Intertemporal Choice with Health-Dependent Discounting

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June 2020

Abstract. In this paper I discuss a standard model of life cycle consumption behavior when the discount rate depends on the state of health and health deteriorates with increasing age. I show that this feature allows the introduction of time-consistent discounting at a non-constant rate and to model, in a convenient way, the notion that individuals discount future payoffs at higher rates when the risk of death increases. I show that the model generates an empirically plausible age-consumption pattern even when perfect annuity markets exist.

Keywords: discount rates, aging, risk of death, consumption behavior.

JEL: D11, D81 D91, I10, I12.

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1. Introduction

Economists agree that the standard assumption of intertemporal choice theory that future gains and losses are discounted at a constant rate exists mainly for convenience and several proposals have been discussed to model more realistic discounting behavior (Frederick et al., 2002). In this paper, I focus on one aspect in this domain of research, namely the notion that individuals discount the future at higher rates when they grow older and, in particular, when death is near. I capture this phenomenon by introducing health-dependent discounting and physiological aging into a standard life cycle model. Conceptualizing the discount rate as a function of the state of health, time-consistent solutions of intertemporal choice are easily obtained. As the state of health deteriorates, death becomes more likely, and the pure rate of time preference increases. In order to evaluate their survival probability, individuals consider their physiological age (their state of health) instead of their chronological age. The feature that the state of health is endogenous but pre-determined at any age enables the unconventional result that decisions are time-consistent although the discount rate is not constant.

As a measure of health, I use the health deficit index developed by Mitnitski et al. (2001) "as an individual state variable, reflecting severity of illness and proximity to death." (ibid., p. 323). This measure, also known as frailty index, is an established methodology used by countless studies in gerontology. It has been introduced by Dalgaard and Strulik (2014) into economics (see also Hosseini et al., 2019). The health deficits index computes the number of health deficits present in a person relative to the number of potential health deficits. Health deficits are accumulated in a quasi-exponential way as individuals get older (Mitnitski et al. (2002a,b; Abeliansky and Strulik, 2018) and they are a precise predictor of mortality. The prediction of mortality can be so accurate that chronological age adds insignificant explanatory power when added to the regression (Rockwood and Mitnitski, 2007).

A limited number of studies have investigated how aging affects discounting. Huffman et al. (2019) find that, among the elderly, discount rates increase with age. Read and Read (2004) consider individuals from a larger range of ages between 19 and 89 and find the lowest discount rate for individuals of middle age, and thus, a u-shaped age-pattern of discounting. Sozou and Seymour (2002) show that such a u-shaped pattern can be motivated by an evolutionary theory of discounting. Chao et al. (2009) find evidence for a u-shaped association of the discount rate with health deficits and that age loses its predictive power for the discount rate when the state of health is taken into account. Falk et al. (2019) confirm for a large cross-country data set, comprising 80,000 individuals in 76 countries, that increasing life expectancy as well as better individual perception of health status is associated with higher discount factors (i.e. lower discount rates). A recent study by Gassen et al. (2019) argues in favor of an evolutionary channel from the physical condition of the body to time preference and finds a negative association between inflammatory activity (as a measure of health deficits and cellular distress) and the ability to delay gratification.

I apply the new discounting method to motivate a hump-shaped age-consumption profile in a life cycle model. The literature has developed several explanations for such a non-monotonous consumption profile (see e.g. Gourinchas and Parker, 2002) and a particularly related proposal is built on age-dependent mortality (Büttler, 2001, Feigenbaum, 2008). This channel, however, breaks down when individuals are allowed to finance old age consumption with annuities. In order to establish health-dependent discounting as an independent pathway, I assume a perfect annuity market to shut off the imperfect-annuities channel. Strulik (2017) shows that the consideration of health in the utility function could also motivate a consumption hump. In contrast to the earlier studies, which were designed to motivate a consumption hump, health-dependent discounting is a more encompassing refinement of preferences that could potentially inspire a host of other applications for which proximity to death influences human behavior.

