This way, the model takes into account that family firms may be transferred together with the ability to manage them, an argument that could be put forward by supporters of inheritance tax relief. Inheritance of ability does not necessarily have to be conceptualized as the transmission of a "manager-gene". It may also include the transfer of tacit management knowledge within the family. In modelling ability inheritance we follow Caselli and Gennaioli (2006). Specifically, we assume that there is a fraction λ of high-ability individuals in the population and that the correlation coefficient of parent's and children's ability is given by μ , $0 \le \mu < 1$. A stationary distribution of ability requires then that the probability to inherit one's parent high ability is $p^H = \lambda + \mu - \lambda \mu$ whereas the probability to inherit low ability is $p^L = 1 - \lambda + \lambda \mu$.

2.2. **Dynasties.** Dynasties are linked through intentional transfers of wealth. A family member i of generation t has preferences over consumption $c_t(i)$ and the net amount bequeathed to the offspring, reflecting a "joy-of-giving" bequest motive (Andreoni, 1989). Taking an inheritance tax at rate τ into account the net bequest b_t^{net} enters a quasi-linear utility function together with consumption.

$$U_t(i) = c_t(i) + v(b_t^{net}(i)), \tag{1}$$

where $v(b) = \beta(1-\eta)^{-1}b^{1-\eta}$ for $\eta \neq 1$ and $v(b) = \beta \ln b$ otherwise; $\beta > 0.5$ Quasi-linearity of the utility function allows us to solve the model analytically and to work out important mechanisms. The form of bequests is conditional on occupation. For workers, bequests consist of foregone lifetime consumption whereas for entrepreneurs they consist of the capital stock of their firm. Thus, the prospect that the firm remains in the ownership of the family serves as a second motive (besides making profits) for investment of entrepreneurs.⁶

Given that capital depreciates at rate $\delta \in [0,1)$, an entrepreneur i in t bequeaths an amount $(1-\delta)k_t(i)$ of the capital stock. We assume that heirs do not assign a particular non-pecuniary value to family firms implying that they sell an inherited firm whenever this appears to be financially

⁵Under these preferences, bequests are always positive. Although this is not true in reality, positive bequests are required to make the analysis meaningful.

⁶The view that the incidence of family firms originates from family values is supported by the evidence compiled in Bertrand and Schoar (2006).

worthwhile. Depending on provenance and occupation individuals in our model-society can be classified into four types:

- heirs of entrepreneurs who continue a family business
- heirs of entrepreneurs who sell an inherited firm and become workers
- heirs of workers who start up a new enterprise
- heirs of workers who continue to be workers.

2.3. Investment and Firm Sale. If a member of dynasty i inherits a firm and remains entrepreneur, he decides upon how much to invest into that firm. When he invests $z_t(i)$ in period t the capital input in t is given by

$$k_t(i) = (1 - \delta)k_{t-1}(i) + z_t(i). \tag{2}$$

If he prefers to give up the inherited firm, in period t the capital stock $(1 - \delta)k_{t-1}(i)$ is sold to the world market at a price $q \in (0,1]$ per unit of capital. The price q may be smaller than one because capital is attached to the specific environment of the firm and is less valuable for an outside buyer than within the particular firm. Alternatively, one may think of costs to deinstall capital and install it elsewhere. In general, q is an inverse measure of transaction costs associated with the sale of a firm, i.e., a low value of q indicates large transaction costs per unit of capital. Because transaction costs are modelled as an extra deprivation of capital through sale, they imply foregone output and impose a real efficiency loss on the economy, a loss that would not occur if the firm were continued.

If a descendant of a worker decides to become entrepreneur, he has to incur a fixed cost $k \ge 0$ so that after investing $z_t(i)$ the amount of capital employed in the production process of a newly founded firm i in t is given by

$$k_t(i) = z_t(i) - \bar{k}. \tag{3}$$

Here the parameter \bar{k} stands as a catch all for startup costs as well as costs stemming from liquidity constraints experienced by entrants because they cannot use an inherited firm as collateral. Fonseca, Michaud and Sopraseuth (2007), show that indices for startup costs and liquidity constraints are usually positively correlated across Western European countries.

Investments are made at the beginning of the period. We consider a small open economy in

⁷We discuss the possibility of external management in the working paper version (Grossmann and Strulik, 2008) and, briefly, in the Conclusion.

an environment with developed financial markets and internationally mobile capital. Simplifying we assume that there are no borrowing costs besides \bar{k} and that the desired capital stock can be financed at an internationally given interest rate, denoted by r. r also equals the rate of return for lenders (i.e., workers leaving financial bequests). Because our article is an investigation of the continuation problem of family firms and not of international tax competition, we exclude the possibility of tax avoidance through firm relocation. For that purpose we assume that owners of family firms who consider to escape inheritance taxation have to move their residence along with their firm (in order to supervise production) and that mobility costs, which may involve mental and social costs of moving abroad, are sufficiently high so that entrepreneurs prefer to stay at home.

2.4. **Production.** Output produced by an entrepreneur of family i in period t is determined by a Cobb-Douglass production function

$$y_t(i) = a_t(i)l_t(i)^{\alpha}k_t(i)^{1-\alpha},\tag{4}$$

 $0 < \alpha < 1$, where $l_t(i)$ is labor input and $k_t(i)$ is capital input. Ability of entrepreneurs complements capital and labor inputs and operates like a measure of total factor productivity. An entrepreneur of high ability $(a_t(i) = a^H)$ produces more output for a given combination of inputs than a less able one.

Workers supply one unit of labor of identical quality to a perfect labor market and receive a wage w_t . Entrepreneurs are the residual claimants to income net of wage payments. Firms are price-takers and output prices are normalized to one. Thus earnings of an entrepreneur i are given by

$$\pi_t(i) = a_t(i)l_t(i)^{\alpha}k_t(i)^{1-\alpha} - w_t l_t(i).$$
 (5)

- 2.5. Government. The government levies proportional taxes on inheritances and redistributes the revenue in form of lump-sum transfers T_t . The latter assumption implies that all groups of society are affected by a change in macroeconomic performance in the aftermath of a tax reform.⁸ The government budget is balanced in each time period. In order to investigate our main policy problem we allow the taxes to depend on the type of asset inherited. The tax rate is
 - $\tau_k \in [0,1)$ for descendants of firm owners who continue the family business.

⁸Alternatively, one may introduce other tax instruments which adjust if bequest taxation is reformed. In contrast to lump-sum transfers, however, this would typically affect investments and bequests as well as entry and exit decisions. We preferred to isolate the effects of bequest tax reforms.

- $\tau_s \in [0,1)$ for descendants of firm owners who sell the family firm.
- $\tau_b \in [0,1)$ for descendants of workers.

In many countries, the effective tax rate applied to the capital stock of an inherited firm depends on institutional depreciation rules. If the tax treatment allows faster depreciation than the one physically taking place (i.e., the tax law allows the book value of the capital of a firm to depreciate at a higher rate than δ), then in effect $\tau_k < \tau_b$. An effectively lower τ_k follows also from the deferral of tax payments for inherited family businesses which is permissible in many European countries and in the U.S.

The legislator's underlying motivation for establishing a preferential tax treatment of inherited business capital is possibly not to privilege the heirs of firm owners but to foster firm continuation by alleviating the succession problem. Thus, a more sensible, fine-tuned tax policy consists of a preferential treatment of inherited family firms contingent on the continuation of the business. In many countries such a policy is already in place or movements in this direction are high on the policy agenda. This motivates our assumption of a third tax rate, τ_s , that applies when an inherited firm is sold, which may differ from τ_k .

Arguments in favor of such tax relief for continued family businesses are captured by two elements in our model. Continued firms entail no startup costs \bar{k} and the value of capital is not diminished in a process of firm dissolution (no sale of capital at price q < 1). Because the continuation of family firms prevents these agency- and transaction costs, i.e. real efficiency losses of the economy, it may be desirable from a macroeconomic viewpoint and a preferential tax treatment seems to be worthwhile. However there is also an efficiency argument speaking against tax allowances for continued firms.⁹

The trade off occurs because ability is transferred imperfectly between generations ($\mu < 1$). Motivated by a preferential tax treatment some less able heirs may be inclined to continue a family business. As explained above, entrepreneurs of low ability make inferior use of factor inputs and reduce efficiency of the economy. The negative effect is amplified further if the presence of less able heir-managers blocks entry of highly able descendants of workers. In that case the survival of low-ability firm owners reduces the number of active high-ability entrepreneurs. In other words, a preferential tax treatment of family firms may slow down the Schumpeterian process of creative destruction. The investigation of the trade off between transaction costs saved and creative

⁹Our discussion focusses on efficiency arguments and largely neglects distributional issues.

destruction prevented is at the center of the following discussion of the effects and desirability of alternative inheritance tax schemes.

3. Career Choices

3.1. Entrepreneurs. Consider a member of family i with ability $a_t(i)$ inheriting a firm with $(1-\delta)k_{t-1}(i)$ units of capital (being equal to the tax base) who continues the family business and invests $z_t(i)$. His consumption is given by $c_t(i) = \pi_t(i) - (1+r)z_t(i) - \tau_k(1-\delta)k_{t-1}(i) + T_t$. When he retires or dies he leaves an amount $(1-\delta)k_t(i)$ of productive capital in the family firm, which he bequeaths to his offspring. Inserting (2) and (5) into consumption, we see that utility (1) is maximized subject to

$$c_t(i) = a_t(i)l_t(i)^{\alpha}k_t(i)^{1-\alpha} + (1+r-\tau_k)(1-\delta)k_{t-1}(i) + T_t - w_t l_t(i) - (1+r)k_t(i), \quad (6)$$

$$b_t^{net}(i) = (1 - \tau_k)(1 - \delta)k_t(i),$$
 (7)

where we implicitly assume that entrepreneurs believe that the firm is continued, i.e., that tax rate τ_k applies.¹⁰

An entrepreneur with ability $a_t(i)$ who operates a newly founded firm and who has a parent with foregone consumption $b_{t-1}(i)$ as bequest (with rate of return r and being taxed at rate τ_b), a worker, maximizes utility subject to

$$c_t(i) = a_t(i)l_t(i)^{\alpha}k_t(i)^{1-\alpha} + (1-\tau_b)(1+r)b_{t-1}(i) + T_t - w_t l_t(i) - (1+r)(k_t(i) + \bar{k}),$$
 (8)

and (7), where we used (3) and (5) to obtain (8).

