Preferences, Income, and Life Satisfaction: An Equivalence Result

Holger Strulik*

First Version: August 2013. This Version: April 2014.

Abstract. In this paper I investigate the nexus between life time utility (life satisfaction) and income predicted by the standard model of endogenous economic growth under different behavioral assumptions. The solution rationalizes why the empirical association between income and life satisfaction is approximately log-linear. I show that the solution is observationally equivalent when individuals compare their consumption (i) with others, (ii) with their own past consumption achievements, and (iii) not at all (ordinary preferences). This finding suggests that the observed slope of the income – life satisfaction curve is uninformative about the presence and strength of habits or reference-dependent utility. The theory offers also an explanation why empirical studies have difficulties to identify a positive impact of long-run economic growth on life satisfaction.

Keywords: status; habit formation; happiness; economic growth.
JEL: D90, E21, O40.

* University of Göttingen, Department of Economics, Platz der Göttinger Sieben 3, 37073 Göttingen, Germany; email: holger.strulik@wiwi.uni-goettingen.de.

1. Introduction

Early research on the nexus between income and life satisfaction (or happiness) focussed on a piece-wise linear relationship between both variables and concluded that at sufficiently high income levels, additional income fails to buy more happiness (e.g. Easterlin, 1974; Frey and Stutzer, 2002; Layard, 2003). Inspired by these results economists introduced reference-dependent preferences into economic models. Recent research on the nexus between income and life satisfaction, in contrast, suggests that the relationship is log-linear (Deaton, 2008, Stevenson and Wolfers, 2008) and that there is no point of satiation (Stevenson and Wolfers, 2013). From these observations it has been concluded that relative income comparisons are of lesser importance for individual life satisfaction than previously thought (Stevenson and Wolfers, 2008).[1]

The flattening of the income-happiness curve, however, could as well be a simple consequence of ordinary preferences, represented by a concave utility function. In fact the empirical association of income and life satisfaction may be uninformative about the presence and strength of reference-dependent preferences. In order to derive this result I follow Carroll, Overland, and Weil (1997, 2000) who introduced comparison utility as formalized by Abel (1990) into a standard endogenous growth model.[2] I show that the model can be solved to provide an explicit solution for life time utility (life satisfaction) as a function of current income. The solution fits the cross-country data on average income and average life satisfaction reasonably well and supports the assumption that the association between these variables is approximately log-linear. The slope of the income-utility curve, however, supports equally well reference-dependent and ordinary preferences. Since the cardinal utility function is only defined up to an affine-linear transformation, the result suggests that it is not feasible to derive conclusions about the strength of status concerns or consumption habits from the empirical association of income and life satisfaction.

[1] I follow the happiness literature and regard the empirical observation of life satisfaction from surveys as a reasonable approximation of life time utility (Stutzer and Frey, 2004; Clark et al., 2008, Stevenson and Wolfers (2008). Most of the economics literature uses happiness and life satisfaction interchangeably but it has been argued that happiness describes better the instantaneous component of subjective well-being while life satisfaction is the more appropriate measure of its evaluative, long-term component (Deaton, 2008; Stevenson and Wolfers, 2008).

[2] The model has become a kind of benchmark for theoretical and quantitative analyses of the impact of consumer reference stocks on growth and has been developed further in several other papers, for example, Futagami and Shibata (1998), Grossmann (1998), Fisher and Hof (2000), Corneo and Jeanne (2001), Alvarez-Cuadrado et al. (2004), Cozzi (2004), and Alonso-Carrera et al. (2005) The issue of the present paper, however, remained unexplored.

2. Setup of the Model and Steady-State Solution

This section contains a brief recap of Carroll et al.'s model. Consider an economy populated by identical households (of measure one) maximizing an infinite stream of utility derived from consumption c_i relative to a reference stock h_i. Instantaneous utility is given by $u_i = \left[c_i^{1-\gamma}(c_i/h_i)^{\gamma}\right]^{1-\sigma}/(1-\sigma)$, where σ denotes the coefficient of relative risk aversion, $\sigma \neq 1$, and $\gamma \in (0,1)$ measures the strength of reference-based consumption.[3] The implied intertemporal (life-time-) utility is given by

$$V(t) = \int_t^{\infty} \frac{(c_i/h_i^{\gamma})^{1-\sigma}}{1-\sigma} \cdot \mathrm{e}^{-\theta(\tau-t)}\mathrm{d}\tau, \tag{1}$$

