Limited Self-Control and Longevity: Appendix

1. Appendix A: Solution of the Model

The costate equations for problem (1)–(2) are:

$$r\lambda_k = \rho\lambda_k - \dot{\lambda}_k \tag{A.1}$$

$$\mu\lambda_D = \rho\lambda_D - \dot{\lambda}_D. \tag{A.2}$$

The system (7)–(9) and (A.1) to (A.2) can be simplified by eliminating the co-state variables. This leads to the following solution:

$$\frac{\dot{h}}{h} = \frac{r-\mu}{1-\gamma}. \tag{A.3}$$

$$\text{If} \quad \left(q + \frac{pB}{\gamma A h^{\gamma-1}}\right)\frac{\beta}{1-\beta} \geq \left(\frac{c}{\zeta}\right)^{\frac{1}{1-\psi}}, \ \text{then} \quad u=0,\ \frac{\dot{c}}{c} = \frac{r-\rho}{\sigma + \frac{(1-\sigma-\psi)(1-\beta)(c/\zeta)^{-\psi}}{\beta+(1-\beta)(c/\zeta)^{-\psi}}}. \tag{A.4}$$

Otherwise:

$$u = c\left\{\left[\frac{pB}{\gamma A h^{\gamma-1}} + q\right]\frac{\beta}{1-\beta}\right\}^{\frac{1}{\psi-1}} - \zeta \tag{A.5}$$

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left\{r - \rho - (1-\sigma-\psi)\frac{(1-\beta)x^{\psi}}{\beta+(1-\beta)x^{\psi}} \cdot \frac{r-\mu}{(1-\gamma)(\psi-1)\left(1+\frac{q\gamma A h^{\gamma-1}}{pB}\right)}\right\} \tag{A.6}$$

with $x \equiv (u+\zeta)/c$. Equation (A.3) is the "Health Euler" for the lifetime trajectory of health expenditure, as derived and explained in Dalgaard and Strulik (2014). The condition in (A.4) determines whether individuals consume the unhealthy good. The unhealthy good is not consumed if it is sufficiently expensive (q is sufficiently high), or sufficiently harmful (B is sufficiently high), or if it is sufficiently little desired ($\beta/(1-\beta)$ is sufficiently high). Ceteris paribus, the unhealthy good is more likely to be consumed if health neutral consumption is high (because the two goods are not perfect substitutes), if the price of health care p is low, and if health investment is low such that marginal return of health expenditure, $\gamma A h^{\gamma-1}$, is high. In this case, damages from unhealthy consumption are relatively cheaply repaired, which provides an incentive to make unhealthy consumption choices.

Inspection of (A.5) shows that if the unhealthy good is consumed, higher q, p, B, and h lead to lower consumption while higher c induces more consumption, with the same intuition as provided with respect to the extensive margin. Equations (A.4) and (A.6) are the Euler equations for health neutral consumption. The standard Euler equation is augmented to take the effect from (potential) unhealthy consumption into account. For the special case of $\beta = 1$, there is never an incentive for unhealthy consumption and (A.4) is reduced to the standard Ramsey rule $\dot{c}/c = (r - \rho)/\sigma$.

An interesting feature of the reduced-form dynamic system (A.3)–(A.6) is that it is independent from ω. This does, of course, not mean that the solution is independent from self-control. Self-control enters through the terminal condition, which requires that the Hamiltonian at the time of death is zero, which can be equivalently written as in (A.7):

$$0 = \bar{H} \equiv \frac{H(T)}{1+\omega} = U(c(T), u(T)) - \frac{\omega}{1+\omega} U(c_s, u_s) \tag{A.7}$$
$$+ \frac{\lambda_k(T)}{1+\omega} \left[w(T) + rk(T) - c(T) - qu(T) - ph(T)\right] + \frac{\lambda_D(T)}{1+\omega} \mu \left[D - Ah(T)^{\gamma} + Bu(T) - a\right].$$

Lower self-control, i.e. higher ω, amplifies the negative impact of short-run desires $U(c_s, u_s)$ on $\bar{H}$ and it reduces the positive impact of savings and health deficit accumulation. From this, we expect that it leads to life cycle decisions resulting in a shorter life. Self-control thus affects the level of expenditure allocated to the two goods and savings and health expenditure but it does not affect the slope of life cycle trajectories, which are obtained from the Euler equations. Intuitively, it makes sense that the slope of life cycle trajectories is determined by intertemporal trade-offs faced by the long-run self, based on r, ρ, and other prices, while short-run desires impact on the "within period" allocation, i.e. on the level of the expenditure paths. How exactly life cycle plans are affected by self-control can only be determined by numerical analysis. The other boundary conditions are that the optimal life cycle trajectory has to fulfil are $k(0) = k_0$, $D(0) = D_0$, $k(T) = \bar{k}$, and $D(T) = \bar{D}$, implying that the individual dies when $\bar{D}$ health deficits have been accumulated.

