Patience and Prosperity

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Abstract. This paper introduces wealth-dependent time preference into a simple model of endogenous growth. The model generates adjustment dynamics in line with the historical facts on savings and economic growth in Europe from the High Middle Ages to today. Along a virtuous cycle of development more wealth leads to more patience, which leads to more savings and further increasing wealth. Savings rates and income growth rates are thus jointly increasing during the process of development until they converge towards constants along a balanced growth path. During the transition to modern growth an economy in which the association of wealth and patience is stronger overtakes an otherwise identical economy and generates temporarily diverging growth rates.

Keywords: economic growth, savings, time preference, poverty trap, moral consequences of economic growth.

JEL: O11, O41, D90, P48.

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1. Introduction

This article proposes a simple preference-driven theory of economic growth. It establishes a savings channel through which the character trait of patience operates simultaneously as cause and consequence of economic growth.

The theory is helpful in explaining the historical evolution of savings, interest rates, and economic growth in the very long-run, i.e. from the Middle Ages until today. In order to see how standard neoclassical growth theory has problems in getting the historical adjustment dynamics right recall that, according to the famous Ramsey rule, economic growth in a neoclassical framework is obtained as the difference between the interest rate r and the time preference ρ scaled by the intertemporal elasticity of substitution $1/\theta$, $g = (r - \rho)/\theta$. Far below the steady-state, capital per capita is low and the return to capital $r(k)$ is high. Consequently the incentive to invest (or save) is high and the model predicts high growth far off the steady-state. Imposing a constant time preferences rate, the standard model then predicts that $r(k)$ declines with ongoing capital accumulation such that the savings rate and economic growth jointly decline as people get richer. These predictions are at odds with the facts of our historical economic development.

Savings rates are higher for richer people across individuals [20] and for richer countries across countries [33]. Within countries, savings rates usually rise with economic development. In England, for example, investment rates have been estimated as about 4% in 1688 [16] and then rising during industrialization from around 7% in 1760 to 14% in 1800 [14]. Across 142 contemporaneous countries, David Weil [47] documents that the savings rate is about 5 percent on average for countries in the lowest income decile (i.e. those most closely to subsistence and most similar to England and other European countries in the Middle Ages). The savings rate then gradually increases with income. It is about 10 percent in countries in the second decile, about 20 percent for seventh decile and somewhat above 30 percent for tenth decile.

Likewise the historical evolution of economic growth rates has been gradual. The annual rate of GDP per capita growth for England, for example, was 0.0% from year 1 to 1000, 0.1% from 1000 to 1500, 0.2% from 1500 to 1700. In the 19th century growth gains momentum; it is computed as 0.3% from 1700 to 1820, 1.3% from 1820 to 1870, 1.0% from 1870 to 1913, 1.2 % from 1913 to 1960, and 2.1% from 1960 to 2000 (calculated from the Maddison [34] data). While the process of gradual take off was roughly the same everywhere in Western Europe (and the Western offshoots) there are also important differences discernable across countries. In particular there was a European “reversal of fortune". Output in Italy, France, and Belgium, was higher than in England and the Netherlands until the 16th century while the opposite was true after the 17th century.

The present theory explains these phenomena by wealth-dependent time preference. As already observed by Irving Fisher [23], people become more patience as they get richer. Since it is historically true that interest rates declined during long run development, the theory has to explain not only that time preference rates decline but also that they decline more strongly with wealth accumulation than interest rates such that declining interest rates are observed together with increasing savings rates and increasing growth of income per capita.[1] In this framework the overtaking of countries is explained by cultural differences in the association between wealth and patience. When the development process gets momentum it is further amplified through increasing savings rates in countries where the association between wealth and patience is particularly strong. This mechanism could express itself, for example, in that the population in these countries becomes more open towards doctrines that reward savings vis a vis current consumption (the Protestant belief system [46]). This way, small differences in time preferences can generate temporarily large cross-country differences in economic performance.

Empirically it has been found that time preference rates decrease with income and wealth in panel data analysis ([32], [40]) and large-scale experiments [18]. Unfortunately, many empirical studies on intertemporal consumption behavior impose a constant time-preference rate. In that case we expect the fact of a positive association between wealth and the time preference rate to be reflected in the estimated intertemporal elasticity of substitution. Studies that control for wealth effects have indeed found that the intertemporal elasticity of substitution is higher for richer individuals (e.g. [2], [26]).

Investigating the evolutionary aspects of long-run economic development Gregory Clark [13] observes that people became substantially more patient before the onset of the industrial revolution. Inspired by Galor and Moav [25] he discards improvements of institutions and life-expectancy for being not sufficient to having provoked such a substantial drop of time preference and provides instead an evolutionary argument based on Rogers [39]. According to this view the institutionally stable agrarian societies of the Middle Ages created a selective pressure towards lower time preference. More patient people were wealthier which allowed them to have more surviving children and to bequeath more wealth to them, a fact that amplified the genetic transmission of patience.

[1] According to Homer and Sylla [27] interest rates where between 10 and 20 percent in Europe during the high Middle Ages and about as high as interest rates in the ancient Roman, Greek and Babylonian empires. In the later Middle Ages interest rates started to decline towards low levels of about 4 to 6 percent, i.e. towards rates that are observed in modern times as well.

Doepke and Zilibotti [17] argue in favor of a related yet conceptually different mechanism. They propose that patience is transmitted through cultural evolution. Parents expecting a steep lifetime income profile for their children are more inclined to invest resources in their children's patience. As a consequence their children are more likely to become skilled and rich. Children of the already rich in turn are facing a high but flat income profile and their parents are more inclined to invest in their taste for leisure implying that "success carries the seed of its own destruction" i.e. the rise and downfall or patient and rich dynasties.[2]

Of course, economists acknowledge that the rate of time preference is not a natural constant. As observed by Obstfeld [37], "mathematical convenience rather than innate plausibility has always been the main rationale for assuming time-additive preferences in economic modeling." While this convenience is certainly welcome and justified for many research questions, it is particularly limiting when the focus of investigation is on the evolution of savings rates over time. As a result of this insight there already exists a literature of endogenous time preference to which the present paper contributes.[3] Most closely related to the present work is the paper by Schumacher [42] which integrates wealth-dependent time preference into the neoclassical growth model and demonstrates the possibility of multiple equilibria of stagnation. Long-run growth, however, is not considered. Offering a theory of endogenous growth and endogenous patience that explains the gradual take-off from stagnation to modern growth is the novel contribution of the present article.