in which θ denotes the rate of pure time preference. The maximized intertemporal utility will be called life satisfaction. In order to square this approach with finite individual life one needs, as usually, an altruistic head of a dynasty. Consumption comparisons may be formed with respect to household i's own past consumption (inward-looking preferences) or with respect to consumption of others (outward-looking preferences). In the latter case the reference stock evolves according to

$$\dot{h}_i = \rho(c - h_i). \tag{2}$$

The case of inward looking preferences implies the law of motion

$$\dot{h}_i = \rho(c_i - h_i). \tag{3}$$

The speed at which the reference stock adjusts to current consumption is given by ρ. The larger ρ the more important is consumption of the recent past. The important difference between (2) and (3) is that there is strategic interaction between consumption and habits in the case of inward-looking preferences but not in the case of outward-looking preferences. When making their utility maximizing consumption plans, individuals with inward-looking preferences take the feedback of their consumption c_i on the evolution of their habit stock into account. Individuals with outward-oriented preferences take consumption of others, which equals average consumption c, as given.

The equation of motions (2) and (3) can also be interpreted differently, taking into account the debate about whether individuals are aware of the adjustment of reference stocks (Clark

[3] If there were no long-run growth, then $c = h$ at the steady state. In this sense γ is the share of consumption which matters only in relative terms, either compared to the consumption of others or compared to own achievements.

et al., 2008). Equation (3) would then capture the case of outward oriented preferences when individuals are realizing the fact that the Joneses are just like them and will thus follow the same consumption strategy. Equation (2) would represent inward looking consumers which are not aware of the adjustment of their consumption habits.

Output is produced using capital and a linear production function such that income of household i is $y_i = Ak_i$ where k_i is capital (wealth) of household i and A is capital productivity. Capital depreciates at rate δ, implying that the capital stock of household i evolves according to $\dot{k}_i = (A-\delta)k_i - c_i$. In case of inward looking preferences we impose $\sigma > (1+\gamma)/\gamma$ in order to ensure that the utility function is concave in both elements such that the Hamiltonian is concave in control and states (The condition $\sigma > 1/(1-\gamma)$ in Carroll et al. (1997) is overly restrictive). In case of outward looking preferences h is exogenous and $\sigma > 0$ is sufficient for concavity of the Hamiltonian. Moreover we assume that the time preference rate is sufficiently large in order to get finite life-time utility. As shown below, this requires

$$\theta > (A-\delta)(1-\sigma)(1-\gamma). \tag{4}$$

It is the conventional condition for bounded utility (e.g. Barro and Sala-i-Martin, 2005, Ch. 4.1), scaled by factor $(1-\gamma)$.

Carroll et al. show that the steady-state solution of the optimization problem is the same irrespective of whether habits are formed outward-looking or inward-looking and is given by

$$g_c^* \equiv \frac{\dot{c}}{c} = \frac{A-\delta-\theta}{\gamma(1-\sigma)+\sigma}, \tag{5a}$$

$$x \equiv \frac{c}{h} = 1 + \frac{1}{\rho}\left(\frac{A-\delta-\theta}{\gamma(1-\sigma)+\sigma}\right) = 1 + \frac{g_c^*}{\rho}, \tag{5b}$$

$$p \equiv \frac{k}{h} = \frac{1}{\rho}\left[\frac{\rho(\gamma(1-\sigma)+\sigma)+(A-\delta-\theta)}{(A-\delta)\left[(1-\sigma)\gamma+\sigma-1\right]+\theta}\right], \tag{5c}$$

in which g_c^* denotes the balanced growth rate. Individual indices have been dropped since all households are identical.

3. Income and Life-time Utility (Life Satisfaction)

While the results so far are well known, the implied association between income and life satisfaction remained unexplored. To solve this problem we begin with noting that at the steady state consumption grows at rate g_c^*, such that $c(\tau) = c(t)\mathrm{e}^{g_c^*(\tau-t)}$. Inserting this information

and (5a) and (5b) in (1) provides (6).

$$V(t) = \frac{x^{-\gamma(1-\sigma)} c_0^{(1-\gamma)(1-\sigma)}}{1-\sigma} \int_0^{\infty} \mathrm{e}^{[g_c^*(1-\gamma)(1-\sigma)-\theta]t} \mathrm{d}t. \tag{6}$$

Inserting g_c^* from (5a) into the exponent of the integrant confirms that condition (4) is needed for bounded utility. Using the fact that $c = (x/p) \cdot k$ and inserting (5b) and (5c) provides utility as a function of the current capital stock per capita. Finally, substituting $y(t) = Ak(t)$ as implied by the production function we get maximized intertemporal utility (life satisfaction) as a unique function of current income per capita.

