Intertemporal Choice with Health-Dependent Discounting: Appendix

Holger Strulik*

June 2019

In this Appendix, I consider a more complex model in which the accumulation of health deficits can be slowed down by deliberate health investments such that equation (2) from the main text is replaced by

$$\dot{D} = \mu\left(D - Ah^{\gamma} - a\right), \tag{A.1}$$

where h is health investment and A and γ are parameters describing the available medical technology. This functional form has been introduced and extensively discussed in Dalgaard and Strulik (2014). The budget constraint (3) from the main text is replaced by

$$\dot{k} = w + (r+m)k - c - ph, \tag{A.2}$$

in which k is financial wealth, w is labor income, and p is the price of health care. Aside from private health investment, the presence of health deficits incurs also inevitable health expenditure $h^e = p\alpha D^{\beta}$, which is covered by health insurance, i.e. h is conceptualized as out-of-pocket health expenditure and total health expenditure is given by $h + h^e$. The objective function remains the same as in the basic model. For convenience, it is here restated as (A.3)

$$V = \int_0^{\infty} S(D(t))u(c(t))\mathrm{e}^{-\int_0^t \rho(D(v))\mathrm{d}v}\mathrm{d}t. \tag{A.3}$$

Individuals maximize (A.3) subject to (A.1) and (A.2). Again, the easiest way to solve this problem is to apply a transformation of variables. Define $q \equiv \int_0^t \rho(D(v))\mathrm{d}v$ such that $\mathrm{d}q/\mathrm{d}t = \rho(D)$ and $\mathrm{d}t = \mathrm{d}q/\rho(D)$. This implies $\dot{k} \equiv \mathrm{d}k/\mathrm{d}t = (\mathrm{d}k/\mathrm{d}q)/(\mathrm{d}q/\mathrm{d}t)$ such that $\mathrm{d}k/\mathrm{d}q = \dot{k}/\rho(D)$ and likewise for $\dot{D} \equiv \mathrm{d}D/\mathrm{d}t$. The time-transformed problem (A.1)–(A.3) thus reads

$$\max \int_0^{\infty} \frac{S(D)u(c)\mathrm{e}^{q}}{\rho(D)}\mathrm{d}q \qquad \text{s.t. } \frac{\mathrm{d}k}{\mathrm{d}q} = \frac{\dot{k}}{\rho(D)}, \frac{\mathrm{d}D}{\mathrm{d}q} = \frac{\dot{D}}{\rho(D)} \tag{A.4}$$

* University of Goettingen, Department of Economics, Platz der Goettinger Sieben 3, 37073 Goettingen, Germany; email: holger.strulik@wiwi.uni-goettingen.de.

The associated Hamiltonian reads

$$H = \frac{S(D)u(c)}{\rho(D)} + \frac{\lambda_k}{\rho(D)}\left[w + (r+m) - c - ph\right] + \frac{\lambda_D}{\rho(D)}\left[D - Ah^{\gamma} - a\right], \tag{A.5}$$

with costate variables λ_k and λ_D. The first order conditions and costate equations are:

$$\frac{\partial H}{\partial c} = S\frac{u'}{\rho} - \frac{\lambda_k}{\rho} = 0 \tag{A.6}$$

$$\frac{\partial H}{\partial h} = -\frac{\lambda_k}{\rho} - \frac{\lambda_D \mu}{\rho} A\gamma h^{\gamma-1} = 0 \tag{A.7}$$

$$\frac{\partial H}{\partial k} = \frac{\lambda_k r}{\rho} = \lambda_k - \frac{\mathrm{d}\lambda_k}{\mathrm{d}q} \tag{A.8}$$

$$\frac{\partial H}{\partial D} = \frac{S'\rho - \rho' S}{\rho}u + \frac{\lambda_D \mu}{\rho^2}\left[\rho - \rho'(D - Ah^{\gamma} - a)\right] - \frac{\lambda_k \rho'}{\rho^2}\left\{\left[w + (r+m)k - c - ph\right]\right\} = \lambda_D - \frac{\mathrm{d}\lambda_D}{\mathrm{d}q}. \tag{A.9}$$

