

Fair Pension Policies with Occupation-Specific Aging*

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Abstract. We study the optimal design of a fair public pension system in a multi-period overlapping generations model with occupation-specific morbidity and mortality that depends on the retirement age. The fairness constraint acts as institutional device ensuring that lifetime returns to contributions are equal across occupational groups. We consider group-specific replacement rates and a calculatory interest rate for early contributions as policy instruments. Calibrating the model to Germany, we find that switching to optimal fair pension policies may induce early retirement of blue-collar workers and significantly raises their lifetime pension benefits and welfare. Aggregate welfare increases in all fair pension scenarios.

Keywords: Fair Pensions; Early Retirement; Occupation-Health Gradient; Life Expectancy; Replacement Rate.

JEL: H55, I14, I24.

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1. INTRODUCTION

Many studies have observed that remaining life expectancy at the age of 65 differs in terms of lifetime income, education, or occupation (e.g., Case and Deaton, 2005; Cambois, 2011; Marmot, 2015; Chetty et al., 2016; Haan et al., 2020). One reason for these differences is that physically demanding or stressful blue-collar jobs are associated with faster aging than physically less-demanding and less stressful white-collar jobs (Abeliansky and Strulik, 2022). Occupational health risks are at the center of recurring political debates about a universal increase in the statutory retirement age (SRA) in several European Union (EU) countries such as Italy, Belgium and Finland.

A number of statutory pension systems already provide special entitlements for workers who have been in arduous and hazardous jobs for an extended period of time, including early retirement incentives and extraordinary replacement rates (for an overview, see Natali, 2016, and European Commission, 2021). For instance, Finland has gradually since 2018 raised the minimum retirement age, except for workers in arduous jobs with at least 38 years of service. Italy has recently extended its Early Retirement Allowance program for workers who have been in arduous and hazardous jobs in at least seven of ten years before applying for retirement or if they have been in such jobs for at least half of their career. The automatic linking of retirement conditions to changes in life expectancy has been suspended until 2025.¹ Austria, Croatia, Greece, Finland, France, Luxembourg, Italy, Portugal, Romania, and Spain grant preferable early retirement conditions for workers in arduous and hazardous jobs, whereas Estonia, Greece, Romania, and Slovenia generally provide higher replacement rates.²

However, many countries have not yet fully adjusted their pension systems to the large and widening occupational gap in life expectancy. A prime example is Germany, which largely follows the so-called “equivalence principle” in its conventional form, which means that annual pension benefits are proportional to lifetime contributions (e.g., Bönke et al., 2019; Eggert, 2021). The equivalence principle is considered as an implementation of the principle of fair taxation whereby a tax distribution is considered fair if every citizen pays taxes to the extent that corresponds to their share of the state benefits (Eggert, 2021). However, as we show below, a German-style (Bismarckian) pension system is only fair if all occupational groups face the same life expectancy at the age of retirement, a condition that is clearly violated.³

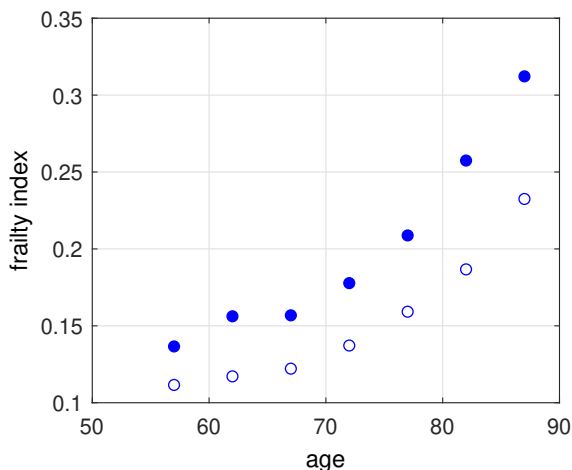
¹Belgium has concrete reform plans in similar directions (European Commission, 2021).