$$\begin{aligned} V(t) &= \beta_0 \cdot y(t)^{\beta_1} \qquad (7) \\ \beta_0 &\equiv \frac{1}{1-\sigma} \cdot \left(\frac{g_c^* + \rho}{\rho} \right)^{\gamma(1-\sigma)} \cdot \left\{ \frac{[\theta - (A-\delta)(1-\sigma)(1-\gamma)]}{\sigma(1-\gamma)+\gamma} \right\}^{(1-\gamma)(1-\sigma)-1} \cdot A^{-(1-\gamma)(1-\sigma)} \\ \beta_1 &\equiv (1-\gamma)(1-\sigma). \end{aligned}$$

The term in square brackets is positive when life-time utility is finite, as initially assumed. This means that the sign of β_0 and β_1 is uniquely pinned down by σ and that the sign of both parameters coincides. The main observation is that the curvature parameter of the function, β_1, is a compound of the coefficient of relative risk aversion σ and the strength of habits or status concerns γ. Quantitative comparisons of life satisfaction, like estimating its association with income, require a cardinal concept of utility. Since a cardinal utility function is only defined up to an affine linear transformation (Strotz, 1953), no inferences about the underlying preferences can be made from the observation of β_0. The curvature parameter β_1, however, is uninformative about the strength of habits or status concerns unless the coefficient of relative risk aversion is known.

Proposition 1 (Equivalence Result). *For any coefficient of relative risk aversion σ and any strength of habits or status concerns γ there exists a coefficient of risk aversion that supports the same association between income and life satisfaction without habits or status concerns.*

This means that inferences on status concerns or consumption habits cannot be derived from an estimate of the empirical association of income and life satisfaction if the workhorse model of endogenous growth augmented by status concerns or consumptions habits is a reasonable approximation of reality. Notice that this conclusion is robust against the existince of a reporting

function that translates actual life-satisfaction into reported life-satisfaction (Oswald, 2008).

The conjecture of the earlier literature that people are subject to status concerns and consumption habits because the observed empirical association of income and life satisfaction is flat at high income levels is not supported by theory. Likewise the conclusion that status concerns or habits play a relatively minor role, as recently derived from the observation that the empirical association between income and life satisfaction is not as flat as previously thought, is not supported by theory, at least not by general equilibrium theory that puts consumption habits or status concerns in the framework of endogenous growth.

Notice furthermore that the growth rate g_c^* shows up only in the constant β_0. Since the cardinal utility function is defined only up to an affine linear transformation, the constant β_0 cannot be identified with data, which implies the following result.

COROLLARY 1 (Economic Growth). *It is impossible to identify the nexus between long-run economic growth and life-satisfaction implied by the present model with real data on life-satisfaction and long-run growth.*

This result offers an explanation for the seemingly puzzling finding that empirical studies provide a strongly significant association between income levels and life-satisfaction and simultaneously fail to identify a positive impact of long-run growth on life satisfaction (see, for example, Easterlin et al., 2010). The present paper suggests that the weak empirical results on growth and life-satisfaction do not necessarily mean that the association is not there. In (7) it is clearly present and positive but it is not identifiable because of measurement problems resulting from the notion of cardinal utility. Notice that this holds true irrespective of the presence, nature, and strength of habits or reference-dependent utility. The result thus advises caution concerning the interpretation of the finding of a non-significant association between long-run growth and life-satisfaction. Notice that a similar argument applies to the speed of adaptation ρ, which also shows up only in the constant β_0. Again, this means that no inferences on the actual speed of adaptation can be made from the observed association between income and life satisfaction.