Entrepreneurs maximize utility through the optimal choice of employment of labor $l_t(i)$ and capital $k_t(i)$. It is easy to check that an interior solution requires that $w_t > \left(\frac{1-\alpha}{1+r}\right)^{\frac{1-\alpha}{\alpha}} \alpha(a^H)^{1/\alpha} \equiv \underline{w}$. We focus on this case throughout. Inserting (6), (7), and (8), respectively, in (1), stating the first order conditions, and solving for factor inputs we obtain

$$k_t(i) = \frac{\beta^{1/\eta} \left[(1 - \tau_k)(1 - \delta) \right]^{\frac{1 - \eta}{\eta}}}{\left(1 + r - (1 - \alpha) \left(\frac{\alpha}{w_t} \right)^{\frac{\alpha}{1 - \alpha}} a_t(i)^{\frac{1}{1 - \alpha}} \right)^{1/\eta}} \equiv \tilde{k}(a_t(i), w_t, \tau_k), \tag{9a}$$

¹⁰The assumption simplifies the analysis considerably. As will become apparent, it is sufficient to assume that entrepreneurs believe that their offspring has high entrepreneurial ability. When higher tax rates discourage investments (which is the focus of our quantitative analysis) the assumption gives preferential tax treatment of continued firms ($\tau_k < \tau_s$) the best chance to improve macroeconomic performance.

$$l_t(i) = \left[\frac{\alpha \cdot a_t(i)}{w_t}\right]^{\frac{1}{1-\alpha}} \tilde{k}(a_t(i), w_t, \tau_k) \equiv \tilde{l}(a_t(i), w_t, \tau_k). \tag{9b}$$

Inspection of the solution shows that the size of an inheritance (k_{t-1} or b_{t-1} , respectively) does not affect the choice of factor inputs, i.e. the size of the family firm. This outcome is a consequence of the assumed constant marginal utility from consumption. It prevents that lucky dynasties for which nature draws several a^H 's after another amass disproportionate wealth and firm sizes. This way the range of possible types of dynasties is finite and an analytical solution of the equilibrium is possible.¹¹ The size of a bequest will be "only" responsible for the decision to continue an inherited firm or not.

The size of the inheritance tax τ_k , however, may matter for the size of firms because it affects the current firm owner's desire to leave bequests and through this channel affects investment and capital accumulation. Magnitude and sign of the effect of higher taxes are generally ambiguous. On the one hand, a substitution effect reduces the incentive to invest. One the other hand, there is also a wealth effect because higher taxes reduce the net amount inherited by offsprings. For $\eta = 1$, the wealth effect exactly counterweighs the substitution effect and taxation does not affect factor inputs of a family firm. If $\eta < 1$, the substitution effect dominates and higher taxes reduce the incentive to invest into family businesses. Allowing for $\eta < 1$ we take a frequently heard anti-inheritance tax argument into account (Holtz-Eakin, 1999, Prescott, 2006). Since $\eta \leq 1$ seems to be supported empirically, and in order to limit case differentiation, we ignore in our numerical analysis the third possibility of $\eta > 1$ which would imply that higher capital taxes trigger higher investments (but see Uhlig and Yanagawa, 1996).

Finally, the size of a firm, irrespective of whether inherited or not, depends on labor costs and the ability of its owner-manager. Inspection of (9a) and (9b) shows that factor demand is inversely related to the wage rate w_t , an outcome that reflects the neoclassical shape of the production function. Inspection shows also that less able entrepreneurs lead smaller firms. Other things equal, they prefer to install less machines and employ less workers. This outcome reflects the complementarity of managerial skills and factor inputs.

3.2. Workers. A worker i who sells an inherited firm and foregoes consumption b(i) to leave a

¹¹The simplifying assumption entails the opportunity cost of a less rich wealth distribution. This cost seems to be acceptable because we focus on the problem how inheritance taxation affect efficiency of an economy. For efficiency, the distribution of wealth is possibly of second order compared to the viability of low-ability entrepreneurs and the transaction costs of firm dissolution and establishment.

bequest to his offspring consumes

$$c_t(i) = w_t + (q - \tau_s)(1 - \delta)k_{t-1}(i) + T_t - b_t(i).$$
(10)

If the worker is the offspring of a worker, he consumes

$$c_t(i) = w_t + (1 - \tau_b)(1 + r)b_{t-1}(i) + T_t - b_t(i).$$
(11)

Thus, from utility maximization of workers we obtain that an optimal bequest requires that $v'(b_t^{net}(i))(1+r)(1-\tau_b) = 1$, where $b_t^{net}(i) = (1-\tau_b)(1+r)b_t(i)$. Thus, irrespective of social provenance a worker bequeaths

$$b_t(i) = \beta^{1/\eta} [(1 - \tau_b)(1 + r)]^{(1 - \eta)/\eta} \equiv \bar{b}(\tau_b). \tag{12}$$

As for entrepreneurs, there is no long-run path dependency of wealth within dynasties since bequests do not depend on inheritances.

3.3. Exit. Heirs of family firms abandon the business if they can enjoy higher utility as a worker (and living off the receipts for the sold firm). Technically they compare utility (1) for (6), (7), (9a) and (9b) with utility (1) for (10) and (12). In conclusion, a member of family i sells an inherited firm in period t if and only if

$$g(a_t(i), w_t, \tau_k) + \Delta(1 - \delta) \cdot \tilde{k}(a_{t-1}(i), w_{t-1}, \tau_k) < w_t + B(\tau_b)$$
(13)

where

$$\Delta \equiv 1 + r - q + \tau_s - \tau_k, \qquad B(\tau_b) \equiv \frac{\eta}{1 - \eta} \cdot \bar{b}(\tau_b),$$
$$g(a_t(i), w, \tau_k) \equiv \frac{\eta}{1 - \eta} \cdot \beta^{1/\eta} \cdot \left(\frac{(1 - \tau_k)(1 - \delta)}{1 + r - (1 - \alpha) \left(\frac{\alpha}{w_t}\right)^{\frac{\alpha}{1 - \alpha}} a_t(i)^{\frac{1}{1 - \alpha}}} \right)^{(1 - \eta)/\eta}.$$

Here, B is the net utility received from making a bequest as a worker $(v(b^{net}) - b)$. Adding to it the income of a worker (w) gives us the right hand side of (13). Likewise, the first term on the left hand side, $g(a, w, \tau_k)$, is the sum of an entrepreneur's income (π) and net utility from passing on the firm $(v(b^{net}) - (1+r)k)$.

¹²For the special case of $\eta = 1$ we have to redefine $B \equiv \beta \ln[(1 - \tau_b)(1 + r)]$ and $g(a, w, \tau_k) \equiv \beta \ln[(1 - \tau_k)(1 - \delta)] - \beta \ln\left(1 + r - (1 - \alpha)\left(\alpha/w\right)^{\frac{\alpha}{1 - \alpha}} a^{\frac{1}{1 - \alpha}}\right)$.

The second term on the left hand side of (13) sums up the missing items. The parameter Δ can be conceptualized as the continuation value of a unit of business capital. If there are no transaction costs of firm dissolution (q = 1) and no tax advantage of keeping the firm $(\tau_k = \tau_s)$, then the continuation value is equal to the interest rate. Δ is increasing in transaction costs (1 - q), the tax advantage from firm continuation $(\tau_s - \tau_k)$ and the interest rate (r). Whenever $\Delta > 0$, the incentive to continue a family business increases with the size of the bequest. Note that this implies that both high-ability and low-ability heirs are more inclined to continue a family business if they have received it from a high-ability parent because, as explained above, high-ability entrepreneurs lead large firms.

Because highly able entrepreneurs generate more profits, they get more utility out of their entrepreneurship than their low-ability counterparts. To verify this observe that $g(a, w, \tau_k)$ is strictly increasing in ability a. Running a firm is also, ceteris paribus, more worthwhile if the wage rate w_t is low, i.e. cash flow and profits are high, and if the inheritance tax rate τ_k is low, i.e. utility experienced from bequeathing the firm to the offspring is high.

3.4. **Entry.** Now consider the entry decision of descendants of workers. They compare utility (1) for (7), (8), and (9a) with utility (1) for (11) and (12). Thus, an offspring of a worker-parent i becomes an entrepreneur if and only if

$$g(a_t(i), w_t, \tau_k) - (1+r)\bar{k} \ge w_t + B(\tau_b).$$
 (14)

As above, the right hand side of (14) comprises, for a worker, income plus net utility from making a bequest and the first term on the left hand side is the analogous expression for an entrepreneur. With contrast to heirs of family firms, heirs of workers cannot experience any continuation value from keeping a business. Instead they have to bear startup costs \bar{k} . Not surprisingly, higher entry costs mitigate the incentive to enter. Workers are also less inclined to enter if wages are high because then, ceteris paribus, income of entrepreneurs is low and labor income of workers is high. Inheritance taxes have, with respect to their tax base, opposing effects on entry. A higher tax rate applied to the bequests of descendants of workers (τ_b) raises the incentive to enter, whereas a higher tax rate on bequeathed firms (τ_k) reduces it.

4. Equilibrium Analysis

At a steady-state, the number of exiting and entering firms coincide. To avoid only mildly interesting case distinctions, we focus on parameter constellations such that there is entry and exit in equilibrium. Low-ability descendants of workers, however, will never enter entrepreneurship. To see this, conclude from $\Delta \geq 0$ and $\bar{k} \geq 0$ that according to (13) and (14) the incentive to set up a new firm is never larger than the incentive to continue an inherited firm for any given type of ability. Thus, in an equilibrium with exit of low-ability heirs of family firms there cannot be simultaneously entry of low-ability heirs of workers. As will become apparent, in such an equilibrium there will not be any transitional dynamics in the economy. That is, the economy immediately jumps into a steady state state where the wage rate, the distribution of assets, and the fraction of high- and low-ability entrepreneurs are stationary.