2. The Model

Consider an economy populated by a mass one of individuals consuming c and holding wealth k. As usual individuals maximize discounted intertemporal utility from consumption. In deviation from the standard model, however, the discount rate depends positively on wealth. As individuals get richer they become more patient.

$$\max_c V_0 = \int_0^\infty u(c) \mathrm{e}^{-\int_0^t \rho(k(v))\mathrm{d}v} \mathrm{d}t, \qquad c(t) > 0 \;\; \forall t \tag{1}$$

[2] By way of contrast, if time preference rates differ across individuals in the neoclassical growth model, the economy is predicted to converge towards a steady-state at which all capital is hold by the most patient individual [5]. Integrating cultural evolution a la Doepke and Zilibotti in the neoclassical framework could potentially eliminate this counterfactual prediction. For the following exposition it could thus be helpful to imagine time preference of the representative dynasty as representing time preference of an average dynasty in an economy, in which the identity of rich and poor families depends on cultural evolution and luck and perhaps changes over time, while the average dynasty gets gradually richer and more patient.

[3] Contributions have been made, inter alia, by [22], [37], [19], [4], [38], [15], [11],[12], [41], and [6].

with $\rho(k) > 0$, $\rho'(k) < 0$, $\rho''(k) > 0$ and $\lim_{k\to\infty} \rho(k) = \bar{\rho} > 0$, which is a formal representation of the idea that individuals become more patient as they get wealthier and that the time preference rate approaches a constant from above as wealth goes to infinity. Instantaneous utility is assumed to be iso-elastic, $u(c) = c^{1-\theta}/(1-\theta)$, $\theta \neq 1$.

Individuals supply one unit of labor and receive wage income w and capital income rk, where r is the interest rate (net of depreciation). Income is spent on consumption and investment, implying the budget constraint (2).

$$\dot{k} = w + rk - c \geq 0. \tag{2}$$

With the focus on the take-off to balanced growth we avoid distracting discussion of what could potentially happen below subsistence by requiring consumption to be positive and investment to be non-negative. This implies that any economy not following a path towards balanced growth is stagnating at subsistence level. The setup in continuous time is useful in order to develop an elegant representation of the economy that facilitates the analysis of stability of equilibria and transition dynamics.[4] The solution of maximizing (1) with respect to (2) leads to the Ramsey rule (3), see Appendix for details.

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left[r - \rho - \frac{\rho'}{\rho}\frac{c}{1-\theta} - \frac{\rho'}{\rho}(w + rk - c)\right]. \tag{3}$$

For $k \to \infty$ and thus $\rho \to \bar{\rho} > 0$ and $\rho' \to 0$, the result collapses to the familiar Ramsey rule of the standard neoclassical growth model, $\dot{c}/c = (r - \bar{\rho})/\theta$. Compared with the standard model we get two additional effects of wealth-dependent time preference on growth: a level effect and a growth effect. The level effect is reflected by the term $(-\rho'/\rho)c/(1-\theta) = (-\rho'/\rho)u(c)/u'(c)$. Since $\rho' < 0$ the term is negative if $u(c) < 0$. The level effect is reminiscent of the earlier literature on endogenous time preference where $u(c) < 0$ is frequently assumed in order to generate stability (see e.g. [37]). Here, the level effect is irrelevant for stability and convergence towards balanced growth, as will be shown below. For the case of $\theta > 1$ utility is indeed always negative with a bliss point at zero such that the level effect vanishes with economic growth. For $\theta < 1$ the level effect is positive and determined by two counteracting forces. It is increasing with economic growth through the $u(c)/u'(c)$ term and decreasing with economic growth through the $= (-\rho'/\rho)$ term because ρ' vanishes as people become

[4] Since we are considering economic dynamics over the very long-run, the appropriate notion is that of a continuous time approximation of an infinite series of altruistically linked generations. The accompanying working paper [43] shows how problem (1) – (2) can be derived from the expansion of a two-period model of altruistically linked individuals with finite lifes.

richer. This interaction can produce overshooting behavior of the savings rate along the transition towards balanced growth.

To see that the last term in (4) reflects a growth effect on patience recall that $\dot{k} = w + rk - c$. The growth effect is unambiguously positive. Living in a growing economy makes people, ceteris paribus more patient. As a consequence they choose a steeper profile for consumption growth ($\dot{c}/c$ is higher) implying that they save more. The growth channel implies that patience can be simultaneously conceptualized as a cause and consequence of economic growth. The model thus supports the observation that higher growth causes higher savings. It relates also to Friedman's [24] "Moral consequences of Economic Growth". When people have a sense of "getting ahead" they appreciate more future gains of foregone consumption and are inclined to save more today, a behavior that further amplifies economic growth.

The fact that $\dot{\rho} = \rho' \dot{k}$ allows to write the partial differential equation (3) as an ordinary differential equation as in (4).

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left[r - \rho - \frac{1}{1-\theta}\frac{\dot{\rho}c}{\rho\dot{k}} - \frac{\dot{\rho}}{\rho}\right]. \tag{4}$$

In order to proceed with a solution we introduce a simple functional form for the evolution of time preference.

$$\rho(k) = \bar{\rho} + \rho_0 k^{-\eta} \qquad \Rightarrow \qquad \dot{\rho} = -\eta(\rho - \bar{\rho})\dot{k}/k. \tag{5}$$

The parameter $\eta > 0$ controls the speed at which time preference converges towards $\bar{\rho}$ when the economy is growing. Note that ongoing wealth accumulation has no effect on time preference once ρ has reached $\bar{\rho}$. This feature will cause balanced growth. Since there exists a natural lower bound for time preference at zero, the condition for balanced growth is a natural one.[5]

Output is produced according to a linear-concave production function, as introduced by Jones-Manuelli [29] (see also [3]). This means that production consists of a neoclassical part with diminishing returns and of a linear part with constant returns, i.e. $y = Ak + Bk^{\alpha}\ell^{1-\alpha}$ where ℓ is labor input. This formulation has the advantage that results can be easily compared to standard endogenous growth theory with and without adjustment dynamics. The implied interest rate and wage rate are $r = A + \alpha B k^{\alpha-1}\ell^{1-\alpha}$ and $w = (1-\alpha)Bk^{\alpha}\ell^{-\alpha}$. As usual the model becomes more realistic when we interpret capital broadly, consisting of physical and human capital. Inserting factor prices into the

[5] Evolutionary economists argue that the natural lower bound of time preference is around 0.02; see [39].

budget constraint we get the equation of motion (6).

$$\dot{k} = Ak + Bk^{\alpha} - c. \tag{6}$$

The linear part of production implies the potential for perpetual accumulation-driven growth. In order to allow for positive long-run growth we assume $A > \bar{\rho}$. If we imagine that manufacturing operates with constant returns to scale with respect to the accumulable factors (physical and human capital) while agriculture is subject to diminishing returns because of limiting land, then the setup can be interpreted as a reductionist model of industrialization in which the indicator $\lambda \equiv Bk^{\alpha}/y$ measures the relative contribution of the diminishing-returns part (i.e. agriculture) to GDP.[6]