We next reintroduce age by substituting $\mathrm{d}q = \rho(D)\mathrm{d}t$. Thus (A.8) and (A.9) become

$$\lambda_k r = \lambda_k \rho - \dot{\lambda}_k \tag{A.10}$$

$$\left[S' - \frac{\rho'}{\rho}S\right]u - \frac{\lambda_k \rho'}{\rho}\left[w + (r+m)k - c - ph\right] + \lambda_D \mu\left[1 - \frac{\rho'}{\rho}(D - Ah^{\gamma} - a)\right] = \lambda_D \rho - \dot{\lambda}_D \tag{A.11}$$

Substituting λ_k from (A.6) and λ_D from (A.7), (A.11) becomes:

$$-\left\{\left(\frac{S'}{S} - \frac{\rho'}{\rho}\right)\frac{u}{u'} - \frac{\rho'}{\rho}\left[w + rk - c - ph\right]\right\}\frac{\mu A\gamma h^{\gamma-1}}{p} + \mu\left[1 - \frac{\rho'}{\rho}\left(D - Ah^{\gamma} - a\right)\right] - \rho = -\frac{\dot{\lambda}_D}{\lambda_D}. \tag{A.12}$$

Differentiating (A.7) with respect to age and inserting (A.10) and (A.12) provides:

$$\frac{\dot{h}}{h} = \frac{1}{1-\gamma}\left\{r - \mu\left[1 - \frac{\rho'}{\rho}\left(D - Ah^{\gamma} - a\right)\right] + \frac{\mu\gamma A h^{\gamma-1}}{p}\left[\left(\frac{S'}{S} - \frac{\rho'}{\rho}\right)\frac{u}{u'} - \frac{\rho'}{\rho}\left(w + (r+m)k - c - ph\right)\right]\right\} \tag{A.13}$$

Differentiating (A.6) with respect to age and inserting (A.10) provides the same Euler equation as for the basic model:

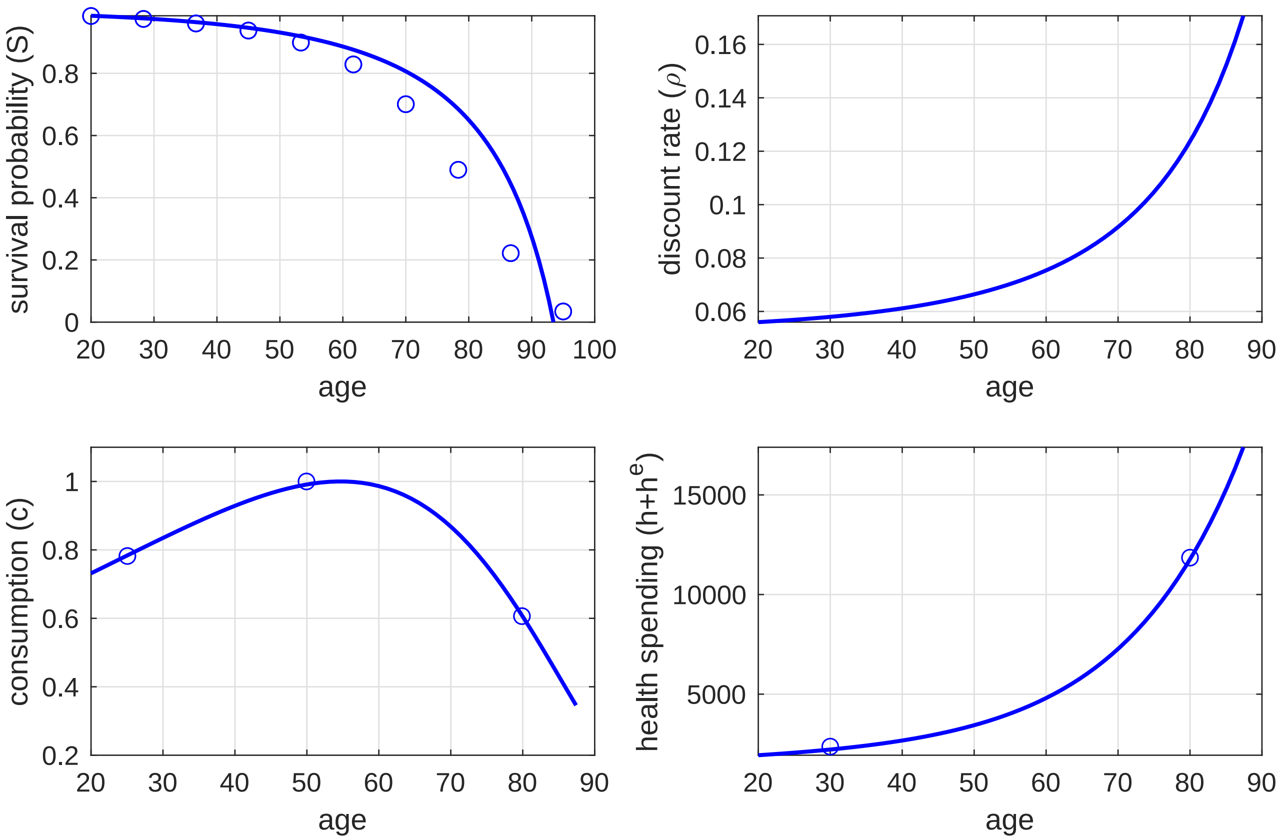
$$\frac{\dot{c}}{c} = \frac{r - \rho(D)}{\sigma}, \tag{A.14}$$