While low-ability types never enter entrepreneurship in equilibrium, high-ability types enter until the utility from setting up and running a business is driven down to the utility from wage work. This is the case when the wage equalizes utility from entering and staying out, i.e. the equilibrium wage rate w^* fulfils

$$g(a^{H}, w^{*}, \tau_{k}) - (1+r)\bar{k} = w^{*} + B(\tau_{b}).$$
(15)

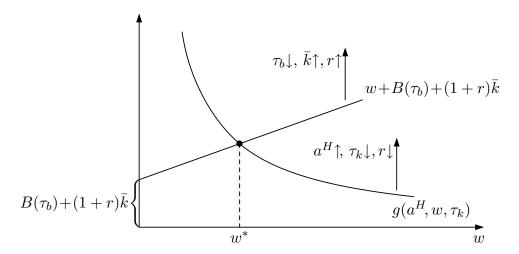
The equilibrium wage w^* is unique because $g(a^H, w, \tau_k)$ is decreasing in the wage rate whereas the utility from wage work is strictly increasing. Figure 1 visualizes the equilibrium. Utility from running a firm increases with ability and decreases with the inheritance tax. In the figure, higher ability a^H and lower taxes on firms τ_k shift the $g(a^H, w, \tau_k)$ curve upwards and the resulting higher demand and lower supply of wage work leads to an equilibrium at a higher wage rate. Likewise, higher start up costs \bar{k} and lower inheritance taxes τ_b (implying higher net utility from bequeathing B) shift the $w + B(\tau_b) + (1+r)\bar{k}$ curve upwards. A career as entrepreneur becomes less attractive and higher supply and lower demand of workers are balanced at a smaller equilibrium wage. An increase in the interest rate r shifts the $g(a^H, w, \tau_k)$ curve downwards and the $w + B(\tau_b) + (1+r)\bar{k}$ curve upwards; thus, w^* is decreasing in r.

Finally, there has to be exit. According to the exit decision rule (13) there will be exit in equilibrium if

$$g(a^L, w^*, \tau_k) + \Delta(1 - \delta)\tilde{k}(a^L, w^*, \tau_k) < w^* + B(\tau_b).$$
 (A1)

Assumption A1 ensures that low-ability descendants of entrepreneurs exit if also their parent had

Figure 1: Entry Decision and Equilibrium Wage



low ability. It is maintained throughout.

Interestingly, assumption A1 leaves scope for two structurally different equilibria, which can alternatively occur depending on the numerical specification of the model's parameters, i.e. depending on the specification of technologies, preferences, institutions, and, most importantly, the underlying inheritance tax policy. At the first equilibrium low-ability heirs of family firms always exit implying that only firms led by high-ability entrepreneurs are participating in the market. At the alternative equilibrium low-ability heirs of family firms continue the business if their parent was of high ability.

Intuitively, the likelihood that an economy is situated at the second equilibrium is high when the continuation value Δ is large. As explained, this is the case if either transaction costs entailed by the sale of capital are high (low q) or if the government rewards a high tax advantage for continued family firms, i.e. if $\tau_s - \tau_k$ is large. In other words, if transactions costs are low and/or the tax advantage is absent or low, then the continuation value is small and low-ability heirs are more inclined to sell the firm and exit immediately irrespective of their parents' ability. This reasoning implies that there exists a threshold for the continuation value below which there are only high-ability entrepreneurs present and above which the market is shared by entrepreneurs of high and low-ability. Before we show that the intuition is indeed true, we first define an equilibrium.

In an equilibrium with entry and exit (assumption A1)

• investments of entrepreneurs and bequests of workers maximize utility,

- workers with high entrepreneurial ability are indifferent whether or not to enter the market (see eq. (15)),
- descendants of entrepreneurs exit if being a worker yields higher utility than staying in the market,
- total labor demand equals supply; i.e., $\int_0^{n_t} l_t(i)di = 1 n_t$.

Let n^L and n^H denote the mass ("number") of firms led by entrepreneurs of type a^L and a^H , respectively. Using (9b) and omitting the time index, labor market clearing implies

$$n^{L} \left[\tilde{l}(a^{L}, w, \tau_{k}) + 1 \right] + n^{H} \left[\tilde{l}(a^{H}, w, \tau_{k}) + 1 \right] - 1 = 0.$$
 (16)

We denote the wage rate which is implicitly defined in (16) by $\tilde{w}(n^L, n^H, \tau_k)$. It is strictly increasing in both n^L and n^H (to see this, recall that $\tilde{l}(a, w, \tau_k)$ is decreasing in w). A larger number of entrepreneurs of either kind raises labor demand and reduces labor supply; thus, the equilibrium wage rate rises. Moreover, the effect of an increase in τ_k on \tilde{w} is negative if $\eta < 1$ and zero if $\eta = 1$. The following proposition specifies the threshold value for Δ that determines the type of equilibrium and the number of participating firms of each type. (All proofs are relegated to the Appendix).

Proposition 1. There is a threshold value

$$\hat{\Delta} \equiv \frac{w^* + B(\tau_b) - g(a^L, w^*, \tau_k)}{(1 - \delta)\tilde{k}(a^H, w^*, \tau_k)}$$
(17)

such that in equilibrium the following holds: 13

- (i) For $\Delta < \hat{\Delta}$, there are only high-ability entrepreneurs in the market (i.e., $n^L = 0$). The number of firms, $n = n^H$, is given by $\tilde{w}(0, n^H, \tau_k) = w^*$, with w^* as defined by (15). In each period, all firm-heirs who have drawn low ability, i.e., $(1 p^H) \cdot n^H$ firms, exit. (Type 1 equilibrium.)
- (ii) For $\Delta > \hat{\Delta}$, there are $n^L = (1 p^H) \cdot n^H > 0$ firms led by low-ability entrepreneurs in the market and the number of high-ability entrepreneurs, n^H , is given by $\tilde{w}((1-p^H) \cdot n^H, n^H, \tau_k) = w^*$. In each period, all descendants of low-ability entrepreneurs who have low ability themselves, i.e., $p^L \cdot n^L$ firms, exit. (Type 2 equilibrium.)

It is thus apparent that (under assumption A1) the following holds.

Corollary 1. A steady state equilibrium with entry and exit exists and is unique; there are no transitional dynamics.

¹³We will not consider the knife-edge (non-generic) case where $\Delta = \hat{\Delta}$.

The next corollary shows how preferential tax treatment of continued businesses affects the type of equilibrium assumed by an economy.

Corollary 2. Starting from a Type 1 equilibrium where $\tau_k = \tau_s$, introducing a sufficiently pronounced preferential tax treatment of continued businesses $(\tau_s > \tau_k)$ by raising tax rate τ_s induces a transition to a Type 2 equilibrium.

It is interesting to examine in which type of equilibrium there are more firms led by high-ability entrepreneurs. Using (16), this question is addressed in the next proposition.

Proposition 2. The number of firms led by high-ability entrepreneurs in an equilibrium of Type 1 and Type 2 are given by

$$n^{H} = \frac{1}{\tilde{l}(a^{H}, w^{*}, \tau_{k}) + 1} \equiv \hat{n}^{H1}, \tag{18}$$

$$n^{H} = \frac{1}{(1 - p^{H})\tilde{l}(a^{L}, w^{*}, \tau_{k}) + \tilde{l}(a^{H}, w^{*}, \tau_{k}) + 2 - p^{H}} \equiv \hat{n}^{H2},$$
(19)

respectively, where w^* is given by (15). In a Type 2 equilibrium there are more firms in total but less firms led by high-ability entrepreneurs than in a Type 1 equilibrium ($\hat{n}^{H2} < \hat{n}^{H1}$).

The result of Proposition 2 implies that tax incentives for continuing family firms, possibly established with the intention to save transaction costs entailed by firm dissolution and startup, have a negative side-effect on performance of the economy. If the economy assumes a Type 2 equilibrium as a consequence of preferential tax treatment, firms are continued although heirs have low entrepreneurial ability. This continuation deters entry of high-ability descendants of workers such that the equilibrium number of high-ability entrepreneurs is lower than without such tax incentives. Crowding out of high-ability entrepreneurs, however, is not perfect because staying low-ability heirs run smaller businesses due to the managerial skill complementarity with factor inputs. This implies that the impact of a staying low-ability entrepreneur on labor demand and the wage rate is smaller than the impact of an entering high-ability entrepreneur, i.e. $\partial \tilde{w}/\partial n^L < \partial \tilde{w}/\partial n^H$. In words, two staying low-ability heirs of family firms prevent entry of less than two descendants of workers with high ability.

The partial crowding out of high-ability descendants of workers by low-ability owners of family firms incurs a twofold burden on the economy. High-ability entrepreneurs invest more, which has a positive effect on economic performance. Furthermore high-ability entrepreneurs produce more output for any given input combination. These losses of scale and productivity do not necessarily

imply the conclusion that a continuation-friendly tax system should be abandoned (for efficiency reasons). The losses have to be compared with the potential gains from saved transaction costs. And, of course, a continuation-friendly tax policy does not automatically imply that a Type 2 equilibrium is assumed since the continuation value Δ may be still below the threshold.

Finally, note that Corollary 2 and Proposition 2 compare equilibria under the ceteris paribus condition of holding τ_k constant. A clear-cut conclusion on theoretical grounds is thus only possible if the preferential treatment of continued businesses results from a discriminatory tax increase for sold businesses. In this case, τ_s rises at constant τ_k leaving equilibrium wages and employment unaffected. Thus, if a transition from Type 1 to Type 2 equilibrium has been caused by an increasing τ_s , there will be unambiguously fewer high-ability entrepreneurs. In this case we can furthermore prove the following result concerning aggregate welfare, $\int_0^1 U(i) di$.

Proposition 3. Introducing preferential tax treatment of continued businesses, $(\tau_s > \tau_k)$ by raising tax rate τ_s , leaves aggregate welfare in a given type of equilibrium unaffected. Moreover, there exists a critical level \bar{q} such that welfare is reduced (raised) if the economy turns from a Type 1 to a Type 2 equilibrium and $q > \bar{q}$ $(q < \bar{q})$ holds.

If, however, the preferential treatment has (also) been caused by a tax cut for continued businesses τ_k , we may observe counteracting forces to the negative crowding-out of high-ability entrepreneurs, in particular if costs of firm dissolution are low $(q > \bar{q})$. This is because a decrease in τ_k affects factor inputs of entrepreneurs (directly and through raising the equilibrium wage rate w^*) and thereby may also change the number of firms in a given type of equilibrium.¹⁴ The theoretical indeterminacy in this empirically particularly relevant case makes the subsequent quantitative analysis all the more important. We thus continue with a calibration of the model in order to further assess the role of tax schemes and transaction costs on the continuation of family firms and on output, investment, and utility of the individual types of entrepreneurs and workers.

5. Calibration

With respect to family firm friendliness, our model differentiates between unconditional preferential treatment of family firms ($\tau_k = \tau_s < \tau_b$) and preferential treatment of family firms contingent on their continuation ($\tau_k < \tau_s$, $\tau_k < \tau_b$). This distinction is epitomized almost ideally by the recent inheritance tax reform in Germany, which followed an intensive debate among legislature,

¹⁴See (9a), (9b), (15), (18) and (19).

jurisdiction, and business associations. According to the inheritance tax law in place until end of year 2008, real estates and businesses received a preferential treatment vis-à-vis other forms of bequests, irrespective of their continuation. Because this procedure did not conform to the principle of equality the Federal Constitutional Court has ruled against it. The Constitutional Court left, however, the possibility to tax certain kinds of wealth transfers preferentially if this is justifiably desirable from a general economic viewpoint.