2. The Basic Model

Consider an individual with uncertain lifetime. Following the biological foundation of aging, the the probability to be alive at age t does not directly depend on chronological age t but on the accumulated health deficits at that age, $S(D(t))$, $S' < 0$, in which D is the health deficit index.[1] Accumulated health deficits are thus an informative indicator of the proximity of death. Individuals are aware of this fact and discount the future at a higher rate when they expect death to be near, i.e. when many health deficits have been accumulated. There exists a health deficit index $\bar{D}$ above which survival is impossible, $S(\bar{D}) = 0$. Let $u(c(t))$ be the utility experienced from consuming c at age t. Expected lifetime utility is then given by

$$V = \int_0^\infty S(D(t))u(c(t))\mathrm{e}^{-\int_0^t \rho(D(v))\mathrm{d}v}\mathrm{d}t. \tag{1}$$

[1]In gerontology, aging is defined as the intrinsic, cumulative, progressive, and deleterious loss of function that eventually culminates in death (Arking, 2006, p. 11) and in a successful theory of aging, there should be no role for chronological age in explaining death (ibid., p. 10).

The feature that the discount rate $\rho(D)$ depends on the state of health, which is – although endogenous – a predetermined state variable at any age, implies time consistency of decisions and avoids any complications that may arise from the recursiveness of the utility functional.[2] Intuitively, the discount rate is stationary over time and over age, *conditional* on the state of health of the individual. The state of health, however, is not a choice variable. This is true even when health investment is a choice variable since the current state of health is predetermined by health investments earlier in life (see Appendix). By assuming that $\rho' > 0$ we capture the idea that individuals discount utility more heavily when death is near. The mortality rate m is defined as the rate of change of the survival rate, $m \equiv -\dot{S}/S = S'\dot{D}/S$.[3]

We measure health deficits by the health deficits index, also called frailty index (Mitnitski et al, 2001). Mitnitski et al. (2002a) show that the relative number of health deficits $D(t)$ increases with age t in a quasi-exponential way such that $D(t) = a + be^{\mu t}$. This "law of deficit accumulation" explains around 95 percent of the variation in the data and its parameters are estimated with great precision. For most nations, the force of aging μ is found to be around 3 to 4 percent (Mitnitski et al., 2002a, Harttgen et al., 2013, Abeliansky and Strulik, 2018). Here, we differentiate $D(t)$ with respect to age and obtain the law of motion for health deficits:

$$\dot{D} = \mu\left(D - a\right). \tag{2}$$

Individuals can freely save and borrow and face the budget constraint

$$\dot{k} = w + (r + m)k - c, \tag{3}$$

in which k is financial wealth and w is a flow of non-financial income. We consider perfect annuity markets such that the interest rate is a compound of the return on capital r and the mortality rate m and individuals inherit no wealth and leave no bequests. As explained in the introduction, this is an interesting benchmark since it has been shown that the feature of mortality as such is capable to generate a hump-shaped age-consumption pattern only when a (perfect) market for annuities is absent.

The easiest way to solve (1) subject to (2) and (3) is to apply a transformation of variables. Define $q \equiv \int_0^t \rho(D(v))\mathrm{d}v$ such that $\mathrm{d}q/\mathrm{d}t = \rho(D)$ and $\mathrm{d}t = \mathrm{d}q/\rho(D)$. This implies $\dot{k} \equiv \mathrm{d}k/\mathrm{d}t =$

[2] See, for example, Obstfeld (1990) for recursive utility when the discount rate depends on consumption. Strulik (2012) explores the idea that the discount rate (of an infinitely long living individual) depends on wealth.

[3] In the following, I omit the age argument (t) of all variables whenever this can be done without causing confusion.

$(\mathrm{d}k/\mathrm{d}q)/(\mathrm{d}q/\mathrm{d}t)$ such that $\mathrm{d}k/\mathrm{d}q = \dot{k}/\rho(D)$. The transformed problem is thus given by $\max \int_0^\infty S(D)u(c)\mathrm{e}^q/\rho(D)\mathrm{d}q$ subject to $\mathrm{d}k/\mathrm{d}q = \dot{k}/\rho(D)$. The associated Hamiltonian reads:

$$H = \frac{S(D)u(c)}{\rho(D)} + \frac{\lambda_k}{\rho(D)}\left[w + (r+m)k - c\right], \tag{4}$$

with costate variable λ_k. The first order condition and costate equation are:

$$\frac{\partial H}{\partial c} = S(D)u'(c) - \lambda_k = 0 \tag{5}$$

$$\frac{\partial H}{\partial k} = \frac{\lambda_k(r+m)}{\rho(D)} = \lambda_k - \frac{\mathrm{d}\lambda_k}{\mathrm{d}q}. \tag{6}$$