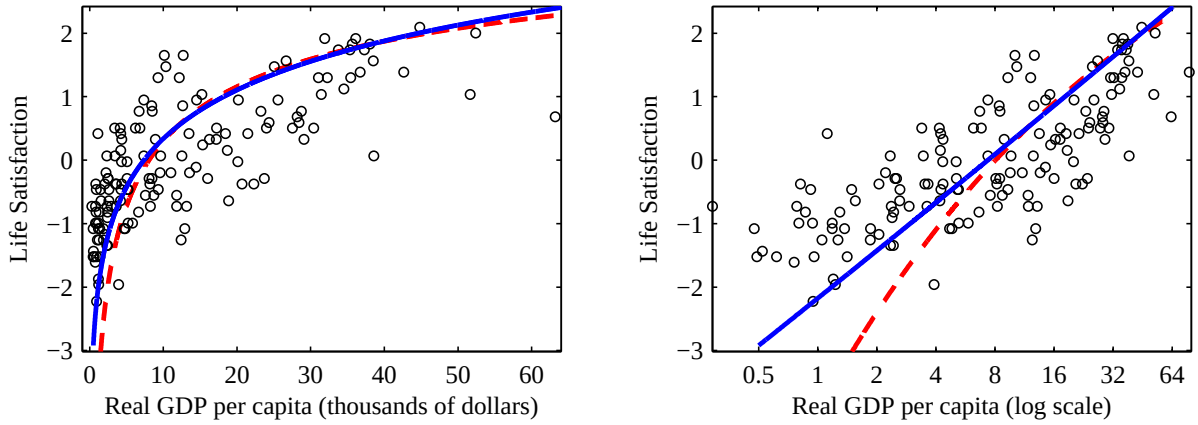
In order to visualize the result I take data for life-satisfaction across countries from the most recent Gallup (2013) poll and data on income per capita (real PPP GDP per capita) from the most recent Penn World Tables (Fenstra et al., 2013). Most of the Gallup data and all of the income data is for the year 2011. Altogether we have data points for 148 countries, represented by dots in Figure 1. In order to normalize data and model predictions I take up the suggestion

of the happiness literature and compute z-scores. In case of the model I feed income data into (7) and compute from the resulting life time utility V,

$$\tilde{V} = \frac{V - \mu(V)}{s(V)} + b,$$

in which $\mu(V)$ is the mean of V and $s(V)$ its standard deviation. Notice that this constitutes an affine linear transformation of V. The life satisfaction data is normalized in a similar fashion. The parameter b is another shifter needed in order to get the origin right. It is set to 1.3 for the model predictions and to zero for the data.

Figure 1: Life Statisfaction across Countries: Data and Model Prediction



Data for 148 countries from Gallup (2013) and Feenstra et al. (2013). Solid line: model prediction for $\beta_1 = 0.01$ (i.e. quasi-log). Dashed line: model prediction for $\beta = -0.26$.

The panels on the left and right hand side of Figure 1 show the same information. The only difference is the scale of the income axis. I took up the idea from Stevenson and Wolfers (2008) and present results for income measured in absolute terms and as well as in logs. The blue line is the prediction of the model for $\beta_1 = 0.01$ that is, for example, for $\gamma = 0$ and $\sigma = 0.99$ (no status concerns) or $\gamma = 0.7$ and $\sigma = 0.96$ (strong status concerns). The case of $\beta_1 \to 0$ is interesting because the recent empirical happiness has imposed (rather than estimated) a log association between income and life satisfaction (Deaton, 2008; Stevenson and Wolfers, 2008). We see that the log assumption, here approximated by $\beta = 0.01$ fits the data quite well irrespective of whether there are strong status concerns or non at all. Imposing strong status concerns modifies the estimated coefficient of relative risk aversion only marginally; σ declines from 0.99 to 0.96.

Dashed red lines in Figure 1 show the model prediction for another important case. Layard

et al. (2008) use results from six different social surveys (mostly on life satisfaction but also on happiness) and estimate the associated "income elasticity of marginal utility" here denoted by ϵ. The point estimates are remarkably consistent across surveys and vary between 1.19 and 1.34 with quite narrow 95% confidence intervals. The "combined estimate" is 1.26. In the present context this means that $\beta_1 = 1 - \epsilon = -0.26$. The model provides a theoretical foundation of the iso-elastic functional form imposed by Layard et al. In Figure 1 Layard et al's combined estimate is reflected by red dashed lines, which represent, for example, values of $\gamma = 0$ and $\sigma = 1.26$ (no status concerns) or $\gamma = 0.7$ and $\sigma = 1.87$ (strong status concerns). The model predictions fit the income and life satisfaction data of the present study somewhat less well than the "quasi log"–prediction, a detail which, however, becomes only visible when the data is inspected in log-linear scale.

For all cases considered the implied range of values for σ is remarkably narrow and compatible with values imposed in other calibration studies as well as with recent empirical estimates (e.g. Chetty, 2006). This observation holds irrespective of the presence and strength of status concerns or consumption habits. Table 1 shows results over the whole range of feasible γ and conceivable curvatures of the utility function. For that purpose I take up the terminology of Layard et al. and represent the slope of the curve by the income elasticity of marginal utility $\epsilon \equiv 1 - \beta_1$. This figure provides the information by how many percent marginal utility (marginal life satisfaction) declines when income increases by 1 percent. The entries in the Table show the coefficients of relative risk aversion (σ) that provides a given ϵ for alternative values γ.