Consequently, the recently launched tax reform has abolished the general preferential treatment of family firms and has tightened depreciation and accounting rules. Arguing, however, that continuation of family firms is welfare-enhancing, the new law has also created tax incentives for heirs to continue the business of their parents. Heirs who aim at running the family firm without major reductions in the firm's wage bill for at least 7 years are eligible to reduce the tax base by 85 percent. Alternatively, if they continue the firm at least 10 years and meet strict requirements regarding the wage bill, they are fully exempted from inheritance tax. Heirs have to decide in advance which of the two options to take and cannot switch afterwards.

To examine the implications of the tax reform, we calibrate our model with German data. We start out with the situation before 2009, where real estates and businesses got a preferential treatment vis-à-vis cash, shares, bonds, and other bequests, i.e., $0 < \tau_k = \tau_s < \tau_b$. This will be our policy case 1. The preferential treatment of continued firms is approximated by two further cases. Case 2 keeps that $\tau_k < \tau_b$ but raises the tax rate for heirs who sell the firm to the level applied to other bequests, i.e., $\tau_s = \tau_b$. Case 3 assumes that not only the tax rate for sold firms is raised compared to the initial policy (case 1), but also taxes on continued firms are abolished entirely; i.e., $\tau_s = \tau_b$ and $\tau_k = 0$.¹⁵

The typical firms that we have in mind when conducting the policy experiments are the small and medium enterprises (SMEs) of the so called German "Mittelstand", which encompasses 99 percent of all German companies and employs about 70 percent of the labor force. ¹⁶ Four percent of the German population lives in entrepreneur households, which would suggest to match n to 0.04. On the other hand, the average SME owner employs 10 workers, which would suggest that n equals $1/11 \approx 0.09$. We solve this dilemma by matching n = 0.07 (which is the percentage of

¹⁵Both cases 2 and 3 are interesting from an US perspective as well. In the US, there is significant estate tax alleviation for businesses that are continued at least for 10 years and there are initiatives to abolish the tax on continued firms entirely.

¹⁶For this and the following data on Germany's SME, see Deutsche Bank (2007) and BDI (2006). According to the EU definition, a SME has less than 250 employees and sales revenue not exceeding EUR 50 Mio.

self-employed households in Germany) for the scenario prior to tax-reform (case 1).

Currently about 30 percent of Germany's family businesses are planning on a succession of the firm within the next years. According to a poll in the manufacturing sector 43 percent of firm owners state that the recent inheritance tax reform is of "very high" importance for their solution of the succession problem. A further 27 percent state that the tax reform is of "high" importance. About 30 percent of entrepreneurs are reckoning on solving the succession problem by selling or closing their firm. We thus match an (initial) exit rate of 0.3 with our calibration.¹⁷

According to a study by ZEW (2004) the market value of the average German non-corporation is 4.4 million Euros and the marginal tax rate on an inheritance of this size (if inherited by a son or daughter) is 19 percent. Yet, inherited family businesses were treated favorably under the old law. Besides the possibility to defer tax payment, family firms were also entitled to a 35 % discount of the tax base, and other forms of relief. According to ZEW's calculations the effective average inheritance tax rate on a family firm of average size was just 3.8%. For the model's calibration we are, however, interested in marginal taxes. We thus set $\tau_b = 0.19$ and account for the beneficial treatment of firm wealth by setting $\tau_k = \tau_s = \tau_b/2$ in our policy case 1.

We calibrate the marginal elasticity of utility from bequests, η , according to the estimates in Kopczuk and Slemrod (2001). The most applicable of their results is probably the correlation of the reported estates with the estate tax at ten years before death. The elasticity of the bequest with respect to 1 minus the tax rate is estimated as 0.10, implying $(1 - \eta)/\eta = 0.10$. This leads to the specification of $\eta = 0.91$ in our benchmark setup. By sensitivity analysis we take into account that Kopczuk and Slemrod have reported different estimates for alternative specifications, sometimes insignificantly different from zero, and that their study was anyway carried out with data for U.S. households. The parameter β affects the scale of the economy but leaves rates between variables unaffected (see below). It can thus not be used for calibration and we begin with setting $\beta = 1$. Likewise we thus begin by setting r = 0.2, which implies an annual riskless interest rate of about one percent (when the length of a generation is 20 years).

The specification of managerial ability and the intergenerational inheritance of managing skills are based as closely as possible on the innovative calibration of these parameters by Caselli and Gennaioli (2006). They show that for a steady-state distribution of ability the probabilities to

¹⁷Many European countries (but not the U.S.) show similar characteristics. 99 percent of the European enterprises are SMEs. The average European SME employs 6 people and 66% of the European labor force are employed in SMEs (but only 33% of the U.S. labor force). The contribution of European SMEs to GDP is about 50 percent. See European Commission (2003) and Deutsche Bank (2007).

inherit one's parent ability must fulfil $p^L = 1 - \lambda + \lambda \mu$ and $p^H = \lambda + \mu - \lambda \mu$ for a given share of high-ability types in the population λ and a given intergenerational correlation of talent μ . Based on the psychological literature about the inheritance of IQ (and hoping that transmission of managerial talent behaves not too differently) they fix $\mu = 0.4$. Interestingly this value is not too far away from Galton's (1877) famous 1/3 observed for height and other personal characteristics that are inherited by nature. Casselli and Gennaioli then set $\lambda = 0.1$ and use the implied values of p^H and p^L together with Perez-Gonzales' (2001) estimate that dynastic successions in the U.S. lead to an average decline in the return on assets of 20 percent to come up with the result that $a^H = 1.33 \cdot a^L$ (all parameter names are adjusted to the present paper's notation).

Of course, we cannot adopt all parameter values from their study because we are dealing with a different model. Since Caselli and Gennaioli admit to know relatively little about λ , the population share of managerially talented people, we take this as our "degree of freedom". We thus set $\mu = 0.4$ and $a^H = 1.33 \cdot a^L$ and take over the two equations determining p^L and p^H but use λ to adjust our model to the empirical exit ratio of 0.3.

On average the startup of a new business in Germany takes 42 days and costs 15.7 percent of GDP per capita, according to Djankov et al. (2002). In order to relate these numbers to our specification of \bar{k} we have to take into account that Djankov et al.'s figures are based on GDP per capita per year whereas our model economy produces GDP per generation. The length of a generation is best conceptualized as the length of the time period spend by a member of a dynasty of entrepreneurs as head of the business. Imagining that he has inherited the firm from his father when he was 50 years old and will bequeath the firm to his son when he is 70 gives an estimate for period length of 20 years. Comparing the monetary start-up costs with our model-GDP per capita at the initial steady-state (which is 2.33 such that the annual GDP per capita equals 2.3/20 = 0.11) we get an estimate of $\bar{k} = 0.11 \times 0.157 = 0.018$. However, we may also want to include time costs taking account of lost opportunities for the non-working firm founder. Djankov et al. estimate total startup costs, including monetary and time cost, as 32.5 percent of GDP per capita. This renders an estimate of $\bar{k} = 0.038$. We take this as our benchmark value and conduct sensitivity analysis.

A parameter we know relatively little about in the context of family firms is q. An indication about the magnitude of q can be derived from recent work by Officer (2007), who estimates that unlisted stand-alone firms and subsidiaries of other firms are sold at an acquisition discount of 15-30

percent relative to comparable publicly-traded targets. The discount seems to rise with the extent of the liquidity problems of the seller. Because in our model firms are not sold due to liquidity problems, we calibrate the discount at the lower end of the estimated range by Officer (2007). More precisely, we conduct two experiments. Given the numerical specification of the model, it turned out that the equilibrium threshold of Proposition 1 is crossed for a value of q between 0.8 and 0.85 (discount of 15-20 percent). We first consider the case of 0.85 for which the economy is situated at a Type 1 equilibrium, i.e. given the initial tax policy all firms are led by high-ability entrepreneurs. When q = 0.80 the benchmark economy is situated at a Type 2 equilibrium where the market is shared by entrepreneurs of high and low ability. Qualitatively, the first scenario captures the notion that transaction costs are relatively low and that Germany's Mittelstand entrepreneurs are of high ability. The notion that high transaction costs in the process of firm dissolution are an important structural problem and that there are also low-ability entrepreneurs present in Germany's economy is captured by the second scenario.

Following numerous previous calibration exercises we set the capital share $(1 - \alpha)$ to 0.4. We fix the remaining three parameters, the value of low-ability skills a^L , the share of the managerially talented people λ , and the depreciation rate δ so that the model matches three statistics: the share of entrepreneurs in the population (n = 0.07), the exit rate, i.e. the number of exiting entrepreneurs relative to the number of entrepreneurs (0.3 as explained above), and the wealth share hold by entrepreneurs. Denote the capital stock employed by high-ability and low-ability entrepreneurs by k^{H} and k^{L} , respectively. According to the model, the wealth share is easily be found by comparing total (after tax) wealth of entrepreneurs $(n^H k^H + n^L k^L)(1 - \delta)(1 - \tau_k)$ and total wealth of workers $(1-n)(1-\tau_b)\bar{b}$. Empirically, however, we were not able to find the appropriate statistics. In the U.S. entrepreneurs hold 40% of total wealth according to Quadrini (2000). For Germany we know that the self-employed hold 15% of all wealth (ZEW, 2005). One explanation for the unexpectedly huge cross-country difference is that not all self-employed are entrepreneurs, certainly not in the spirit of the current model. Another known statistics is that the highest decile of Germany's wealth distribution holds 47% of total wealth. Perhaps this value fits our model better. Again, we meet the parameter uncertainty by choosing an "intermediate" value of 27%. From benchmark settings we obtain the missing parameters as $a^L = 1.96$, $\lambda = 0.5$ and $\delta = 0.56$ (implying an annual depreciation rate of capital of 2.3 percent if the length of a generation is 20 years). Together with $\mu = 0.4$, $\lambda = 0.5$ implies $p^L = p^H = 0.7$. If the economy starts out in a Type 2 equilibrium this implies that

a high-ability entrepreneur at the end of his or her life holds eightfold the wealth of a low-ability entrepreneur and 12-fold the wealth of a worker.

