Knowledge and Growth in the Very Long Run: Online Appendix: A Network Theoretic Foundation of Knowledge Diffusion

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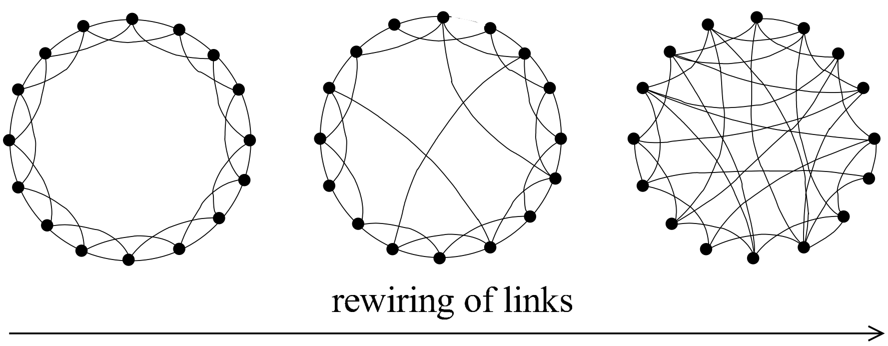
This section illustrates how the knowledge diffusion function $\sigma(k)$ can be derived from an evolving network, the Small World model. The Small World model was originally developed by Watts and Strogatz (1998) and has since been empirically verified for a plethora of biological and social phenomena.[1] The Small World model is particularly suited for the analysis of economic integration because a measure of localization, the clustering coefficient, can be calculated analytically. This way, the impact of network evolution on knowledge diffusion can be conveniently analyzed without any concrete specification of the network itself.

Consider a network in which vertices (nodes) are persons and edges (links) indicate whether any two persons know each other. A stylized version of such a network, a network on a one-dimensional lattice, is shown in Figure 3. The Small World model interpolates smoothly between a regular network, i.e. a network with the highest degree of local connections (left hand side of Figure 3), and a random network, i.e. a network with the highest degree of global connections. This is done by rewiring edges with other edges that are randomly chosen from the net. The process of network evolution and increasing globalization is approximated by continuously increasing from zero to unity the fraction p of rewired edges (movement from the left to the right in Figure 3).

Figure 3: Small World Model



Left hand side: one-dimensional lattice with connections between all vertex pairs by two or fewer lattice spaces away. A Small World is created by randomly rewiring edges.

The clustering coefficient is an indicator for the connectivity of local communities. It gives the average probability that two persons which are connected to any person are also directly connected with each other, that is the probability that someone's friends (trading partners) are also friends (trading partners) with each other. A high clustering coefficient indicates a collectivistic society, in which information is exchanged mainly between neighbors, whereas a low clustering coefficient indicates an individualistic society, with long-distance information exchange. Based on insight from other research we assume that the degree of knowledge diffusion σ is high when the clustering

[1]See Newmann (2003), for an overview. Serrano and Beguna (2003) show that the world trade trade-web is well represented by the Small World network characteristics

coefficient is low.[2]

Generally, the clustering coefficient depends on network size and the number of links per node. Normalizing the clustering coefficient by measuring it relative to the highest possible clustering, however, eliminates all network specifics and yields the clustering coefficient $\tilde{c}$ as a simple function of the fraction of long distance links, $\tilde{c}(p) = (1-p)^3$, see Newman (2003). The inverse relation between the degree of knowledge diffusion and the clustering coefficient can thus be written as $\sigma = 1 - \tilde{c}(p) = 1 - (1-p)^3$. In contrast to Watts and Strogatz' original approach, in which nature randomly created long-distance links, we assume that the formation of long-distance link depends on economic activity. Specifically, it seems reasonable to assume that the relative number of long-distance links depends positively on aggregate capital stock because capital consists partly of effective distance reducing devices (like horses, ships, cars, or airports). A larger capital stock thus increases the probability that a particular long-distance link between two nodes exists. Because $p \in (0,1)$, the most parsimonious and general formal notation of this notion is given by (1).

$$p = \frac{g(k)}{\omega + g(k)}, \quad g(0) = 0, \quad g'(k) > 0. \tag{1}$$

This generates a monotonous mapping from k to $p \in (0,1)$ in which the constant ω controls for capital-independent factors (for example, spatial size of the economy and other geographic factors). Inserting the probability of a long-distance link into the formula for the clustering coefficient and then into the degree of knowledge diffusion we get

$$\sigma(k) = 1 - \left(\frac{\omega}{\omega + g(k)}\right)^3. \tag{2}$$

The final step is to check whether the network-based notion of knowledge diffusion supports the proposed theory of knowledge and growth, that is to check whether it is consistent with Assumption 3. Inserting $\sigma(k)$ and its derivative into $b(k)$ and simplifying we obtain

$$b(k) = \left(\frac{\omega}{\omega + g(k)}\right)^3 \cdot \left(\frac{3g'(k)}{\omega + g(k)} k \log k - 1\right).$$

For Assumption 3 to be fulfilled, $b(k)$ has to change its sign exactly once. Since the first term is always positive and the second term is negative for $k < 1$, it is sufficient to show that the term

$$\frac{g'(k)k}{\omega + g(k)} \log k$$

rises monotonously for $k \geq 1$. This is a very mild requirement. For example, it is straightforward to show that it is fulfilled for linear ($g(k) = Bk$), exponential ($g(k) = \exp(k)$) and iso-elastic ($g(k) = k^\theta, \theta > 0$) "production functions". It is actually hard to come up with a function $g(k)$ not fulfilling Assumption 3. The theory is thus largely independent from structure. The essential assumption is that more capital is good for long-distance link formation, $g'(k) > 0$. This seems to be a relatively mild constraint and intuitively plausible.

References

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[2] Hofstede (2001) and Fogli and Feldkamp (2011). This notion is consistent with the finding of predominantly local knowledge diffusion in LDCs (Foster and Rosenzweig, 1995) and global knowledge diffusion in fully developed countries (Irwin and Klenow, 1994).

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