TABLE 1. Life Satisfaction Equivalence

	income elasticity of marginal utility							
γ	3.00	2.50	2.00	1.50	1.25	0.99	0.75	0.50
0.00	3.00	2.50	2.00	1.50	1.25	0.99	0.75	0.50
0.30	3.86	3.14	2.43	1.71	1.36	0.99	0.64	0.29
0.60	6.00	4.75	3.50	2.25	1.63	0.98	0.37	-0.25
0.90	21.0	16.0	11.0	6.00	3.50	0.90	-1.50	-4.00

The entries in the table report the coefficient of relative risk aversion (σ) that provides a given income elasticity of marginal utility ($\epsilon = 1 - \beta_1$) for alternative strengths of reference-dependent utility (γ).

For an understanding of the results it is helpful to recall that individuals compare consumption with the reference stock now and in the future. The curvature of the utility function, summarized in σ determines whether present or future consumption comparisons are more important. If σ

is small, the intertemporal elasticity of substitution is high, and individuals give up future consumption in order to improve present relative consumption (current status). If σ is large, individual are induced to smooth heavily their consumption comparisons with the Joneses or own achievements. This means that individuals subjected to status concerns or habits give up consumption today in order to better compete with the reference stock in the future. The threshold is where σ equals unity such that both effects cancel each other and the presence of status concerns or habits has no impact on observed behavior.

Table 1 shows that the observation of an income elasticity of marginal utility close to one allows to make an inference about σ, which is in this case predicted to be close to one irrespective of the strength of status concerns γ. In principle the model does not allow any inference about γ. But some specific (γ, ϵ) combinations can be excluded by plausible constraints on σ. For example, it is impossible for individuals to display a low income elasticity of marginal utility and to be simultaneously risk averse ($\sigma > 1$) and strongly subjected to status concerns or consumption habits ($\gamma \geq 0.6$). Likewise the combination of a high income elasticity and strong status concerns or habits can be excluded because it would require an implausibly large coefficient of risk aversion. For income elasticities in the empirical plausible range between 1.0 and 1.5 (Layard et al., 2008), however, it impossible to draw conclusions about status concerns or habits because there is (yet) to little precision in the estimate of σ such that all possible γ values are compatible with empirically supported values of σ.

4. Discussion

This paper has calculated the implied life satisfaction (life time utility) when the basic model of endogenous growth is extended by status concerns or consumption habits. This way it has been shown that the estimated iso-elastic or log-linear association between and income and life satisfaction has a theoretical foundation. The slope of the estimated relationship however is found to be equally well supported by ordinary preferences and (strong) status concerns or consumption habits. Inferences from the observed association of income and life-satisfaction on the presence and strength of reference-based utility, made by the earlier (Easterlin, 1972) and recent (Stevenson and Wolfers, 1998) happiness literature, are not supported by dynamic general equilibrium theory.

In order to arrive at an explicit solution for life satisfaction the economic framework was chosen

to be deliberately simple, built upon the Ak growth model. Based on Rebelo (1991), it has been argued by Carroll et al. (2000) in a similar context that the linear Ak production function is the ultimate structure of all endogenous growth models. The model has been modified to allow for wealth heterogeneity and status derived from wealth or education (e.g. Futagami and Shibata, 1998; Kawamoto, 2008) with little effect on the qualitative results discussed in the present paper (as long as $\sigma > 1$). Likewise the empirical estimate of ϵ has been shown to vary little across socioeconomic strata (Layard et al., 2008). These facts indicate that the equivalence result is more general than shown in the present paper. In fact, any model employing utility function (1) would display the result at a steady-state along which consumption grows at a constant rate.

The empirical estimates of an income elasticity of marginal utility above unity imply a coefficient of risk aversion σ greater than unity. For $\sigma > 1$ the rate of economic growth is a positive function of γ, as shown in (5a), replicating the earlier literature (Carroll et al., 1997, 2000). Put into general equilibrium context, the empirical evidence thus suggests that strong status concerns or consumption habits are good for economic growth. In a related paper (Strulik, 2013) I show that status concerns may also, through the growth channel, be conducive to the experience of a happy life.

References

Abel, A.B., 1990, Asset pricing under habit formation and catching up with the Joneses, *American Economic Review* 80, 38-42.