For policy case 1, the calibration implies that \bar{q} is slightly above 0.6, which means that switching from Type 1 to Type 2 equilibrium after raising τ_s leads to a welfare gain if less than 40 percent of firm capital is lost due to dissolution of a family business (Proposition 3).

6. The Quantitative Impact of Inheritance Tax Reforms

As explained, we investigate three policy cases for an economy that is initially situated at one of two types of equilibria. For each case and type of equilibrium we evaluate several statistics: the number of firms and the number of high-ability entrepreneurs, the exit rate and entry rate, i.e. the share of descendants of workers who become entrepreneur, investment rates, and the utility experienced by the different groups in our model society. We also report the change in consumption for each group which is required to make an individual equally well-off before and after a tax reform (when case 1 constitutes the status quo policy). Moreover, we compute the rate of change of GDP and aggregate total factor productivity (TFP). Finally, we report aggregate welfare as the weighted sum of utilities where the weights are the population shares of the different groups.

6.1. Effects of Switching from Type 1 to Type 2 equilibrium. Table 1 summarizes the results when there are only high-ability entrepreneurs initially. The first row shows the performance of the economy under policy case 1. According to Proposition 1, the exit rate equals $1 - p^H$ in Type 1 equilibrium (matched to 30 percent initially, given a probability to inherit one's parent high managerial skills of $p^H = 0.7$) and $p^L n^L/n$ in Type 2 equilibrium. The entry rate is $(1 - p^H)n^H/(1 - n^H)$ in Type 1 and $p^L n^L/(1 - n)$ in Type 2 equilibrium. In policy case 1 about 2.3 percent of descendants of workers become entrepreneur. One sees also that the model is able to reflect the empirical regularity found by Gentry and Hubbard (2004) that investment of new entrants (i_W) is much higher than investment of heirs of family firms (i_E) .

In a Type 1 equilibrium all exiting entrepreneurs are low-ability sons and daughters of high-ability entrepreneurs. Their utility u_E^{HL} is significantly below that of their high-ability counterparts who continue the family business (u_E^{HH}) . But – thanks to their large inheritance – utility of low-ability heirs of family firms is significantly higher than that of their new colleagues, the sons and daughters of workers (u_W) . Recalling that exiting heirs earn the same wage and bequeath the same amount as their fellow workers, the result impressively demonstrates the high utility gained by low-ability

heirs from living off their sold family businesses.

Note also that – by construction of equilibrium – utility of descendants of workers must be independent from career choice. This in turn implies a large difference in the *composition* of their utility. For heirs of workers who remain workers utility originates to a relatively large extent from current consumption while for heirs of workers who establish a new business utility originates to a relatively large extent from bequeathing the family firm.

Table 1: Inheritance Tax Policy Scenario I: Only high-ability entrepreneurs initially

case	n	n^H	exit	entry	y	TFP	W- r	i_E	i_W	u_E^{HH}	u_E^{HL}	u_E^{LH}	u_E^{LL}	u_W	welf.
1	7.0	7.0	30.0	2.3	_	_	27.0	21.9	33.8	17.6	15.7	_	_	12.7	13.4
2	8.8	6.7	6.9	0.7	-1.9	-5.7	27.4	21.9	33.8	17.6	15.6	12.5	12.2	12.7	13.1
										(0.2)	(8.3)	(-111)	(-31)	(0.7)	
3	8.7	6.7	6.9	0.7	-1.8	-5.7	29.4	20.4	33.8	18.2	16.2	12.6	12.1	12.7	13.0
										(10)	(19)	(-110)	(-33)	(-1.3)	

Parameters: $q=0.85, \ \alpha=0.6, \ \beta=1, \ \delta=0.56, \ \eta=0.90, \ \lambda=0.5, \ \mu=0.4, \ \bar{k}=0.038, \ r=0.2, \ a^L=1.94, \ a^H=1.33a^L, \ \tau_b=0.19.$ y is the relative deviation of GDP from initial state in percent, TFP is the relative deviation of aggregate total factor productivity from initial state in percent, W-r denotes the wealth share of entrepreneurs, i_W and i_E denote investment rates workers and of high-ability entrepreneurs who are descendants of entrepreneurs, respectively, u_E^{xz} denotes utility of a z-ability entrepreneur who is descendant of an x-ability entrepreneur, $x,z\in\{H,L\}$. welf. denotes aggregate welfare. In parentheses: consumption equivalents w.r.t. to case 1. See text for further explanations.

Policy case 2, the preferential tax treatment of continued firms triggers a structural break. Motivated by tax alleviation low-ability heirs continue the family business unless they have inherited it from a low-ability parent. As a consequence, the market is now shared by high-ability and low-ability entrepreneurs. Recall that coming from case 1 our case 2 implies an increase of τ_s at constant τ_k , i.e. the scenario which has been already covered analytically by Corollary 2 and Proposition 2 and 3. Since the policy change is obviously strong enough to initiate the threshold crossing we know already from formal analysis that we can expect more firms in total, less firms led by high-ability entrepreneurs, and - since $q = 0.85 > \bar{q}$ — lower aggregate welfare. Besides staying and exiting low-ability entrepreneurs there occurs a further new species at a Type 2 equilibrium: high-ability heirs of low-ability entrepreneurs. These are the talented grandsons of talented founders and sons of untalented fathers.

Overall, we observe some crowding out of high-ability entrepreneurs, but the more pronounced effect is the increased total number of firms in the market. We observe also a huge drop of exit and entry rates showing that the continuation-friendly policy is indeed very effective with respect of preventing exit.

Unfortunately, lower exit and entry implies also less creative destruction and entails detrimental effects on output per capita and aggregate total factor productivity. On average, firms are now smaller and employ less workers per unit of capital. Columns y and TFP show the rate of change of GDP per capita and aggregate total factor productivity, respectively, relative to case 1 in percent. The low average efficiency and scale of production causes a drop of GDP by about 2 percent. Consequences on total factor productivity are even more severe; it drops by 5.7 percent relative to the initial state.

The policy change leaves utility of descendants of workers (u_W) almost unaffected. With contrast, turning towards the descendants of entrepreneurs, we see that the policy is indeed utility enhancing for a low-ability keepers of family firms (whose fathers had high ability). These persons would have chosen to become worker in policy case 1. In case 2 they derive utility $u_E^{HL} > u_W$. The utility gain for this group is equivalent to a 8.3 percent increase in the consumption level relative to that of a worker.¹⁸

It is also interesting to note that the numerical analysis reveals a large multiplier of entrepreneurial talent. Compared to their high-ability counterparts, low-ability entrepreneurs hold only $k^L/k^H = 0.11$, i.e. about a tenth of firm worth (not reported in Table 1), although they are equipped with 75 percent of the talent, according to our calibration.

The most interesting and unpleasant effects of the considered tax reforms are experienced by heirs of low-ability entrepreneurs who were made to continue a family business. This "third-generation" effect seems to be completely overlooked in the public debate. Interestingly, a considerable utility loss occurs irrespective of whether the third generation is again of low-ability and exits or whether it is of high ability and continues the family business. Both types are disadvantaged by their low inheritance because market forces caused their low-ability father to reduce the scale of the family business. Exiting entrepreneurs are now suffering not only from loss through transaction costs but also from the fact that they have inherited their firm from a low-ability parent who was caused by fiscal policy to continue the family business inherited from a high-ability grandparent. This makes the inheritance of exiting descendants of low-ability entrepreneurs and therefore their utility u_E^{LL} smaller than that of descendants of workers.

The staying high-ability heirs of low-ability fathers fare only little better than the exiting ones

¹⁸Consumption equivalents are reported in parentheses below utility. Note that heirs of low-ability entrepreneurs do not exist under case 1. We thus compute their consumption equivalents relative to workers because, in the end, they would have been workers if the policy would have made their low-ability fathers to abandon the firm.

and worse than entering entrepreneurs (workers) although the latter have to bear the startup cost. Expressed in consumption equivalents, the loss is even more dramatic than that of exiting descendants of entrepreneurs. We compute a consumption equivalent of more than 100 percent, which means that the *additional* consumption needed to make high-ability heir of a run-down firm as well off as a worker is higher than the initial level of consumption of a worker. Altogether these results strikingly demonstrate the power of a low-ability entrepreneur in driving down business wealth.

With case 3 we consider the complete abolishment of taxes on continued family firms. The case is very similar to the previous one. The economy assumes again a Type 2 equilibrium. Compared to case 1 utility improves for low-ability heirs of high-ability entrepreneurs and deteriorates for heirs of low-ability entrepreneurs irrespective of whether they continue or abandon the firm. The similarity of cases 2 and 3 indicates that the absolute size of tax change is of minor importance compared to the effect of a Type 1 – Type 2 threshold crossing.

TABLE 2: INHERITANCE TAX POLICY SCENARIO I: Mixed entrepreneurs initially

case	n	n^H	exit	entry	y	TFP	W- r	i_E	i_W	u_E^{HH}	u_E^{HL}	u_E^{LH}	u_E^{LL}	u_W	welf.
1.0	7.0	4.5	30.0	2.3	0.0	0.0	27.0	23.9	33.6	20.9	18.2	12.8	12.5	13.1	13.7
2.0	7.0	4.5	30.0	2.3	0.0	0.0	27.0	23.9	33.6	20.9	18.2	12.8	12.5	13.1	13.7
										(0.0)	(0.0)	(-0.3)	(-3.0)	(0.0)	
3.0	6.9	4.5	30.0	2.2	0.1	0.0	29.0	22.8	33.6	21.8	19.1	12.9	12.4	13.1	13.7
										(10.3)	(10.0)	(2.5)	(-5.0)	(-1.5)	

Parameters: as for Table 1 except q=0.8 which requires recalibration: $\delta=0.66, a^L=2.30, \lambda=0.10.$

6.2. Effects When the Economy Starts in Type 2 Equilibrium. We next consider the scenario where the economy is situated already in a Type 2 equilibrium initially. This situation is induced by reducing the price of sold firms from q = 0.85 to q = 0.8 so that transaction costs become large enough for the first generation of low-ability heirs to keep an inherited firm already for policy case 1. Thus, low-ability heirs keep the firm although the inheritance tax policy is not continuation-friendly. Since, for our benchmark calibration, the threshold between Type 1 and Type 2 equilibrium lies between q = 0.85 and q = 0.8, all scenarios of relatively low firm dissolution costs ($q \ge 0.85$) can be expected to be similar to that shown in Table 1 whereas all scenarios of relatively high dissolution costs ($q \le 0.8$) resemble the one shown in Table 2.¹⁹

¹⁹The fact that low-ability entrepreneurs stay implies that we have to recalibrate the three "free parameters" in order to meet the imposed statistics. The strongest effect is here on λ . For matching an exit rate of 30 percent we have to assume that only 10 percent of the population are endowed with high managerial ability. As consequence, the

Given that the economy is already at a Type 2 equilibrium initially, no tax reform induces a threshold crossing and, consequently, policy effects are comparatively small. The fact that our macroeconomic variables assume almost identical values under case 1 and case 2 suggests that a transition towards a continuation-friendly policy is indeed capable to preserve the status quo outcome if the unconditioned tax advantage of family firms has to be abandoned. The case 3 policy, i.e. the abolishment of all taxes on continued firms improves slightly welfare of heirs of high-ability entrepreneurs at the expense of workers and heirs of low-ability entrepreneurs. The continuation-friendly policy cannot affect the exit rate (at least not when there is exit according to our assumption A1, i.e. when two low-ability heirs in a row always imply exit). Thus, interestingly, at a Type 2 equilibrium a policy intended to be continuation-friendly is indeed continuation-neutral (and possibly entry-deterring).