We next reintroduce age by substituting $\mathrm{d}q = \rho(D)\mathrm{d}t$. Thus (6) becomes $\lambda_k(r+m) = \lambda_k\rho(D) - \dot{\lambda}_k$. Differentiating (5) with respect to age provides $\dot{S}/S + (u''/u')\dot{c} = \dot{\lambda}_k/\lambda_k$. Substituting $\dot{\lambda}_k$, λ_k, and the mortality rate $m \equiv -\dot{S}/S$, provides the Euler equation:

$$\frac{\dot{c}}{c} = \frac{r - \rho(D)}{\sigma}, \tag{7}$$

in which $\sigma \equiv -(u''/u')c$ denotes the intertemporal elasticity of substitution. For constant $\rho(D)$ the solution collapses to the standard Ramsey rule and consumption evolves monotonously with age. Equation (7) also reflects the well-known result that the survival probability plays no role for the age-profile of consumption when individuals have access to annuities (Bütler, 2001; Feigenbaum, 2008). In contrast, non-monotonous age-profiles of consumption can be motivated by a health-dependent discount rate. For example, if $\rho' > 0$, $\rho(0) < r$, and $\rho(\bar{D}) > r$, consumption exhibits a hump-shaped age-profile.

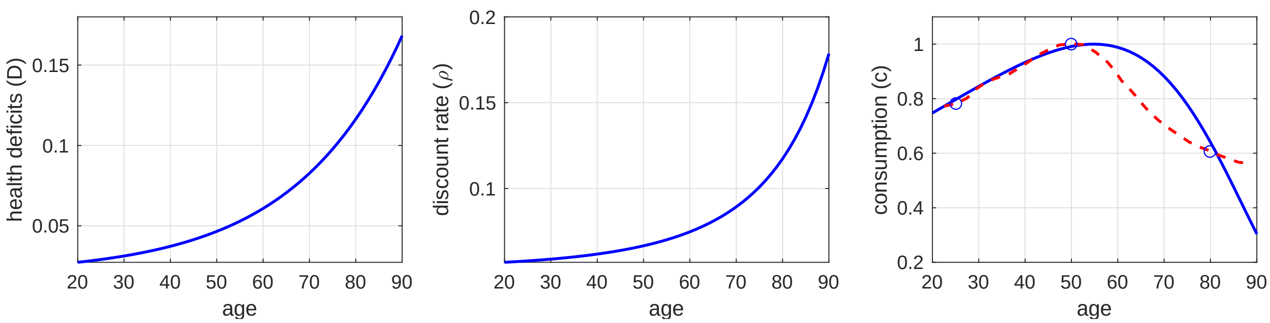
In order to explore the quantitative features of the result, we begin by considering the following parsimonious specification of the discount rate:

$$\rho = \bar{\rho}\mathrm{e}^{\phi(D-D_0)}, \tag{8}$$

such that the discount rate equals $\bar{\rho}$ at the initial age and is exponentially increasing with the accumulation of health deficits. As a benchmark, I set $r = 0.07$ according to the long-run interest rate estimated in Jorda et al. (2019). For health deficit accumulation, I take from the estimates of Mitnitski et al. (2002a), $\mu = 0.043$, $a = 0.02$, and $D_0 = 0.027$ at the initial age of 20 years. I then calibrate σ, $\bar{\rho}$, and ϕ to approximate three points of age-specific consumption as estimated by Fernandez-Villaverde and Krueger (2007), namely at age 25, 50 (peak consumption), and 80. This leads to the estimates $\sigma = 0.99$, $\phi = 8.1$, and $\bar{\rho} = 0.056$. The estimated elasticity of intertemporal substitution is close to unity (log-utility), in line with studies suggesting that the "true" value of σ is probably close to unity (Chetty et al., 2006; Layard et al., 2008).