Since there is potential for long-run growth of c and k, we introduce for steady-state analysis the auxiliary variable $x \equiv c/k$, solve (5) for k, $k = [(\rho - \bar{\rho})/\rho_0]^{-1/\eta}$, and transform (4)–(6) into (7).

$$\dot{\rho} = g(\rho, x) = -\eta(\rho - \bar{\rho}) \left[A + B \left(\frac{\rho - \bar{\rho}}{\rho_0} \right)^{\frac{1-\alpha}{\eta}} - x \right]. \tag{7a}$$

$$\dot{x} = h(\rho, x) = \frac{x}{\theta} \left\{ A + \alpha B \left(\frac{\rho - \bar{\rho}}{\rho_0} \right)^{\frac{1-\alpha}{\eta}} - \rho + \eta \left(\frac{\rho - \bar{\rho}}{\rho} \right) \left(A + B \left(\frac{\rho - \bar{\rho}}{\rho_0} \right)^{\frac{1-\alpha}{\eta}} + \frac{\theta x}{1-\theta} \right) \right\}$$
$$- \left[A + B \left(\frac{\rho - \bar{\rho}}{\rho_0} \right)^{\frac{1-\alpha}{\eta}} - x \right] x. \tag{7b}$$

The model is fully described by this dynamic system in ρ and x. Note that the time preference rate is a (quasi)-state variable. It is tied down by the state of the economy k and cannot jump instantaneously.

3. Equilibrium Analysis

3.1. **Simplified Model.** Before we investigate the full model it is instructive to start with a simplified version by assuming $B = 0$. The simplified model entails the shortcoming of a constant interest rate, which is – as noted in the Introduction – inconsistent with the historical fact of declining interest rates in the very long-run. Inspecting the simplified version is nevertheless useful because it allows us to obtain conveniently the main mechanics of savings and growth. The same mechanics are also driving the full model, which is harder to analyze analytically. For $B = 0$ the model reduces

[6] This reductionist approach is only justified because industrialization in the sense of structural change is not the focus of the present paper. See, for example, [45] for a proper model on structural change during the industrial revolution.

to system (8).

$$\dot{\rho} = -\eta(\rho - \bar{\rho})(A - x). \tag{8a}$$

$$\dot{x} = \frac{x}{\theta}\left[A - \rho + \eta\left(\frac{\rho - \bar{\rho}}{\rho}\right)\left(A - x + \frac{x}{1-\theta}\right)\right] - (A - x)x. \tag{8b}$$

Note that the model collapses to the textbook Ak growth model if we assume that $\rho = \bar{\rho}$ for all t. In contrast to the textbook model, the version with endogenous time-preference displays adjustment dynamics. The next propositions, proven in the Appendix, establish the existence of two distinct equilibria, a steady-state of stagnation and a steady-state of balanced growth.

Proposition 1. *There exists a steady-steady-state of stagnation at subsistence level where*

$$x = x^* = A, \qquad \rho = \rho^* = \frac{A + \phi}{2} + \sqrt{\frac{(A+\phi)^2}{4} - \phi\bar{\rho}}, \quad \phi \equiv \eta A/(1-\theta). \tag{9}$$

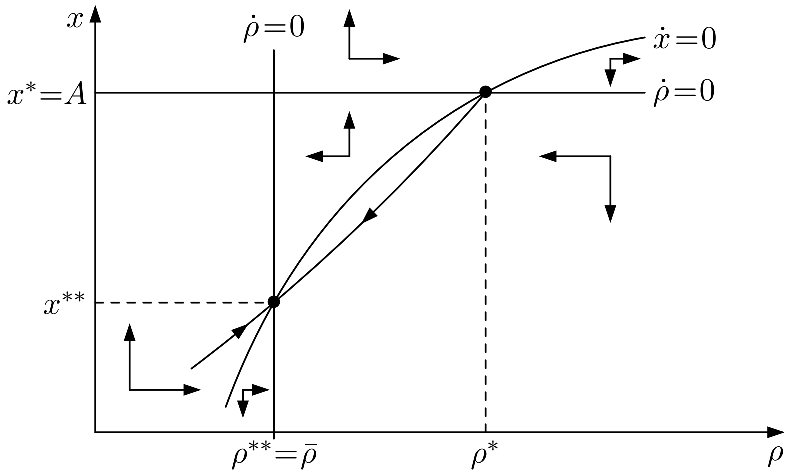
Proposition 2. *If $\bar{\rho}/A > 1 - \theta$, then there exists a unique balanced growth path along which the economy grows at rate $(A - \bar{\rho})/\theta$ and where x^{**} and ρ^{**} are given by (10).*

$$x = x^{**} = \frac{(\theta - 1)}{\theta}A + \frac{\bar{\rho}}{\theta}, \qquad \rho = \rho^{**} = \bar{\rho}. \tag{10}$$

The condition for balanced growth to exist is the same as for the standard Ak-model. It requires that utility is bounded such that x^{**} assumes a constant positive value (cf. [3]). Naturally, since ρ converges towards a constant, the balanced growth path coincides with the one obtained for the standard Ak growth model. The value added of the present model does not lie in its steady-state characteristics but in the new transitional dynamics.

In order to analyze transitional dynamics and stability of equilibria within a phase diagram infer from (8a) that there are two loci where $\dot{\rho} = 0$. In a (ρ, x) – diagram, the equilibrium of stagnation is found along the horizontal isocline where $x = A$. The equilibrium of balanced growth is found along the vertical isocline where $\rho = \bar{\rho}$. Inspect (8a) to conclude that $\dot{\rho} < 0$ in the area to the right of the $\bar{\rho}$–line and below the A–line. The opposite is true if x is above A and if ρ is below $\bar{\rho}$. Next compare x^* and x^{**} and conclude that $x^{**} < x^*$. From this follows that the $\dot{x} = 0$-curve has negative slope. Its intersection with the horizontal $\dot{\rho} = 0$-isocline identifies x^* and the intersection with the vertical $\dot{\rho} = 0$-isocline identifies x^{**}. The arrows of motion point towards lower x below the $\dot{x} = 0$ curve and towards higher x above (see Appendix for details).