Appendix B: Details of the Calibration

The model is calibrated to match initial deficits D_0, final deficits $\bar{D}$, longevity T, and life-cycle health investments $h(t)$ for 20-year-old U.S. American men in the year 2010. From Mitnitski et al. (2002), I take the estimate for the rate of aging, $\mu = 0.043$. I set r to 0.07 according to the long-run real interest rate (Jorda et al., 2017) and $\rho = r$ as in Dalgaard and Strulik (2014). In the year 2010, the average life expectancy of a 20-year-old American male was 57.1 years, i.e the expected age at death was 77.1 (NVSS, 2014). From Mitnitski et al. (2002), I infer terminal health deficits $\bar{D} = D(75.6) = 0.106$ and initial health deficits $D(0) = D(20) = 0.0273$.* In order to get an estimate of a, I assume that before the 20th century, the impact of medical technology on adult mortality was virtually zero. In the year 1900, the life expectancy of a 20-year-old American was 42 years (death at 62; Fries, 1980), implying that a 20-year-old expected to live for 46 additional years. I set a such that a person who abstains from unhealthy consumption and has no access to life prolonging medical technology expects $T = 46$. From this value, I get the estimate $a = 0.0135$.

When the individual is between 20 and 65 years old, I set $w = 27,928$, which is the average labor income for single men in the year 2010 (BLS, 2012). For older individuals, I set $w = 0.45 \cdot 27,928$ using an average replacement rate of 0.45 from the OECD (2016). In order to assure that the savings motive is confined to that of health and consumption expenditure, I assume that the initial and final capital stock are zero. I assume that all labor income and pension payments are liquid but that capital income is illiquid, i.e. for the benchmark case $\tilde{w} = w$. Furthermore, I normalize the price of health and the unhealthy good to unity, $p = q = 1$. This is an interesting benchmark case because it eliminates any price channel through which poor individuals may have an incentive to consume more unhealthy goods or spend less on health. We later investigate the sensitivity of results with respect to alternative prices.

While the model is applicable for unhealthy consumption in general, we need to specify the unhealthy good for quantitative analysis. Since most of the available empirical literature on consumption of unhealthy goods is on cigarettes and tobacco, I begin with a calibration that tries to capture characteristics of cigarette consumption and investigate alternative assumptions with a sensitivity analysis. On average, single male Americans spent \$ 364 on cigarettes in the year 2010 (BLS, 2012). Doll et al. (2004) estimate that for men born between 1900 and 1930, cessation at age 60, 50, 40, or 30 gained, respectively, about 3, 6, 9, or 10 years of life expectancy. Jha et al. (2013) arrive at an even higher estimate and suggest that life expectancy is shortened by more than 10 years among current smokers compared with those who have

*Mitnitski et al. estimate health deficit accumulation for Canadian men. Deficit accumulation within the USA and Canada appears to be similar enough to justify it as a good approximation for the U.S. (Rockwood and Mitnitski, 2007).

never smoked. Schauer et al. (2015) estimate that the mean age of cessation (of those who quit smoking) is about 40 years. Of course, some individuals never quit. According to the CDC (2012), 8 percent of American men aged 65 and over were smoking in the year 2010. These considerations illustrate that any calibration of a Reference American will be a compromise that tries to capture the essence of these stylized facts.

Obviously, the degree of self-control ω is difficult to quantify, mostly due to measurements problems. To deal with this problem, I consider the sensitivity of results with respect to this parameter. In an attempt to assess ω, I relate the benchmark specification to the estimate provided by Kovacs (2016). This study estimates the temptation parameter for Gul and Pesendorfer (2001) preferences, which can be mapped into an estimate of ω. In order to see this, note that maximizing $U(c,u) - \omega\left[U(c_s,u_s) - U(c,u)\right]$ is the same as maximizing $U(c,u) - \frac{\omega}{1+\omega}U(c_s,u_s)$. This means that the utility function is structurally identical with temptation preferences as in Gul and Pesendorfer (2001), where $\omega/(1+\omega) \equiv \lambda$ is the temptation parameter. Using consumer expenditure data for the US, Kovacs (2016, Table 5) estimates $\lambda = 0.2$, implying $\omega = 0.25$.