²Cyprus, Czech Republic, Poland, and Germany have permanent early retirement provisions for miners. In Germany, the pensionable age for miners is between 60 and 62, depending on the cohort. Statutory pension insurance in the UK does not provide special pension provisions for workers in arduous and hazardous jobs, but provides disability benefits (Natali et al., 2016). In some countries, there also exist occupational pension schemes with special provisions, e.g. for construction workers in the Netherlands.

³A recent pension reform in Germany aims at departing from that principle by granting extra benefits for those having worked 33 years or longer if they earned modest income. However, supplemental benefits are projected to be rather limited (an amount of EUR 75 per month on average, which is 4% of median disposable income of the population aged 65 or older). The reform benefits only 6-7% of the elderly population (80% of those eligible for the supplemental benefits are women) and can thus not be viewed as significant departure from the current proportionality of annual benefits to lifetime contributions (Börsch-Supan et al., 2022).

Figure 1 shows the frailty index for blue-collar and white-collar workers in Germany. The frailty index measures the fraction of health deficits of individuals out of a long list of potential health deficits and is highly related to mortality risk (see Searle, 2008, for methodological background). Figure 1 shows that the frailty index increases exponentially in age with a kink around the typical retirement age. Blue collar workers have accumulated more health deficits at any age and they appear to benefit more from retirement in terms of health. Kibele et al. (2013) report that these differences in health imply that remaining life expectancy of German, white-collar, male workers at age 65 was about 2.6 years longer than that for their blue-collar counterparts in 2003-2004.⁴

FIGURE 1. Frailty Index: Blue- vs. White-Collar Workers



Data for Austria, Belgium, Switzerland, Germany, Denmark, Spain, France, Italy, Netherlands and Sweden; males and females. Blue dots refer to blue-collar workers and white dots refer to white-collar workers. Data source: Abeliansky and Strulik (2022).

The convexity of the age–frailty nexus reflects the feature that existing health deficits are conducive to the development of new health deficits (Mitnitski et al., 2006). The self-productive nature of health deficit accumulation implies that blue collar workers continue to age faster in retirement than individuals in other occupations. Abeliansky and Strulik (2022) show that occupational health differences increase before and after retirement for workers stratified by education, collar color, and physical and psychosocial job-burden. The evidence in Abeliansky and Strulik (2022) combined with the observation of a strong association between the frailty index and mortality (e.g., Mitnitski et al., 2002; Dalgaard et al., 2022) suggests that blue collar workers (workers with high occupational health burden) will experience longer life expectancies if they retire earlier — a hypothesis that is supported by recent evidence for Germany in Giesicke (2019).

⁴Figure 5 in Appendix B shows the difference in life expectancy at age 65 from 1996 to 2004, calculated using occupation-specific mortality data obtained from the Research Data Centre of the German Pension Insurance (FDZ-RV). After 2004, the FDZ-RV discontinued the computation of occupation-specific mortality rates. Murin et al. (2022) document substantial life expectancy gaps for 18 OECD countries in 2011 across education groups.

Motivated by the evidence in Figure 1, our study proposes public pension policies that take occupational differences in health, aging, and longevity into account. We present welfare-maximizing pension policies that fulfill well-defined fairness constraints, which can be perceived as institutional devices ensuring that expected lifetime returns to contributions are not lower for workers from faster aging occupation groups. The proposed policies implement a “strict equivalence principle”: the pension system is non-redistributive from an *ex ante* point of view, i.e. behind the veil of ignorance, even under life expectancy differences between occupational groups. More precisely, the institutional devices impose an equal benefit-contribution ratio across occupations, where the benefit-contribution ratio is defined as the present discounted value (PDV) of pension payments expected at retirement age divided by lifetime contributions to the pension system.