Figure 1: Phase Diagram



Inspect the resulting field of arrows of motion in Figure 1 to conclude that the equilibrium (x^*, ρ^*) is unstable and the equilibrium $(x^{**}, \bar{\rho})$ is a saddlepoint. Any path other than the stable saddlepath leads either to $x = 0$ and thus zero consumption in finite time or to $x = \infty$ and thus zero capital stock in finite time. Intuitively, following a path towards $x = 0$ implies too much savings while following a path towards $x = \infty$ implies too little savings in order to sustain positive consumption in the long-run. Since capital is essential in production these paths imply zero consumption in finite time as well. This means that the first order conditions do not provide an interior optimal solution for $k(0) < k^*$, i.e. for $\rho(k(0)) > \rho^*$. Instead the economy assumes the corner solution, $x^* = A$, implying $c^* = Ak = y$, i.e. stagnation at subsistence level. For $k(0) > k^*$, i.e. for $\rho(k(0)) < \rho^*$ the only path not violating the first order condition and the positive consumption requirement is the stable arm leading to the saddlepoint equilibrium (x^{**}, ρ^{**}). In retrospect, if we observe balanced growth today, the economy must have developed along the stable arm. The savings rate is defined as $s = 1 - c/y$, i.e. $s = 1 - x/A$. It is thus rising along the stable arm from stagnation towards balanced growth. The following proposition summarizes these findings.

Proposition 3. *The growing economy follows a unique path towards balanced growth. During transition the time preference rate decreases and the savings rate increases.*

For an intuitive assessment of adjustment dynamics consider an economy that is initially very close to the steady-state of stagnation. In this situation capital stock and production are small and the time preference rate is large. Almost all income is consumed implying that c is close to

Ak. This in turn means that there are almost no (net) savings and $\dot{k}$ is close to zero. Having only a relatively short time-window of observations at hand one would conclude that the economy stagnates. At glacier speed, however, capital is accumulated and people become more patient with rising wealth. More patience leads to a higher savings rate (lower x) and further rising capital (wealth). Sooner or later economic growth gets momentum and the economy travels with rising savings rates towards the balanced growth path. During the transition both the savings rate and the growth rate are perpetually rising. The feature that produces this realistic outcome is wealth-dependent time preference. Higher economic growth makes people wealthier and thus more patient, which in turn triggers more savings and leads to even higher subsequent growth.

It is interesting to note that the Ak growth model with endogenous time preference reverses the mechanics of the neoclassical growth model. There, the interest rate $r(k)$ is endogenous and the time preference rate $\bar{\rho}$ is exogenous and constant. This implies that economic growth, $g_c = [r(k) - \bar{\rho}]/\theta$, is at its highest level initially, when wealth k is smallest and the return on capital is largest. With ongoing wealth accumulation, the growth rate is continuously falling because of decreasing returns on capital, $r'(k) < 0$. Thus neither the standard Ak model (with no adjustment dynamics at all) nor the neoclassical growth model can explain the historical adjustment dynamics sketched in the Introduction, i.e. the gradual take off with increasing growth rates over time. The present modification of the model gets the historical adjustment dynamics right by holding capital productivity A constant and showing that $\rho(k)$ decreases with increasing k such that economic growth increases with ongoing wealth accumulation..

3.2. **The Full Model.** While the simplified model manages to generate reasonable paths for savings and growth it fails to predict the historical downward trend of interest rates documented by Homer and Sylla [27]. This shortcoming is an inevitable side-effect of a linear technology. The full model repairs the shortcoming through a representation of neoclassical economics in production. This can be seen form (6): as long the economy is growing, i.e. as long as $\dot{k}/k = A + Bk^{\alpha-1} - x > 0$, the interest rate $r = A + \alpha Bk^{\alpha-1}$ is converging towards A from above. Because the influence of the neoclassical part of capital productivity vanishes over time, the long-run growth path is, of course, the same as for the simple Ak growth model.

PROPOSITION 4. *If $\bar{\rho}/A > 1 - \theta$, then there exists a unique balanced growth path along which the economy grows at rate $(A - \bar{\rho})/\theta$ and where x^{**} and ρ^{**} are given by (10).*

At the steady-state of stagnation $x^* = A + B\left[(\rho - \bar{\rho})/\rho_0\right]^{(1-\alpha)/\eta}$. Using this information in (7b) evaluated at a steady-state we can numerically obtain ρ^*. The solution, however, needs not to be unique. In order to avoid distracting case differentiation for the purpose of this paper the following result, proven in the Appendix, is helpful.

Proposition 5. *The steady-state (x^{**}, ρ^{**}) is a saddlepoint.*

This means that there exists, as for the simple model, a unique path towards balanced growth on which we focus the subsequent analysis.[7]

4. The Slow Transition Towards Modern Growth: A Calibration Study

This section shows that the numerically calibrated model can approximate the historical evolution of growth, savings, and interest rates from the Middle Ages to today reasonably well. Since the standard neoclassical model fails in this regard, endogenous time preference is helpful in explaining the historical gradual transition from stagnation to growth. We also address the phenomenon of overtaking of less patient countries in the Middle Ages.

We begin by determining the balanced growth path. The savings rate, $s = 1 - c/y$, along the balanced growth path is $s^{**} = 1 - x^{**}/A$ and the implied growth rate is $(A - \bar{\rho})/\theta$. For the calibration we assume that the balanced growth rate is 2 percent annually and the consumption share of income is 70 percent, implying a savings rate of 0.3. A steady-state growth rate of 2 percent is frequently calibrated for fully developed countries (see e.g. [30]) and a savings rate of 0.3 is observed on average for the richest countries [47]. Imposing $\theta = 2$, a frequently used value in calibration exercises, provides the estimates $A = 0.0667$ for the steady-state return on capital and of $\bar{\rho} = 0.0267$ for the steady-state time preference rate. A real rate of return on capital around 7 percent accords well with the average real return on the stock market for the last century and has been used in other calibration studies as well (e.g. [28]).

The remaining parameters $\alpha = 1/3$, η, B, and ρ_0, do not affect long-run growth. We normalize $B = 1$ and assign values to the remaining parameters such that the model economy can, in its benchmark run, replicate approximately the historical evolution of savings, interest rates, and growth for England. This means that the economy starting with almost zero growth in the High Middle

[7] Note that the stable saddlepath must also go through the steady state (x^*, ρ^*). This proves that the steady-state of stagnation nearest to the steady-state of growth is unstable. If there are several steady-states of stagnation, the steady-state (x^*, ρ^*) separates a steady-state of growth from a locally stable steady state of stagnation (x^{***}, ρ^{***}). Stagnation and the possibility of convergence clubs are discussed in the accompanying working paper [43]. See also, in context of the neoclassical growth model, [42].

Ages reaches its highest acceleration of growth during industrialization in the 19th century, that the savings rate increases from 4 percent in 1700 to 11 percent in 1840 as documented by Crafts [14], and that the interest rate falls from a high initial level (at the steady-state of stagnation) of 10.9 percent as documented by Clark [13] for England in the 13th century towards $\bar{\rho}$. This provides the estimates $\alpha = 0.33$, $\eta = 0.35$ and $\rho_0 = 0.12$.