Alonso-Carrera, J., J. Caballe, and X. Rauch, 2005, Growth habit formation, and catching up with the Joneses, *European Economic Review* 49, 1665-1691.

Alvarez-Cuadrado, F., G. Monteiro, and S.J. Turnovsky, 2004, Habit formation, catching up with the Joneses, and economic growth, *Journal of Economic Growth* 9, 47-80.

Barro, R.J. and X. Sala-i-Martin, 2004, *Economic Growth* (2nd ed.), MIT Press, Cambridge, MA.

Carroll, C.D.; J. Overland, and D.N. Weil, 1997, Comparison utility in a growth model, *Journal of Economic Growth* 2, 339-367.

Carroll, C.D.; J. Overland, and D.N. Weil, 2000, Savings and growth with habit formation, *American Economic Review* 90, 341-355.

Chetty, R., 2006, A new method of estimating risk aversion, *American Economic Review* 96, 1821-1834.

Clark, A.E., P. Frijters, and M.A. Shields, 2008, Relative income, happiness, and utility: An explanation for the Easterlin paradox and other puzzles, *Journal of Economic Literature* 46, 95-144.

Corneo, G. and O. Jeanne, 2001, Status, the distribution of wealth and growth, *Scandinavian Journal of Economics* 103, 283-293.

Cozzi, G., 2004, Rat race, redistribution, and growth *Review of Economic Dynamics* 7, 900-915.

Deaton, A., 2008, Income, Health, and Well-Being around the World: Evidence from the Gallup World Poll, *Journal of Economic Perspectives* 22(2), 53-72.

di Tella, R., J. Haisken-De New, and R. MacCulloch, 2007, Happiness adaptations to income and to status in an individual panel, NBER Working Paper 13159.

Easterlin, R.A., 1974, Does economic growth improve the human lot? Some empirical evidence, in: *Nations and Households in Economic Growth: Essays in Honor of Moses Abramovitz*, R. David and M. Reder (eds.), Academic Press, New York, 89-125.

Easterlin, R.A., 2001, Income and happiness: towards a unified theory, *Economic Journal* 111, 465-484.

Easterlin, R.A., McVey, L.A., Switek, M., Sawangfa, O., and Zweig, J.S., 2010, The happiness-income paradox revisited, *Proceedings of the National Academy of Sciences* 107, 22463-22468.

Feenstra, R.C., Inklaar R., and Timmer, P.M., 2013, The Next Generation of the Penn World Table", available for download at www.ggdc.net/pwt.

Fisher, W.H. and F.X. Hof, 2000, Relative consumption, economic growth, and taxation, *Journal of Economics* 72, 241-262.

Frey, B.S. and A. Stutzer, 2002, What can economists learn from happiness research?, *Journal of Economic Literature* 40, 402-435.

Futagami, K. and A. Shibata, 1998, Keeping one step ahead of the Joneses: status, the distribution of wealth, and long-run growth, *Journal of Economic Behavior and Organization* 36, 109-126.

Gallup (2013), Gallup World View – Life Today, available at https://worldview.gallup.com.

Grossmann, V., 1998, Are status concerns harmful for growth?, *Finanzarchiv* 55, 357-373.

Kawamoto, K., 2009, Status-seeking behavior, the evolution of income inequality, and growth, *Economic Theory* 39, 269-289.

Layard, Richard, 2003, *Happiness: Has Social Science A Clue?*, Lionel Robbins Memorial Lectures, Centre for Economic Performance, London.

Layard, R., Mayraz, G., and Nickell, S., 2008, The marginal utility of income, *Journal of Public Economics* 92, 1846-1857.

Oswald, A.J., 2008, On the curvature of the reporting function from objective reality to subjective feelings, *Economics Letters* 100, 369-372.

Rebelo, S. T., 1991, Long-Run Policy Analysis and Long-Run Growth, *Journal of Political Economy* 99, 500-521.

Stevenson, B. and J. Wolfers, 2008, Economic growth and subjective well-being: Reassessing the Easterlin paradox, *Brookings Papers on Economic Activity*, Spring 2008, 1-87.

Stevenson, B. and Wolfers, J., 2013, Subjective well-being and income: Is there any evidence of satiation?, *American Economic Review* 103, 598-604.

Strotz, R., 1953, Cardinal utility, *American Economic Review* 43, 384-397.

Strulik, H., 2013, How Status Concerns Can Make Us Rich and Happy, Discussion Paper, University of Goettingen.