We integrate the group fairness concept into a multi-period overlapping generations (OLG) model with an age-structured population, stochastic survival, occupation-specific morbidity and mortality, and endogenous labor supply at the intensive margin (hours supplied) and extensive margin (retirement decision). In line with the health deficit approach, our calibrated model measures morbidity by the frailty index and utilizes the fact that the frailty index has high predictive power for death at the individual level (i.e., death is triggered by high frailty rather than being an age-specific event) and for mortality at the group level (e.g., Mitnitski et al., 2002a, 2002b, 2005, 2006, 2007). The theory thus connects the longevity gaps observed across occupations to the occupation-specific evolution of health deficits (displayed in Figure 1). The frailty index as a measure of physiological aging also allows us to conceptualize increasing disutility (pain) of work and the desire for early retirement as motivated by deteriorating health rather than increasing chronological age.⁵

We consider two different institutional devices that fulfill the fairness constraint of the social planner. The first policy implements optimal group-specific replacement rates. The social planner selects from the set of fair and fiscally sustainable replacement rates the welfare maximizing replacement rates for blue-collar and white-collar workers, conditional on occupation-specific retirement ages and accounting for the endogenous labor supply choices at the extensive and intensive margin.

The welfare-maximizing replacement rates may be politically infeasible, for example, due to incentives to mis-classify jobs to profit from more favorable pension terms. We thus propose an alternative (second-best) policy based on occupation-wide replacement rates and a calculatory interest rate that weighs pension contributions in the distant past higher than those closer to the retirement age (Richter and Werding, 2021). The alternative policy achieves fairness by exploiting the less steep age-earnings profiles of blue-collar workers.

⁵The health deficit approach has been introduced into economics by Dalgaard and Strulik (2014) and has been applied to a wide array of research questions. See Strulik (2022a) and Grossmann (2022) for surveys on the concept and applications. Dalgaard and Strulik (2017) use the approach to explore how the long-run gains in labor productivity and medical progress explain the historical evolution of years spent in retirement. Grossmann and Strulik (2019) investigate the interaction between health and pension policy for health inequality and the effects of increased longevity on the optimal pension system and health spending. In contrast to the present study, they neither allow for heterogeneity in earnings nor for an occupation-health gradient. They also do not consider an age-structured population and do not focus on the fairness of pension systems.

To derive the optimal fair policy under both sets of policy instruments we apply both the utilitarian and Rawlsian welfare criterion conditional on the SRA. The applied fairness criterion appears politically feasible, as it implements the original idea of the equivalence principle.⁶ We first characterize the fair pension system for both sets of policy instruments analytically. We then use the analytical results in a welfare analysis based on a calibration of the model for Germany. While the basic question of how to design a fair pension system is universal, Germany appears to be a particularly appropriate choice for the calibration because income of its elderly citizens relies heavily on the provision from the public pension system (e.g. Börsch-Supan, 2000; Börsch-Supan et al., 2015) while, at the same time, there exists a private annuity market.

The benchmark calibration of the model implies that under the current pension system of a unique and occupation-independent replacement rate without calculatory returns to past contributions, the (lifetime) benefit-contribution ratio of blue-collar workers is 16% lower than that of white-collar workers. The figure increases to 22% when raising the SRA by five years. Our analysis also suggests that, under the “conventional equivalence principle”, an increase in the SRA reduces lifetime utility of blue collar workers so much that aggregate welfare declines. This result is of great relevance for the public discussion about a uniform increase in the SRA.

Allowing for occupation-specific replacement rates, we find that in the benchmark run the optimal fair pension system that equalizes the benefit-contribution ratio across occupations for an SRA of 65 years requires that the replacement rate for blue-collar workers exceeds that of white-collar workers by a factor of 1.19. Under a Rawlsian welfare criterion, where the social planner implements the fair policy that maximizes welfare of blue-collar workers, blue-collar workers face a considerably smaller early retirement penalty than in the current system and they prefer to retire early in equilibrium.⁷ For an SRA of 70 years, the optimal fair policy implies early retirement of blue-collar workers under both welfare criteria. When replacement rates for given retirement age are constrained to be the same for both occupations and the early retirement penalty remains as in the current system, for both welfare criteria the optimal fair calculatory return to past contributions up to age 60 is 6% when the SRA is 65 years and 9.6% when it is 70 years.