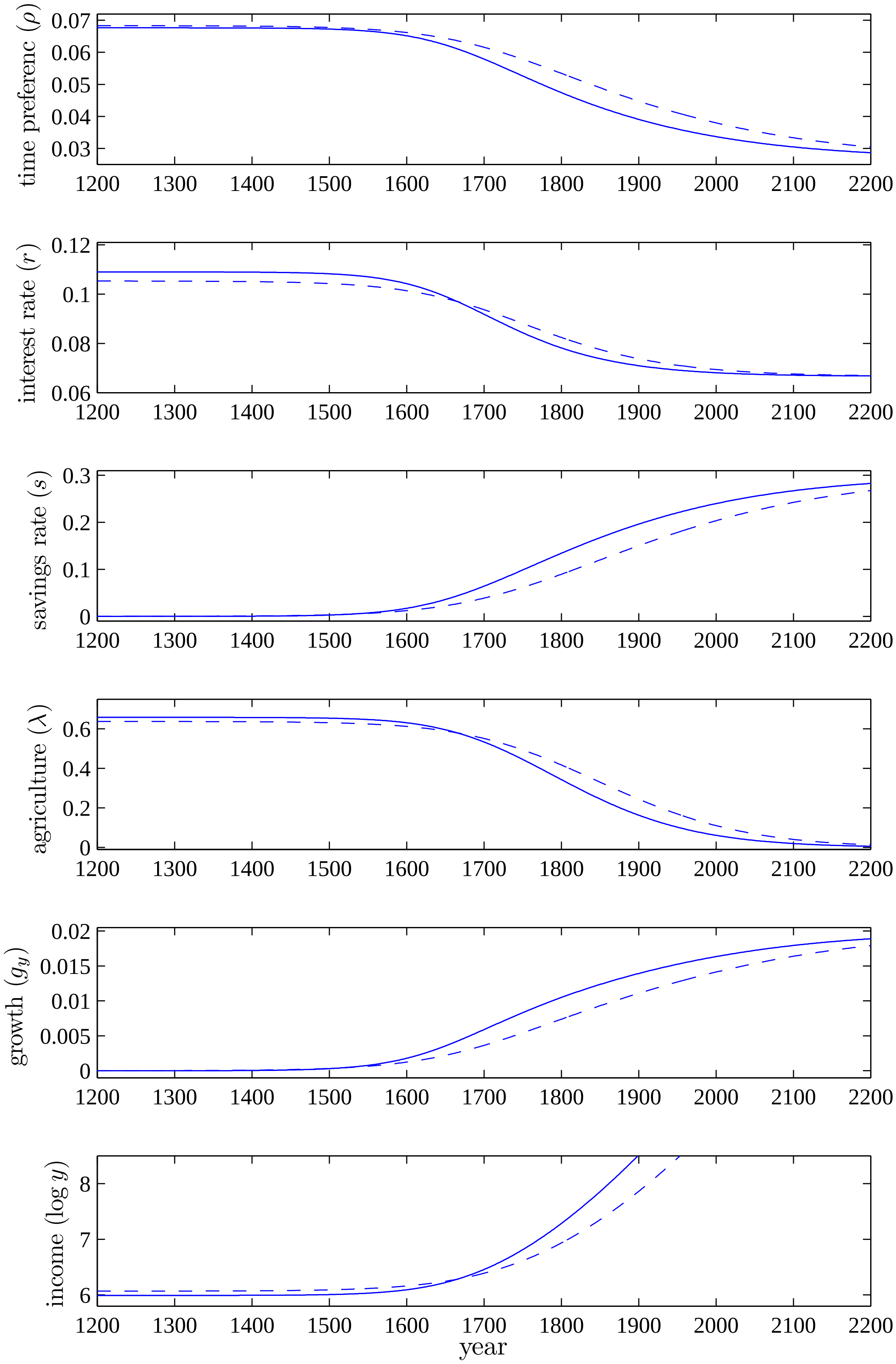
In order to extract the path from stagnation towards balanced growth we employ the method of backward integration. The integration starts when the economy is arbitrarily close to the steady-state of growth $(\bar{\rho}, x^{**})$ and solves the dynamic system (7) backwards until it comes close to the steady-state of stagnation (ρ^*, x^*). The calculation is terminated when $\rho = 0.999\rho^*$. A reversion of time provides the adjustment trajectories for the actual economy. The method solves the non-linearized system up to an arbitrarily small error, see Brunner and Strulik [7] for details.

Figure 2 shows adjustment dynamics. For better comparison with the data, initial time has been normalized such that initially $t(0) = 1200$ and initial income has been normalized such that $y(0) = 400$ (corresponding to England's GDP per capita in year 1200 according to Maddison [34]). The calibration for England is reflected by solid lines. The adjustment process towards modern growth is characterized by slow growth in the Middle Ages. In line with the historical evidence, growth rates are below 0.005 percent before year 1500 and about 0.2 percent around the year 1600. Afterwards, the economy visibly takes off.

While the take-off is visible, compared with modern standards the rate of economic growth is nevertheless small during this period. In line with the empirical observations the model produces a growth rate of about 0.5 percent around 1700, of about 1 percent around 1800 and of about 1.3 percent in 1900. In the early 19th century, when historically the industrial revolution takes place, the model produces the highest momentum, i.e. the *rates of change* of growth, patience, and savings are the highest. Given its simplicity the model predicts the actual evolution of income remarkably well. For the year 1900 it predicts a GDP per capita of 4812 (log(4812)=8.47) while Maddison [34] observes a value of 4492. The model also predicts the historical take-off of savings rates reasonably well. It predicts a savings rate of about 5 percent for 1700 and of about 13 percent in 1800. In the late 19th century the acceleration of growth begins to lose momentum while the savings rate continues to rise. At the end of the 20th century the growth rate and savings rate are close to their long-run steady-state levels.[8]

[8] Some countries seem to have experienced their highest savings rates in the 1950s and 1960s. For particular parameter values the model can replicate such overshooting behavior of the savings rate along the adjustment path to balanced growth. Essential for this behavior is that $\theta < 1$. In that case it can happen that the level effect of time preference

Figure 2: Transitional Dynamics During the 2nd Millennium



Parameters: $A = 0.0667$, $\alpha = 0.33$, $\bar{\rho} = 0.0267$, $\theta = 2$, $B = 1$, $\rho_0 = 0.11985$. Solid lines: $\eta = 0.35$ (Model-England). Dashed lines: $\eta = 0.33$ (Model-Italy).