Given the insight that an economy performs more efficiently at a Type 1 equilibrium, the question may arise whether a reversion from Type 2 to Type 1 can be induced by implementing a continuation-unfriendly policy, i.e. a tax advantage of firm dissolution. For that purposes we investigate the inverse of case 3 by assuming $\tau_k = \tau_b$ and $\tau_s = \tau_k/2$. Indeed the policy manages the threshold crossing and induces relatively large gains of GDP per capita (by 2.7 percent) and of total factor productivity (by 9.5 percent). It causes a small drop of the wealth ratio (to 24 %) and raises aggregate welfare to 13.9. The rise in welfare originates from the rise of utilities of workers and high-ability entrepreneurs at the expense of utility of low-ability heirs of high-ability entrepreneurs who are caused to abandon the family firm.²⁰

6.3. Sensitivity Analysis. A general observation is that tax reforms have only mild consequences if the economy stays at a Type 2 equilibrium before and after the tax change compared to the drastic effects of reforms that induce a threshold crossing. We therefore focus the following sensitivity analysis on the scenario where there are only high-ability entrepreneurs initially and check whether the induced threshold crossing and its effects are robust against parameter variation. Results are reported in Table 3.

We first consider robustness against substantially higher entry costs, $\bar{k} = 0.1$, implying startup costs of more than 80 percent of annual income per capita. The incentive for descendants to enter

probability to inherit one's parent high ability p^H falls from 0.7 to 0.46 whereas p^L rises to 0.94. We also have to assume that low-ability firms are generally more productive, a^L rises to 2.57.

²⁰As the policy entails an increase in τ_k and $\eta < 1$ is assumed, investments are reduced. From an intuitive point of view, continuation-unfriendly policy would be even more desirable if it were not affecting investments. This would be the case if entrepreneurs took tax rate τ_s rather than τ_k into account in their investment decision.

entrepreneurship declines, implying lower equilibrium wages (see Figure 1). There is a small loss of aggregate welfare due to the increasing sunk costs of firm foundation, but the impact of tax reforms is very similar to that reported in Table 1.

Next we discuss the effect of varying the marginal elasticity of utility from bequests. For $\eta=0.99\approx 1$ income and substitution effects of changes in tax rates on investments and bequests approximately balance each other. Thus, also when switching from polity 1 to policy 3, all effects operate through exit and entry. Structurally results remain very similar to the benchmark case indicating, once more, that exit and entry are the dominating effect of policy reforms in Type 1 equilibria. For $\eta=0.75$ we observe similar results.

We next consider alternative assumptions about the intergenerational correlation of managing ability, μ . If we would recalibrate the model to match the statistics there would no change of policy effects at all because λ would adjust. In order to provoke an effect we fix λ at its original value ($\lambda = 0.5$) implying that the model now predicts higher exit rates for lower μ and lower exit rates for higher μ . We consider the limiting case without any inheritance of managing ability ($\mu = 0$) and a very high intergenerational correlation of ability ($\mu = 0.8$). The model predicts a lower loss of GDP and TFP from firm continuation if the IQ correlation is very strong and a higher loss if there is no IQ correlation. Structurally, however, all variables react to tax reforms as for the benchmark calibration. In particularly welfare and individual utilities react as predicted by the benchmark model.

Furthermore, we consider a higher differential between high and low-ability entrepreneurs. For the model to match the statistics we have to recalibrate $a^L = 1.61$, i.e. low-ability entrepreneurs are less talented. Quite intuitively the model predicts larger losses of productivity and GDP from a threshold crossing. Otherwise, results remain unaffected.

Next we consider a significantly higher tax on inheritances other than family firms (τ_b is raised from 0.19 to 0.35). While, of course, aggregate welfare is decreasing in the degree of distortionary taxation, results remain structurally identical to the benchmark case.

The next segment of Table 3 document that our results are robust to lower interest rates. However, if the interest raises significantly (above 0.25) and other parameters are kept, then the Type 1 equilibrium disappears, for two reasons. First, recall that the continuation value per unit of capital Δ is increasing in r, making it more attractive for low-ability descendants of entrepreneurs to stay in the market. Second, an increase in r implies that the equilibrium wage rate declines (recall that $\partial w^*/\partial r < 0$), which gives an additional incentive to continue a business.

Finally, we report results for alternative weights of bequests in utility (β). Of course, this has tremendous effects on the overall magnitude of utility and welfare. The structure of all our results, and predicted magnitude of GDP and TFP change, however, is unaffected.

Summarizing, aggregate welfare, GDP and aggregate productivity under the continuation-friendly policies 2 and 3 are never higher than under the non-preferential policy 1. These indicators remain almost unchanged if the continuation-friendly policy is ineffective with respect to continuation, i.e. when it does not manage to motivate heirs of family firms to keep the business. If it is effective, macroeconomic performance is lower than under the unconditionally firm-friendly policy 1 in all numerical specifications of the model investigated. In other words, the effects of saved transaction-costs from firm continuation never dominate the effects of prevented creative destruction. Moreover, there is a robust negative "third-generation" effect indicating substantial welfare losses for heirs of low-ability entrepreneurs who were caused to continue a family business by tax allowances.

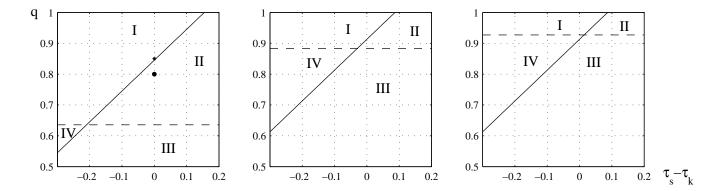
6.4. **Optimal Tax Structure.** In this subsection we discuss two questions that may arise from our results. First, we ask how large a tax disincentive of firm continuation should be to cause an efficient firm sale of low-ability heirs. Second, we investigate whether it can be that implementing a continuation-friendly tax policy raises welfare. That this is at least a theoretical possibility is established in Proposition 3 by showing that welfare increases for a threshold crossing from Type 1 to Type 2 equilibrium if firm dissolution costs are sufficiently high $(q < \bar{q})$. The question is thus whether such a case can actually occur for plausible parameter values of the calibrated model.

Figure 2 helps to answer both questions. It shows four regions of parameter combinations between the tax advantage to continue a firm $(\tau_s - \tau_k)$ and the price per unit of sold capital (q). The dotted line represents the critical $q = \bar{q}$ which separates the case when it is efficient to be in a Type 1 equilibrium (above \bar{q}) from the case where it is efficient to be in a Type 2 equilibrium. The solid line shows for a given tax difference $\tau_s - \tau_k$ the critical value of q above which the economy is actually in a Type 1 equilibrium.²¹

The Segments I - IV indicate which type of equilibrium is socially desirable (i.e., maximizes aggregate welfare) and which type of equilibrium emerges. In segment I it is desirable to be in

²¹According to Proposition 1 and the definition of the continuation value per unit of business capital, $\Delta = 1 + r - q + \tau_s - \tau_k$, the solid line is given by $q = 1 + r - \hat{\Delta} + \tau_s - \tau_k$, where threshold value $\hat{\Delta}$ is given by (17).

Figure 2: Tax Advantage of Firm Continuation and Actual and Efficient Type of Equilibrium



The dotted line shows the critical $q=\bar{q}$ separating the case when it is efficient to be in a Type 1 equilibrium (above \bar{q}) from the case where it is efficient to be in a Type 2 equilibrium. The solid line shows the critical q above which the economy actually is in a Type 1 equilibrium. The middle panel shows the hypothetical case of $\bar{k}=1.5$ and r=0 and the right panel shows the case for $\bar{k}=1.5$, r=0 and $\lambda=0.9$, $\mu=0$. All other parameters as specified below Table 1.

a Type 1 equilibrium, which is also the equilibrium type actually prevailing. In segment II it is desirable to be in Type 1 but the economy is in Type 2 equilibrium, etc. The star in the left panel indicates the policy case 1 analyzed in Table 1, where $\tau_s = \tau_k$ and Type 1 equilibrium prevails, whereas the bullet refers to the initial situation in Table 2. Moving from the star to the right of the solid line, by introducing a sufficiently high tax advantage $\tau_s - \tau_k$, means that the equilibrium switches and welfare declines (policy cases 2 and 3 in Table 1). Likewise, moving from the bullet to the left into segment I by a tax reform which deters continuation ($\tau_s < \tau_k$) raises welfare, in line with the discussion in subsection 6.2.

Observe that the two lines in the left panel intersect at $\tau_s - \tau_k < 0$. This confirms that for our baseline calibration there is no possibility to improve welfare by introducing a continuation-friendly tax policy $\tau_s > \tau_k$. For this to occur we would have to switch from the inefficient segment IV (Type 1 equilibrium prevails although Type 2 would be optimal) to segment III (where a Type 2 equilibrium is desired and prevails). If $q < \bar{q}$ it suffices that the tax deterrence for firm continuation is not too high in order to be in the desired Type 2 equilibrium. However, if $q > \bar{q}$, welfare maximization entails that the tax policy is sufficiently continuation-unfriendly. For instance, if q = 0.75, the tax rate for continued firms should be at least 10 percentage points higher than the tax rate on sold firms $(\tau_s - \tau_k \le -0.1)$.