The remaining seven parameters, A, B, β, γ, σ, ψ, and ζ, are calibrated jointly with ω such that: (i) the model predicts the actual accumulation of health deficits over a lifetime (as estimated by Mitnitski et al., 2002), (ii) death occurs at the moment when $\bar{D}$ health deficits have been accumulated at an age of 77.1 years, (iii-iv) health expenditure matches health care expenditure of American men in 2010 at the age of 35 and 70 (MEPS, 2010), (v) the Reference American spends on average \$ 364 per year on the unhealthy good, (vi) the Reference American quits smoking for good at age 45, and (vii) that consumption of the unhealthy good costs about 7 years of life. This leads to the estimates $A = 0.00177$, $B = 3 \cdot 10^{-6}$, $\beta = 0.365$, $\gamma = 0.19$, $\psi = 0.5$, $\zeta = 650$, and $\sigma = 1.035$.[†]

The model is solved numerically by the relaxation algorithm (Trimborn et al., 2008). The method provides the solution of the non-linear system up to a user specified approximation error (which is set to 10^{-5}).

References

BLS (2012). Consumer Expenditure Survey. U.S. Bureau of Labor Statistics, September 2012 (`http://www.bls.gov/cex/csxcross.htm#y1011`).

[†]The health data from MEPS (2010) represent total health services including inpatient hospital and physician services, ambulatory physician and nonphysician services, prescribed medicines, home health services, dental services, and various other medical equipment and services that were purchased or rented during the year.

CDC (2002). Morbidity and Mortality Weekly Report July 26, 51(29), 642-645, Centers for Disease Control and Prevention, U.S. Department of Health and Human Services.

Centers for Disease Control and Prevention (2012). Current Cigarette Smoking Among Adults United States, Weekly Report 61(44);889-894.

Chaloupka, F.J. and Warner, K.E., 2000, The economics of smoking, *Handbook of Health Economics* Vol. 1, Elsevier, Amsterdam, 1539–1627.

Dalgaard, C-J., and Strulik, H. (2014). Optimal aging and death: Understanding the Preston Curve, *Journal of the European Economic Association* 12, 672-701.

Doll, R., Peto, R., Boreham, J., and Sutherland, I. (2004). Mortality in relation to smoking: 50 years' observations on male British doctors. *British Medical Journal* 328, 1519-1528.

Fletcher, J.M., Deb, P., and Sindelar, J. L. (2009). Tobacco use, taxation and self control in adolescence (No. w15130). National Bureau of Economic Research.

Fries, J.F. (1980). Aging, natural death, and the compression of morbidity. *New England Journal of Medicine* 303(3), 130-135.

Gul, F. and Pesendorfer, W. (2001). Temptation and self-control, *Econometrica* 69, 1403-1435.

Jha, P., Ramasundarahettige, C., Landsman, V., Rostron, B., Thun, M., Anderson, R. N., ... and Peto, R. (2013). 21st-century hazards of smoking and benefits of cessation in the United States. *New England Journal of Medicine* 368(4), 341-350.

Jorda, O., Knoll, K., Kuvshinov, D., Schularick, M., and Taylor, A.M. (2017). The Rate of Return on Everything, 18702015. National Bureau of Economic Research Working Paper w24112.

Kovacs, A. (2016). Present Bias, Temptation and Commitment Over the Life-Cycle: Estimating and Simulating Gul-Pesendorfer Preferences, Oxford University, Discussion Paper No. 796.

MEPS (2010). U.S. Department of Health & Human Services. Center for Financing, Access and Cost Trends, Agency for Healthcare Research and Quality: Medical Expenditure Panel Survey, 2010 (`https://meps.ahrq.gov/mepsweb/data_stats/quick_tables_results.jsp?component=1&subcomponent=0&year=-1&tableSeries=1&searchText=&searchMethod=1&Action=Search`)

Mitnitski, A.B., Mogilner, A.J., MacKnight, C., and Rockwood, K. (2002). The accumulation of deficits with age and possible invariants of aging. *Scientific World* 2, 1816-1822.

NVSS (2014). US department of health and human services, national vital statistics system. National Vital Statistics Report 63(7).

OECD (2016), Gross pension replacement rates. OECD Data (`https://data.oecd.org/pension/gross-pension-replacement-rates.htm`

Rockwood, K. and Mitnitski, A.B. (2007). Frailty in Relation to the Accumulation of Deficits. *Journals of Gerontology Series A: Biological and Medical Sciences* 62, 722-727.

Schauer, G.L., Malarcher, A.M., and Asman, K.J. (2015). Trends in the Average Age of Quitting Among US Adult Cigarette Smokers. *American Journal of Preventive Medicine* 49(6), 939-944.

Trimborn, T., Koch, K.-J., and Steger, T.M. (2008). Multidimensional transitional dynamics: A simple numerical procedure. *Macroeconomic Dynamics* 12, 301319.