The model predicts in line with the historical record that interest rates decline over time with increasing accumulation of wealth. The result of simultaneously falling interest rates and rising growth rates is far from self-evident since falling interest rates per se lead to falling growth rates. In a standard neoclassical framework with constant time preference rate declining interest rates perpetually reduce the incentive to invest and therewith economic growth (according to $g_c = (r(k) - \rho)/\theta$). Here, time preference decreases more strongly in wealth than interest rates such that savings and growth increase along the transition to balanced growth.

The duration of the transition and the qualitative behavior of adjustment dynamics depends sensitively on the assumed strength of the wealth–patience association. This can be demonstrated by an alternative scenario in which $\eta = 0.33$ (instead of 0.35). All other parameters are taken from the basic run, implying that the alternative economy arrives eventually at the same balanced growth path as Model-England. The fact that η is smaller for the alternative economy implies that its citizens are a little less patient not only along the transition path but also, in particular, close to the equilibrium of stagnation. Formally, ρ^* is higher at stagnation. This implies that the propensity to consume is somewhat higher in the less patient economy, i.e. $c'(k)$ is larger at all levels of k. Because $c''(k) < 0$ and $c/k = A$ at the steady-state of stagnation, it then follows that citizens of the alternative economy have to be somewhat wealthier in the neighborhood of the stagnation. In order to keep the analogy to the historical development of countries, we call the alternative, less patient economy "Model-Italy".

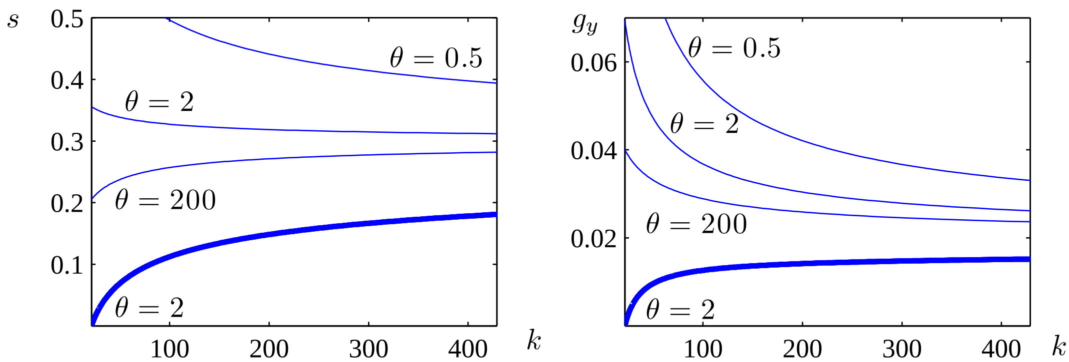
The implied adjustment dynamics are shown by dashed lines in Figure 2. In the 17th century when patience begins to rise visibly in Model-England, it stays almost constant for another century in Model-Italy. The character trait of being more patient and thrifty enables Model-England to overtake Model-Italy's income per capita in the late 17th century, in line with the historical observation. Model Italy experiences industrialization, i.e. a visible take off towards balanced growth by about three generations (75 years) later. During the 19th century, the period of the industrial revolution, the gap between growth rates of Model-England and Model-Italy gets largest and closes slowly afterwards. Actually the model underestimates the catching up process of Model-Italy. Since it has been shown that the model predicts the evolution of income per capita for England quite precisely, this occurs because the model underestimates growth of Model-Italy during the 20th century. For the year 2000 it predicts income per capita of about 9060, a value that was actually reached in 1968

on growth is first increasing (and predominantly affected by rising consumption level c) and then decreasing (and predominantly affected by declining impact of wealth on time preference $\rho'(k)$. See the discussion of equation (3). The Working Paper [43] investigates overshooting behavior in more detail.

[34], indicating that other, neglected mechanisms like, for example international capital flows, are increasingly at work that limit the predictive power of the model for modern economies.[9]

To see that endogenous time preference is essential for getting historical adjustment dynamics right it is instructive to compare with the standard Jones-Manuelli [29] model. The panel on the left hand side of Figure 3 shows the policy function $s(k)$ for the benchmark calibration and alternative specifications of the standard Jones-Manuelli model. The policy function relates the savings rate to the state of the economy (subsumed in state variable k). The bold line shows the policy function as implied by endogenous time preference. Since time preference is constant in the standard case, the only parameter that can be manipulated in order to get the shape of the policy function right is θ, i.e. the degree of relative risk aversion (the inverse of the intertemporal elasticity of substitution).

Figure 3: Savings and Growth: Comparison with Standard Model



Parameters: $A = 0.0667$, $B = 1$, $\alpha = 0.33$, $\bar{\rho} = 0.0267$, $\theta = 2$, $\rho_0 = 0.1198$, and $\eta = 0.35$. k is measured relative to k^*. Bold line: wealth-dependent preference. Thin lines: standard model with constant time preference rate, alternative θ's and otherwise same parameters.

Figure 3 demonstrates that if the patience channel is closed, there is too little curvature of the policy function. Poor people are predicted to save too much. For the benchmark setup of $\theta = 2$ (and lower values) the standard neoclassical model even produces adjustment from the "wrong side", i.e. falling savings rates during economic development. As shown by Barro and Sala-i-Martin [3] this shortcoming can be prevented by imposing (unrealistically) high values for θ, i.e. (unrealistically) low values for the intertemporal elasticity of substitution. But, as the example shows, even if $\theta = 200$

[9] The phenomenon of overtaking by a more patient country has also been investigated by Cavalcanti et al. [10]. There, it is shown that of two countries the one for which it is assumed a higher constant savings rate industrializes earlier. The process of overtaking is initiated through an exogenous parameter change. Specifically, it is assumed that both countries rest at their steady-state and then, in the year 1645, the discount factor for one country shifts from a"Catholic value" to a "Protestant value". By way of contrast, the theory developed in the present paper explains endogenously the gradual, country-specific reduction of time-preference, the differentiated take-off, and the overtaking of Model-Italy. Artige et al.[1] suggest an alternative mechanism for overtaking and (possibly recurring) reversals of fortune based on consumption habits.

(or higher) there is still too little curvature of the savings function such that the model, in general, predicts too fast adjustment from stagnation to balanced growth as compared with the historical evidence.

The panel on the right hand side of Figure 3 shows the associated income growth rates g_y. When the patience channel is closed, diminishing capital productivity is the only impact on growth and the model counter-factually predicts that richer economies grow at lower rates. This means that the standard model gets the positive correlation between savings and growth right for low values of θ but it is then predicting counterfactually that both rates are *falling* during economic development. For high values of θ the standard model counterfactually predicts a negative correlation between savings and growth along the adjustment path to modern growth. Endogenous, wealth-dependent patience is capable to revert these results and to generate adjustment dynamics in line with the historical record.