To give the introduction of a continuation-friendly policy a chance to be welfare-enhancing, we

consider in the middle panel a large increase in entry costs \bar{k} , a measure that makes entry less efficient, accompanied by a large reduction of the interest rate, a measure that makes refinancing less costly for third-generation, high ability heirs of low ability entrepreneurs. These measures raise the threshold level \bar{q} . Intuitively, firm continuation becomes more desirable because it saves transaction costs of firm foundation. As Figure 2 reveals, however, an increase of \bar{k} shifts also the solid line upwards, indicating that for a given tax difference $\tau_s - \tau_k$ a higher q is required for the economy to be in Type 1 equilibrium. In words, not only does a Type 2 equilibrium become desirable but it is also more likely to occur. Consequently, again, the two lines intersect at $\tau_s - \tau_k < 0$, which means that there is no scope for continuation-friendly policy to be welfare-enhancing.

The situation changes in the right panel, where the two lines cross at a positive tax difference $\tau_s - \tau_k$. To construct this case, we had to introduce two additional modifications of the calibrated economy: we assumed a much larger value for λ and set $\mu = 0$. Under these assumptions it is very likely that heirs of low ability entrepreneurs are of high ability. Maintaining the assumption of very high entry costs and very little costs of refinancing a run-down firm, it can under certain parameter constellations be welfare enhancing to make low ability heirs to continue the firm through preferential taxation. Interestingly this would be a world where managing skills are abundant but not inherited with the firm.²²

In sum, although our theoretical analysis shows that giving tax incentives for heirs to continue businesses could in principle be welfare-enhancing, the numerical analysis reveals that this outcome requires some extreme parameter values. The opposite case that a continuation-friendly policy turns out to be harmful is much more likely given reasonable values of the underlying parameters.

7. Concluding Remarks

In this paper we have analyzed the consequences of inheritance tax reforms on the career decision of individuals and on the aggregate performance of economies. We have argued that the transaction-cost channel, which is mostly emphasized in the public debate, interacts with a creative-destruction channel. This way, the continuation of family firms does not only save agency- and start-up costs through less firm dissolution and less setup of new firms, it may also lead to less entry into entrepreneurship by highly able descendants of workers. Within a general equilibrium model we have shown analytically that there are two types of equilibria – one where firms are exclusively led

²²We experimented with changes of other parameters to create this situation, but never succeeded.

by highly able entrepreneurs and one where the market is shared by entrepreneurs of high and low ability – and that the design of inheritance taxes has the power to influence the type of equilibrium that an economy assumes.

Using a numerical implementation of the model we have quantitatively investigated the consequences of tax reforms which treat continued and dissolved businesses differently. We have found that the preferential treatment of continued family firms is either ineffective or affects economic performance considerably, depending on whether it causes a threshold crossing of general equilibrium. If introducing preferential taxation contingent on continuation induces a switch of the equilibrium type, we estimate that gross income generated by family firms shrinks by about two percent. As the contribution of European SMEs to GDP is about 50 percent, this suggests a non-negligible loss of per capita income. Also entry into entrepreneurship is impeded considerably. The most dramatic consequence of the reform is a "third-generation" effect which seems to be completely overlooked in the debate on taxation of inheritances and estates. Welfare of the exiting sons and daughters of low-ability entrepreneurs is cut down substantially irrespective of whether they are themselves of high ability and rebuilt the family firm or of low ability and exit into wage work. If low-ability entrepreneurs continue the business and run down family wealth by investing too little (which is the optimal choice given their low entrepreneurial skills), they make their heirs worse off than the descendants of workers.

Given that our sensitivity analysis has confirmed these results to be robust against parameter variations, we feel save to conclude that our theory does not support preferential tax treatment of continued firms as, for example, suggested by the European Commission and as currently implemented or debated in many countries. Having said this, some qualifications regarding the magnitude of effects are in order. So far, we only managed to derive our results under some simplifying assumptions. Most notably, the empirical distribution of entrepreneurial ability is certainly not bivariate but continuous. However, as long as there are heirs of superior and inferior management skills and as long as a continuation-friendly policy causes some less talented heirs to continue a business, the general mechanism developed in this paper is still at work.

Secondly, a more general utility function would make inheritances path-dependent. If a lucky dynasty experiences several generations of highly able entrepreneurs in a row it may amass disproportionately big fortunes and additional wealth effects occur that are currently ignored. These distributional consequences may be of second order for our theory, however, which is based solely

on efficiency arguments. Anyway, given the empirical evidence mentioned in the introduction regarding the performance of entrepreneurs of second and third generation, several lucky draws in a row seem to be more exception than rule.

Thirdly, we have not captured existence and changes of exemption levels for the bequest tax. Exemptions are currently quite high in the US (but comparatively low in Germany). Although we expect the qualitative effects of continuation friendly tax policy to be similar to the ones obtained in the present study, it could be interesting to examine the quantitative effects of changes in exemption levels contingent on firm continuation in addition to changes in marginal tax rates.

In order to limit the length of this article we have not discussed the possibility that untalented heirs transfer control to hired talented managers. In the associated working paper, Grossmann and Strulik (2008), we show that allowing for external management leaves both our theoretical results and the quantitative analysis on aggregate income, welfare, and the "third-generation" effect unaffected. Intuitively, in an equilibrium with entry and exit it cannot be that external managers are available simultaneously at the same cost as ordinary workers and at the same quality as high-ability entrepreneurs (firm-founders). This fact explains why indeed so many SME's are managed by family members.²³ It also implies that, although low-ability heirs are (unsurprisingly) better off with external management, they nevertheless continue to crowd out firm foundation of high-ability descendants of workers. In others words, the creative destruction channel is still dominating.²⁴

Nevertheless, instead of claiming that the proposed theory should be the last word on the theoretical and empirical investigation of tax induced firm-continuation, we view it as a first step into this new field. Extensions getting rid of the simplifications mentioned above are interesting (yet challenging) tasks for future research. Other interesting further developments could result from the introduction of psychological and sociological elements, for example amenities (ego-rents) from "being entrepreneur", the intergenerational transmission of family values, and peer pressure (from the parent generation) to carry on the business.

²³In Germany's manufacturing sector about 85 percent of all firms are family owned and managed. Of these, 90 percent are fully family-owned (BDI, 2006).

²⁴In Grossmann and Strulik (2008), we also extend the model to a simple endogenous growth framework. This allows us to account for negative growth effects when high-ability entrepreneurs are crowded out by continuation-friendly tax policy.

APPENDIX

Proof of Proposition 1. First, suppose $\Delta < \hat{\Delta}$, which is equivalent to

$$g(a^{L}, w^{*}, \tau_{k}) + \Delta(1 - \delta)\tilde{k}(a^{H}, w^{*}, \tau_{k}) < w^{*} + B(\tau_{b}).$$
(20)

In this case, in equilibrium it is attractive for low-ability descendants of entrepreneurs to exit even if the parent had high ability, according to (13). Thus, all low-ability descendants of entrepreneurs exit, such that $n^L = 0$. This implies $\tilde{w}(0, n^H, \tau_k) = w^*$ in equilibrium. Moreover, as the probability of a high-ability entrepreneur to obtain low ability is $1 - p^H$, the number of exiting firms is $(1 - p^H)n^H$. Second, suppose $\Delta > \hat{\Delta}$, which is equivalent to

$$g(a^{L}, w^{*}, \tau_{k}) + \Delta(1 - \delta)\tilde{k}(a^{H}, w^{*}, \tau_{k}) > w^{*} + B(\tau_{b}).$$
(21)

In this case, low-ability descendants of entrepreneurs remain in the market if their parent had high ability. According to assumption A1, they exit if their parent had low ability. Thus, $p^L n^L$ firms exit, as p^L is the share of low-ability entrepreneurs with low-ability offspring. On the other hand the probability that a high-ability entrepreneur has a low ability offspring who continues the family business is $1-p^H$. This implies $n^L = (1-p^H)n^H$ which together with $\tilde{w}(n^L, n^H, \tau_k) = w^*$ implicitly defines the number of high-ability entrepreneurs in equilibrium.

Proof of Corollary 1. First, note that $\lim_{w\to\underline{w}} g(a,w,\tau_k) \to \infty$. As $g(a,w,\tau_k)$ is strictly decreasing in w and the right-hand side of (14) is strictly increasing in w without bound, there exists a unique $w^* > 0$ as given by (15). One can also show that $\lim_{w\to\underline{w}} \tilde{l}(a,w,\tau_k) \to \infty$ and $\lim_{w\to\infty} \tilde{l}(a,w,\tau_k) = 0$. Using (16), this implies that both $\tilde{w}(0,n,\tau_k)$ and $\tilde{w}((1-\lambda)n,n,\tau_k)$ are increasing as function of n without bound. Observing Proposition 1 confirms the result.

Proof of Corollary 2. Introducing preferential tax treatment of continued businesses $(\tau_k < \tau_s)$ by raising τ_s implies that the continuation value per unit of capital, Δ , rises whereas threshold value $\hat{\Delta}$ remains unchanged, according to (17). Applying Proposition 1 confirms the result.

Proof of Proposition 2. For the Type 1 equilibrium, (18) follows from (16) and $n^L = 0$ (recall part (i) of Proposition 1), where w^* is given by (15). In Type 2 equilibrium, (16) and $n^L = (1 - p^H)n^H$ (recall part (ii) of Proposition 1) imply $(1 - p^H)n^H \left[\tilde{l}(a^L, w, \tau_k) + 1\right] + n^H \left[\tilde{l}(a^H, w, \tau_k) + 1\right] = 1$, which leads to (19). From (18) and (19) we find $\hat{n}^{H1} > \hat{n}^{H2}$. Moreover, in type 2 equilibrium, where $n^L = (1 - p^H)\hat{n}^{H2}$, the total number of firms is given by $(2 - p^H)\hat{n}^{H2}$. It is easy to show that $(2 - p^H)\hat{n}^{H2} > \hat{n}^{H1}$ if and only if $\tilde{l}(a^H, w^*, \tau_k) > \tilde{l}(a^L, w^*, \tau_k)$, which holds according to (9b). This concludes the proof.