Results are shown in Figure 1. The panel on the left-hand side shows the imputed law of health deficit accumulation (2). The center panel shows the "inverse hyperbolic" age-profile of the implied discount rate $\rho(D)$. The panel on the right-hand side shows life cycle consumption, measured relative to peak consumption. Dots display the targeted data points. The full age-consumption profile from Fernandez-Villaverde and Krueger (2007) is shown by red (dashed) lines. The model traces the actual consumption profile quite closely from young to middle age and reasonably well from middle to old age.

Figure 1: Health-Dependent Discounting and Age-Consumption Profile



Blue (solid) lines: calibrated model. Dots indicate targeted data points, see text for details. Red (dashed lines): empirical estimates from Ferndandez-Villaverde and Krueger (2007).

The goodness of fit does not depend on the prevailing interest rate since there are three degrees of freedom to adjust the calibrated discount rate. For example, for $r = 0.05$, a match of the consumption hump with the targeted data points is obtained for $\phi = 10.1$, $\bar{\rho} = 0.038$, and $\sigma = 0.89$.

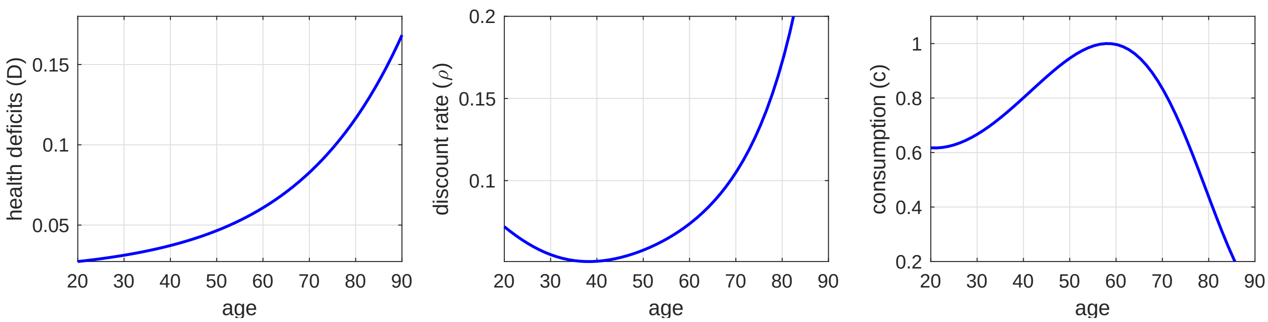
Non-constant discount rates, i.e. non-exponential discounting methods, are usually thought of as implying time-inconsistency (see e.g. Strotz, 1956; Angeletos et al., 2001) although there are exceptions when discounting is multiplicatively separable in planning time and decision time (see Burness, 1976; Strulik and Trimborn, 2018). Conceptualizing the discount rate as health-dependent creates an opportunity to implement non-constant discount rates that lead to time-consistent decisions. This feature makes the approach of more general use than just an amendment to generate a plausible consumption path. It could also be used to implement a declining (hyperbolic) discount rate at young age, capturing the idea that young persons (with almost perfect health) tend to discount the future at a high rate because death is far away. Combined with increasing discount rates at old age this behavior would then imply a u-shaped life-cycle pattern of the discount rate, as motivated in the Introduction.

These ideas can be integrated into the model by the health-dependent discount rate:

$$\rho = \bar{\rho} + \phi_1 \mathrm{e}^{-\phi_2 D} + \phi_3 \mathrm{e}^{\phi_4 D}, \tag{9}$$

which replaces (8). The second term in (9) captures the hyperbolic decline of the time preference rate in young age (when health is very good). The third terms captures the increase in time preference in old age (and bad health). Figure 2 shows the implied discounting and consumption over the life cycle for an example where $\bar{\rho} = -0.03$, $\phi_1 = 30$, $\phi_2 = 250$, $\phi_3 = 0.05$, and $\phi_4 = 12$. In young adulthood, when the hyperbolic part of the discount rate is dominating, consumption is convex in age, it turns into a concave shape in middle age and reaches a maximum, after which it falls in old age.

Figure 2: U-Shaped Discounting and Age-Consumption Profile



3. Conclusion

Modeling discounting as health-dependent provides a straightforward and empirically plausible way to introduce a non-constant discount rate without time-inconsistency problems. This note focused on the age profile of consumption as an application. The Appendix shows an extension towards endogenous health behavior. Interesting further applications include problems where preferences depend on past consumption like habit formation or addiction.

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