5. Discussion

The present article has proposed a theory of endogenous patience that explains the long-run evolution of savings and growth in the Western world reasonably well. The approximation of the actual transition to modern growth is perhaps too good since there is little growth left to be explained for other important drivers of economic development. A reduction of η, for example, would slow down the adjustment speed and open room for factors like structural change, demographic transition, and TFP growth to improve the adjustment dynamics. A reasonable assessment of the results is thus to view endogenous patience as a complementing channel for the explanation of long-run economic development, a channel that has been largely overlooked so far.

Standard growth empirics usually assumes savings as a cause of economic growth [35], but there is now ample evidence for reverse causality, i.e. the view of high savings rates as a consequence of high economic growth (see [21], [33], and the discussion of the literature in Carroll et al. [9]). The present article offers one theory that rationalizes reverse causality. Some of the results, however, can also be generated by introducing habit formation or subsistence consumption into standard growth theory. It seems thus worthwhile to compare.

The modeling of habit formation is technically more involved than the present approach since it requires a second state variable, the habit stock. When augmented with endogenous habit formation already the linear Ak model transforms into a three dimensional dynamic system [8] [9]. The higher formal complexity possibly explains why some interesting features obtained here like the savings rate–interest rate dynamics have not (yet) been explained within the habit formation model. Although ex post adjustment dynamics look similar, they are conceptually different under habit formation. Adaptive behavior implies that the model generates serial correlation of current savings and consumption decisions with those in the past. In contrast, endogenous patience generates serial correlation of current and *future* savings. If an economy traveling along its balanced growth path is hit by a shock we would thus expect current consumption to be correlated with past consumption under habit formation but not under endogenous time-preference. Habit formation also entails a crucial parameter restriction. In order to produce realistic adjustment dynamics it relies on the assumption that $\theta > 1$ whereas the present model generates plausible adjustment dynamics irrespective of the size of θ.

Subsistence consumption in form of Stone-Geary utility, $u(c) = (c - \bar{c})^{1-\theta}/(1-\theta)$, implies that the intertemporal elasticity of substitution rises from zero for $c = \bar{c}$ to $1/\theta$ for infinite c. The changing elasticity produces trajectories similar to the ones for wealth-dependent time preference and the question occurs whether the ad hoc modeling of subsistence is acceptable as a shortcut to endogenous patience. The answer is probably yes on the level of (introductory) macroeconomics textbooks because the "shortcut"-model is formally much less involved. On the level of a serious research question the answer is certainly no. This is instructively demonstrated by Kraay and Raddatz' [31] attempts to calibrate the neoclassical growth model with "subsistence consumption" and, in particular, to find a proper $\bar{c}$.

With respect to the *Ak* growth model with subsistence consumption it can be shown that adjustment dynamics of the savings rate and the rate of economic growth are independent from the specification of $\bar{c}$. In fact, adjustment dynamics of these variables are completely determined by the specification of the balanced growth path [44]. This implies that either all countries adjust alike toward the same balanced growth path or that countries that lag behind will never catch up. The model cannot explain *temporarily* divergent behavior.

Finally, the concept of endogenous time preference is comprehensive. It directly affects all intertemporal decisions alike whereas the subsistence approach affects other choice variables only through urgent consumption needs. For example, there is no "subsistence education" or "subsistence fertility". An extension of the present model towards the analysis of other intertemporal decisions is an interesting subject for future research.

Appendix

Derivation of (3). To solve problem (1) – (2) we parameterize the time integral and define $q \equiv \int_0^t \rho(k(v))\mathrm{d}v$ such that $\mathrm{d}q/\mathrm{d}t = \rho(k)$. The associated Hamiltonian is (A.1).

$$H = \frac{c^{1-\theta}}{1-\theta}\mathrm{e}^{-q} + \nu\,(w + rk - c) - \lambda\rho(k). \tag{A.1}$$

This is equivalent to maximizing the present value Hamiltonian $\tilde{H} = H\mathrm{e}^{q}$.

$$\tilde{H} = \frac{c^{1-\theta}}{1-\theta} + \mu\,(w + rk - c) - \phi\rho(k).$$

with $\mu \equiv \nu^{q}$ and $\phi \equiv \lambda e^{q}$. Note that $\tilde{H}$ is concave in (c, k, ρ). The first order conditions for a maximum are

$$\frac{\partial H}{\partial c} = c^{-\theta}\mathrm{e}^{q} - \nu = 0 \quad \Rightarrow \quad c^{-\theta} = \mu \tag{A.2a}$$

$$\frac{\partial H}{\partial k} = \nu r - \lambda\rho'(k) = -\dot{\nu} \quad \Rightarrow \quad \mu r - \phi\rho'(k) = -\dot{\mu} + \mu\dot{q} = -\dot{\mu} + \mu\rho \tag{A.2b}$$

$$\frac{\partial H}{\partial q} = -\frac{c^{1-\theta}}{1-\theta}e^{-q} = -\dot{\lambda} \quad \Rightarrow \quad -\frac{c^{1-\theta}}{1-\theta} = -\dot{\phi} + \phi\dot{q} = -\dot{\phi} + \phi\rho. \tag{A.2c}$$

$$\lim_{t\to\infty} H = 0. \tag{A.2d}$$

Here, (A.2d) is the necessary transversality condition according to Michel [36]. Following the solution method introduced by Das [15], the costate variables can be eliminated. For that purpose differentiate H with respect to t.

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\partial H}{\partial c}\dot{c} + \frac{\partial H}{\partial k}\dot{k} + \frac{\partial H}{\partial q}\dot{q} + \frac{\partial H}{\partial \nu}\dot{\nu} + \frac{\partial H}{\partial \lambda}\dot{\lambda}.$$

Insert (A.2a)–(A.2c) to see that for the maximized Hamiltonian $\mathrm{d}H/\mathrm{d}t = 0$ for all t. Thus with (A.2d), $H = 0$ for all t. Then, from (A.1)

$$\frac{\phi}{\mu}\rho(k) = \frac{c^{1-\theta}}{(1-\theta)\mu} + w + rk - c = \frac{c}{1-\theta} + w + rk - c. \tag{A.3}$$

The latter equality follows from use of (A.2a). Next differentiate (A.2a) with respect to time and insert (A.2b).

$$\theta\frac{\dot{c}}{c} = -\frac{\dot{\mu}}{\mu} = r - \rho(k) - \frac{\phi}{\mu}\rho'(k). \tag{A.4}$$

Insert (A.3) into (A.4) to get (3) in the text.