Proof of Proposition 3. Denote utility of a high-ability heir of a high-ability entrepreneur (staying in the market in both types of equilibria) by u_E^{HH} and utility of workers (which in equilibrium equals utility of high-ability heirs of workers who become entrepreneurs) by u_W . Moreover, recall that B is the net utility received from making a bequest as a worker $(v(b^{net}) - b)$, and $g(a, w, \tau_k)$ is the sum of the profit of an entrepreneur with ability a and net utility from passing

on the firm $(v(b^{net}) - (1+r)k)$. Taking into account the amount of bequests received as well as transfers, in view of utility function (1), we find that in equilibrium

$$u_E^{HH} = g(a^H, w^*, \tau_k) + (1 + r - \tau_k)(1 - \delta)\tilde{k}(a^H, w^*, \tau_k) + T, \tag{22}$$

$$u_W = w^* + (1 - \tau_b)(1 + r)\bar{b}(\tau_b) + B(\tau_b) + T.$$
(23)

Low-ability descendants of an entrepreneur with high ability derive utility

$$u_{E,1}^{HL} = w^* + (q - \tau_s)(1 - \delta)\tilde{k}(a^H, w^*, \tau_k) + B(\tau_b) + T$$
(24)

in Type 1 equilibrium and

$$u_{E,2}^{HL} = g(a^L, w^*, \tau_k) + (1 + r - \tau_k)(1 - \delta)\tilde{k}(a^H, w^*, \tau_k) + T$$
(25)

in Type 2 equilibrium.

Denote the aggregate welfare level, $\int_0^1 U(i)di$, in Type 1 and Type 2 equilibrium by W_1 and W_2 , respectively. We start by deriving W_1 . In Type 1 equilibrium, there are $p^H \hat{n}^{H1}$ and $(1-p^H)\hat{n}^{H1}$ high-ability and low-ability descendants of (high-ability) entrepreneurs, respectively. Thus,

$$W_1 = p^H \hat{n}^{H1} u_E^{HH} + (1 - p^H) \hat{n}^{H1} u_{E,1}^{HL} + (1 - \hat{n}^{H1}) u_W.$$
 (26)

Substituting expressions (22)-(24) into (26), observing that tax revenue (per capita) equals transfer T, using from (15) that $g(a^H, w^*, \tau_k) = w^* + B(\tau_b) + (1+r)\bar{k}$, and rearranging terms gives us

$$W_{1} = w^{*} + B(\tau_{b}) + p^{H} \hat{n}^{H1} (1+r) \bar{k} + (1-\hat{n}^{H1}) (1+r) \bar{b}(\tau_{b}) + (1-\delta) \tilde{k}(a^{H}, w^{*}, \tau_{k}) \left[(1+r) p^{H} + q(1-p^{H}) \right] \hat{n}^{H1}.$$
(27)

In Type 2 equilibrium, there are $p^H \hat{n}^{H2}$ and $(1-p^H)\hat{n}^{H2}$ high-ability and low-ability descendants of high-ability entrepreneurs, respectively. Moreover, there are $p^L n^L$ and $(1-p^L)n^L$ low-ability and high-ability individuals who inherit a firm from a low-ability entrepreneur, respectively. Their respective utility levels are

$$u_E^{LL} = w^* + (q - \tau_s)(1 - \delta)\tilde{k}(a^L, w^*, \tau_k) + B(\tau_b) + T, \tag{28}$$

$$u_E^{LH} = g(a^H, w^*, \tau_k) + (1 + r - \tau_k)(1 - \delta)\tilde{k}(a^L, w^*, \tau_k) + T.$$
(29)

Noting that $n^L = (1 - p^H)n^H$ in equilibrium, we find that

$$W_{2} = p^{H} \hat{n}^{H2} u_{E}^{HH} + (1 - p^{H}) \hat{n}^{H2} u_{E,2}^{HL} + p^{L} (1 - p^{H}) \hat{n}^{H2} u_{E}^{LL} + (1 - p^{L}) (1 - p^{H}) \hat{n}^{H2} u_{E}^{LH} + \left[1 - (2 - p^{H}) \hat{n}^{H2}\right] u_{W}.$$

$$(30)$$

Analogously to the derivation of (27), this can be rewritten as

$$W_{2} = (1 - (1 - p^{H})\hat{n}^{H2}) \left[w^{*} + B(\tau_{b}) \right] + (1 - \delta)(1 + r)\tilde{k}(a^{H}, w^{*}, \tau_{k})\hat{n}^{H2} + (1 - \delta)(1 + r)\tilde{k}(a^{L}, w^{*}, \tau_{k})(1 - p^{H})(1 - p^{L} + qp^{L})\hat{n}^{H2} + (1 - p^{H})\hat{n}^{H2}g(a^{L}, w^{*}, \tau_{k}) + \left[p^{H} + (1 - p^{L})(1 - p^{H}) \right] \hat{n}^{H2}(1 + r)\bar{k} + \left[1 - (2 - p^{H})\hat{n}^{H2} \right] (1 + r)\bar{b}(\tau_{b}).$$

$$(31)$$

Note that neither W_1 nor W_2 depends on τ_s , which confirms the first part of Proposition 3. To decide in which type of equilibrium welfare is higher (for given tax rates), use (27) and (31) together

with (15) to find that $W_1 < (>)W_2$ if and only if $q < (>)\bar{q}$, where

$$\bar{q} \equiv \frac{1}{N} \left([\hat{n}^{H2} - p^H \hat{n}^{H1}] (1+r) [(1-\delta)\tilde{k}(a^H, w^*, \tau_k) + \bar{k}] + (1-p^H)(1-p^L)\hat{n}^{H2} (1+r) [(1-\delta)\tilde{k}(a^L, w^*, \tau_k) + \bar{k}] - (1-p^H)\hat{n}^{H2} [g(a^H, w^*, \tau_k) - g(a^L, w^*, \tau_k)] + [\hat{n}^{H1} - (2-p^H)\hat{n}^{H2}] (1+r)\bar{b}(\tau_b) \right).$$
(32)

with

$$N \equiv (1 - p^{H})(1 - \delta) \left[\hat{n}^{H1} \tilde{k}(a^{H}, w^{*}, \tau_{k}) - p^{L} \hat{n}^{H2} \tilde{k}(a^{L}, w^{*}, \tau_{k}) \right].$$
 (33)

Note that, according to Proposition 2, we have $\hat{n}^{H1} > \hat{n}^{H2}$ such that N > 0. This concludes the proof. \blacksquare

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Table 3: Sensitivity Analysis

		**					****	***	* **				
case	n	n^H	exit	entry	y	TFP	u_E^{HH}	u_E^{HL}	u_E^{LH}	u_E^{LL}	u_W	welf.	
high entry costs ($\bar{k} = 0.1$)													
1	7.0	7.0	30.0	2.3	0.0	0.0	17.6	15.6	-	-	12.6	13.3	
2	8.8	6.7	6.9	0.7	-1.9	-5.7	17.6	15.6	12.5	12.1	12.6	13.0	
3	8.7	6.7	6.9	0.7	-1.8	-5.7	18.2	16.2	12.7	12.1	12.6	13.0	
	no effect of tax on size of bequest $(\eta = 0.99)$												
1	7.0	7.0	30.0	2.3	0.0	0.0	106.6	104.7	-	-	101.7	104.3	
2	8.8	6.7	6.9	0.7	-1.9	-5.7	106.6	104.6	101.5	101.2	101.7	102.6	
3	8.8	6.7	6.9	0.7	-1.9	-5.7	107.2	105.1	101.7	101.2	101.7	102.6	
large effect of tax on size of bequest $(\eta = 0.75)$													
1	7.0	7.0	30.0	2.3	0.0	0.0	10.4	8.6	-	-	5.6	6.1	
2	8.8	6.8	6.9	0.7	-1.9	-5.7	10.5	8.6	5.3	5.0	5.6	5.9	
3	8.5	6.6	6.9	0.6	-1.6	-5.7	11.3	9.3	5.5	5.0	5.6	5.9	
no intergenerational correlation of IQ $(\mu = 0)$													
1	7.0	7.0	50.0	3.8	0.0	0.0	17.6	15.7	-	-	12.7	13.6	
2	9.9	6.6	16.7	1.8	-3.1	-8.3	17.6	15.6	12.5	12.1	12.7	13.1	
3	9.8	6.5	16.7	1.8	-3.0	-8.3	18.2	16.2	12.6	12.1	12.7	13.2	
high intergenerational correlation of IQ ($\mu = 0.8$)													
1	7.0	7.0	10.0	0.8	0.0	0.0	17.6	15.7	_	_	12.7	13.1	
2	7.6	6.9	0.9	0.1	-0.7	-2.3	17.6	15.6	12.5	12.2	12.7	13.0	
3	7.5	6.9	0.9	0.1	-0.6	-2.3	18.2	16.2	12.6	12.1	12.7	13.0	
	higher ability differential $(a^H/a^L=1.6)$												
1	7.0	7.0	30.0	2.3	0.0	0.0	17.6	15.7	_	_	12.7	13.4	
2	8.8	6.8	6.9	0.7	-2.0	-8.7	17.6	15.3	12.3	12.0	12.7	13.1	
3	8.8	6.7	6.9	0.7	-1.9	-8.7	18.2	15.9	12.4	12.0	12.7	13.1	
				h	igher	initial	$\tan (\tau_b)$	= 0.35)					
1	7.0	7.0	30.0	2.3	0.0	0.0	16.4	14.7	_	_	12.4	13.0	
2	8.8	6.7	6.9	0.7	-1.9	-5.7	16.4	14.4	12.3	12.0	12.4	12.7	
3	8.6	6.6	6.9	0.7	-1.7	-5.7	17.4	15.4	12.6	11.9	12.4	12.8	
				lo	wer in	nterest	rate (r	= 0.15))				
1	7.0	7.0	30.0	2.3	0.0	0.0	17.0	15.5	_	_	12.6	13.2	
2	8.8	6.7	6.9	0.7	-1.9	-5.7	17.0	15.0	12.4	12.1	12.6	12.9	
3	8.7	6.7	6.9	0.7		-5.7	17.6	15.6	12.5	12.0	12.6	12.9	
				less i	mport	ance of	beques	$\operatorname{st} (\beta =$	0.8)				
1	7.0	7.0	30.0		0.0	0.0	13.8	12.3		_	9.9	10.4	
$\overline{2}$	8.8	6.7	6.9	0.7	-1.9	-5.7	13.8	12.2	9.8	9.5	9.9	10.2	
3	8.7	6.7	6.9	0.7	-1.8	-5.7	14.3	12.7	9.9	9.5	9.9	10.2	
								est $(\beta =$					
1	7.0	7.0	30.0	2.3	0.0	0.0	29.5	26.4	-	_	21.3	22.4	
$\frac{1}{2}$	8.8	6.7	6.9	0.7	-1.9	-5.7	29.5	26.1	20.9	20.4	21.3	21.9	
3	8.7	6.7	6.9	0.7	-1.8	-5.7	30.6	27.1	21.2	20.4	21.3	21.9	
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