in which σ denotes the inverse of the elasticity of intertemporal substitution. All increasing complexity thus arises from (A.13), which collapses to the simple health Euler equation in Dalgaard and Strulik (2014) for $S' = \rho' = 0$, i.e. when neither survival nor discounting depends on health. In order to explore how the presence of health expenditure changes consumption behavior, we assume that health-dependent survival is given by the function:

$$S(D) = \psi - \frac{\nu}{1 - \chi D}. \tag{A.15}$$

We estimate the three parameters such that the model predicts a reasonable approximation of the empirical survival function $S(t)$ when we feed in the predicted health deficits $D(t)$, $S(t) = S(D(t))$. This provides the estimates $\psi = 1.75$, $\nu = 0.7$, and $\chi = 3.1$. The upper-left panel in Figure A.1 shows the predicted association between age and survival probability. Dots indicate the survival probability estimated from life tables for U.S. American men in 1975-1999, taken from Strulik and Vollmer (2013). The approximation somewhat overestimates the survival of the elderly and underestimates the survival of the oldest old but, altogether, it fits the data reasonably well.

FIGURE A.1: LIFE CYCLE CONSUMPTION AND HEALTH EXPENDITURE



The model is calibrated for a 20 years old male U.S. American. As for the simple model, I set $r = 0.07$, and $\gamma = 0.2$ as well as $\mu = 0.043$, $a = 0.02$, and $D_0 = 0.027$ from Mitnitski et al. (2002a), and I assume that the discount rate increases exponentially with deteriorating health, according to (8) from the main text. I normalize $p = 1$ and set $w = 27,928$ when the individual is between 20 and 65 years old (i.e. the average labor income for single men in the year 2010; BLS, 2012) and $w = 0.45 \cdot 27,928$ above age 65, according to the an average replacement rate of 0.45 (from the OECD, 2016). I then calibrate the remaining parameters to fit three points, at age 25, 50, and 80, from the empirical age-consumption curve (as in the main text) and two points at age 30 and 80 from health expenditure of American men in the year 2010 as well as an average out-pocket expenditure share of 13.6 (data from MEPS, 2010), and a life expectancy at age 20 of 57.1 years (expected age at death at 77.1; NVSS, 2014). This provides the estimates $\phi = 7.3$, $\bar{\rho} = 0.056$, $\sigma = 0.99$, $a = 0.017$, $A = 0.00031$, $\alpha = 1.9 \cdot 10^5$, and $\beta = 1.4$.

The predicted age-trajectories are shown in Figure A.1. Targeted data points are indicated by circles. The model predicts the age-profiles for health expenditure and consumption reasonably well.

Additional References

BLS (2012). Consumer Expenditure Survey. U.S. Bureau of Labor Statistics, September 2012 (`http://www.bls.gov/cex/csxcross.htm#y1011`).

MEPS (2010). U.S. Department of Health & Human Services. Center for Financing, Access and Cost Trends, Agency for Healthcare Research and Quality: Medical Expenditure Panel Survey, 2010 (`https://meps.ahrq.gov/mepsweb/data_stats/quick_tables_results.jsp?component=1&subcomponent=0&year=-1&tableSeries=1&searchText=&searchMethod=1&Action=Search`)

NVSS (2014). US department of health and human services, national vital statistics system. National Vital Statistics Report 63(7).

OECD (2016), Gross pension replacement rates. OECD Data (`https://data.oecd.org/pension/gross-pension-replacement-rates.htm`

Strulik, H., Vollmer, S. (2013). Long-run trends of human aging and longevity. *Journal of Population Economics* 26(4), 1303-1323.