Proof of Proposition 1. Inspect (8a) to see that ρ is constant when $x = x^* = A$ implying $c^* = Ak^*$. Insert $x = A$ into (8b) and solve for ρ where $\dot{x} = 0$ to obtain the roots $\rho_1 = (A + \phi)/2 - \sqrt{(A+\phi)^2/4 - \phi\bar{\rho}}$ and $\rho_1 = (A+\phi)/2 + \sqrt{(A+\phi)^2/4 - \phi\bar{\rho}}$. For existence of a real solution $(A+\phi)^2/4 > \phi\bar{\rho}$, i.e. $A^2 + 2A\phi + \phi^2 > 4\phi\bar{\rho}$. Since $A > \bar{\rho}$ a sufficient condition for this to hold is $A^2+2A\phi+\phi^2 > 4\phi A$, which can be written as $(A-\phi)^2 > 0$ and is thus fulfilled. Since the minimum ρ that can be reached with non-negative k is $\bar{\rho}$, at least one solution has to be larger than $\bar{\rho}$. From $\rho_2 > \bar{\rho}$ we get $\sqrt{(A+\phi)^2/4 - \phi\bar{\rho}} > \bar{\rho} - (A+\phi)/2$, which can be written as $A - \bar{\rho} > 0$ and is thus fulfilled. For uniqueness the other root has to be unfeasible, i.e. $\rho_1 < \bar{\rho}$. Proceeding as above this can be written as $A - \bar{\rho} > 0$ and is thus fulfilled. The unique solution is at $\rho^* = \rho_2$. From $x = A$ follows $c = Ak$ and thus $\dot{k} = 0$, i.e. stagnation.

Proof of Proposition 2. Inspect (8a) to see that ρ is constant when $\rho = \bar{\rho}$. Insert this information into (8b) and solve for x where $\dot{x} = 0$ in order to obtain the unique x^{**}. Verify that x^{**} is positive for $\bar{\rho}/A > 1 - \theta$. Insert $x = x^{**}$ into $\dot{k}/k = A - x$ to get the balanced growth rate.

Arrows of Motion (Phase Diagram). Compute $\partial(\dot{x}/x)/\partial x = 1 + \eta(\rho - \bar{\rho})/(1-\theta)$ from (8b) and conclude that this expression is positive for $\theta < 1$. In order to obtain the arrows of motion for $\theta > 1$ rewrite (8b) as

$$\theta\frac{\dot{x}}{x} = \underbrace{A(1-\theta) + \theta x - \rho}_{b} + \underbrace{[A(1-\theta) + \theta x]}_{d} \cdot \underbrace{\frac{(\rho - \bar{\rho})\eta}{(1-\theta)\rho}}_{<0}.$$

Observe that $b < d$. Next note that at $x = A$, $d = A > 0$ and thus for $\theta > 1$, $b > 0$ for $\dot{x} = 0$ at x^*. On the other hand, at $x = x^{**}$, $b = 0$ and thus $d > 0$. Conclude $d > b \geq 0$ in the relevant range of x. Finally note that the derivative $\partial(\dot{x}/x)/\partial x$ obtained along the $\dot{x} = 0$ curve can be written as $1 - b/d$, which is positive since $d > b \geq 0$. Thus the arrows of motion point towards lower x below the $\dot{x} = 0$ curve and towards higher x above.

Proof of Proposition 5. Inspect (7) to see that the steady-state of growth is obtained at x^{**} and ρ^{**} as specified in (10). To check for local stability begin with linearizing (7).

$$\frac{\partial g}{\partial \rho} = -\eta\left[A + B\left(\frac{\rho - \bar{\rho}}{\rho_0}\right)^{\frac{1-\alpha}{\eta}} - x\right] - \eta(\rho - \bar{\rho})\frac{(1-\alpha)}{\eta}B\left(\frac{\rho - \bar{\rho}}{\rho_0}\right)^{\frac{1-\alpha}{\eta}-1}$$

$$\frac{\partial g}{\partial x} = \eta(\rho - \bar{\rho})$$

$$\frac{\partial h}{\partial \rho} = \frac{x}{\theta}\left\{\frac{(1-\alpha)}{\eta}\alpha B\left(\frac{\rho-\bar{\rho}}{\rho_0}\right)^{\frac{1-\alpha}{\eta}-1} - 1 + \eta\frac{\bar{\rho}}{\rho^2}\left[A + B\left(\frac{\rho-\bar{\rho}}{\rho_0}\right)^{\frac{1-\alpha}{\eta}} + \frac{\theta x}{1-\theta}\right]\right.$$

$$\left.+\eta\left(\frac{\rho-\bar{\rho}}{\rho}\right)\left[\frac{1-\alpha}{\eta}B\left(\frac{\rho-\bar{\rho}}{\rho_0}\right)^{\frac{1-\alpha}{\eta}-1}\right]\right\} - \left[\frac{1-\alpha}{\eta}B\left(\frac{\rho-\bar{\rho}}{\rho_0}\right)^{\frac{1-\alpha}{\eta}-}\right]x$$

$$\frac{\partial h}{\partial x} = \frac{1}{\theta}\left\{A + \alpha B\left(\frac{\rho-\bar{\rho}}{\rho_0}\right)^{\frac{1-\alpha}{\eta}} - \rho - \eta\left(\frac{\rho-\bar{\rho}}{\rho_0}\right)\left[A + B\left(\frac{\rho-\bar{\rho}}{\rho_0}\right)^{\frac{1-\alpha}{\eta}} + \frac{\theta x}{1-\theta}\right]\right\}$$

$$+\frac{x}{\theta}\left[\eta\frac{(\rho-\bar{\rho})}{\rho}\frac{\theta}{1-\theta}\right] - \left[A + B\left(\frac{\rho-\bar{\rho}}{\rho_0}\right)^{\frac{1-\alpha}{\eta}} - x\right] + x.$$

Evaluating the derivatives at the steady-state of growth provides the elements of Jacobian.

$$J_{11} \equiv \frac{\partial g(\bar{\rho}, x^{**})}{\partial \rho} = -\eta(A - x^{**}) = -\frac{\eta}{\theta}(A - \bar{\rho}) < 0$$

$$J_{12} \equiv \frac{\partial g(\bar{\rho}, x^{**})}{\partial x} = \eta(\bar{\rho} - \bar{\rho}) = 0$$

$$J_{21} \equiv \frac{\partial h(\bar{\rho}, x^{**})}{\partial \rho} = \frac{x^{**}}{\theta}\left[\frac{\eta}{\bar{\rho}}\left(A + \frac{\theta x^{++}}{1-\theta}\right) - 1\right]$$

$$J_{22} \equiv \frac{\partial h(\bar{\rho}, x^{**})}{\partial x} = \frac{1}{\theta}(A - \bar{\rho}) + \frac{\bar{x}^{**}}{\theta}\cdot 0 - (A - x^{**}) + x^{**} = x^{**} > 0.$$

The Jacobian determinant is obtained as $-\eta(A-\bar{\rho})x^{**}/\theta < 0$. Thus the steady-state of growth is a saddlepoint.

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