Compared to the current system, fair pensions result in blue-collar workers experiencing a significant welfare gain and providing less work at the intensive margin. Reduced labor supply and early retirement of blue-collar workers has adverse effects on white-collar workers by reducing their pension benefits and their hourly wage rate due to general equilibrium repercussions. However, the calibrated model predicts that aggregate (i.e., utilitarian) welfare increases in all considered reform scenarios. The result reflects the large gains in pension income of blue-collar workers along with gains in life expectancy caused by early retirement. The predicted welfare gain from switching to the optimal fair policy is higher for an SRA of 70 years than for an SRA of 65 years.

⁶Franco and Tommasino (2020) and Holzmann et al. (2020) discuss more generally and informally the scope of pension reforms that address life expectancy gaps.

⁷In Germany, early retirement is currently disincentivized through two channels. It reduces pension contributions and thus pension income due to the shortened work life, and, according to pension law, it results in the pension income being reduced by 0.3% per month of early retirement.

We also investigate scenarios of equal and unequal improvements in health and longevity. The analysis suggests that fair pension policies are quite insensitive to uniform increases of life expectancy across both occupation groups. When life expectancy rises for white-collar workers only or life expectancy of blue-collar workers declines, the optimal fair policy is characterized by relatively more favorable terms for blue-collar workers and features stronger early retirement incentives for blue-collar workers.

The remainder of the paper is structured as follows. Section 2 discusses the contribution to the literature. Section 3 presents the model and defines the equilibrium. Section 4 characterizes the “equivalence principle” before defining and analytically deriving fair pension policies. Section 5 calibrates the model. Section 6 positively analyzes the calibrated model for the status quo pension system in Germany. Section 7 presents the social planning problems with fairness constraints that are numerically solved in Section 8. Section 9 presents sensitivity analyses to changes in the health deficit accumulation processes, the interest rate, and the time preference rate. The last section concludes.

2. CONTRIBUTION TO THE LITERATURE

Our study contributes to the literature studying the interaction between health and/or longevity, labor supply, and pension systems in quantified life-cycle models (e.g., French, 2005; Ludwig and Reiter, 2010; Fehr et al., 2013; Bloom et al., 2014; Haan and Prowse, 2014; Kuhn et al., 2015; see French and Jones, 2017, for a review). A couple of studies focussed, like us, on the effects of pension reforms on inequality when individuals differ in health risk according to their socioeconomic status. Laun et al. (2019) investigate the implications for employment, income, and welfare of Norwegian pension system reforms that aim at restoring fiscal sustainability in the wake of demographic change. Health is a bivariate state (good or bad) that occurs stochastically depending on education, age, and time. They find that proportionately lowering old-age retirement and disability benefits would be the best response to demographic change in terms of aggregate welfare (vis-à-vis uniformly increasing the early benefit access age combined with lower retirement benefits or raising tax and contribution rates). However, this policy would particularly disadvantage people in poor health and would increase income inequality the most.

Our research question differs significantly from Laun et al. (2019) and other previous studies. Instead of focussing on restoring sustainability of pension systems (i.e. *intergenerational* fairness) in response to rising longevity, we ask how *intragenerational* fairness can be achieved in the pension system on condition that reforms are fiscally sustainable. Because of the different focus, Laun et al. (2019) do not consider policy reforms that aim to redistribute. By design none of the retirement policies they discuss has an impact on health and longevity and all considered policy reforms make the socially disadvantaged group worse off. In contrast, our model accounts for a feedback channel from retirement to health such that in particular workers in burdensome occupations benefit from retirement in terms of health and longevity. We therefore consider occupation-specific policies that create early retirement incentives for workers with high occupational health burden.

Sanchez-Romero and Prskawetz (2017) show that an increasing gap in life expectancy by education (ability) enhances the regressivity of the U.S. pension system and income inequality. Also calibrated to the U.S. economy, Sanchez-Romero et al. (2020) evaluate the effects of several pension reforms on the regressivity of pension systems that arise from income-related life expectancy differences. Although their redistribution focus is similar in spirit to our contribution, they do not define or evaluate optimal fair pension reforms, nor do they consider that the type of work or retirement can affect health or longevity.

Pestieau and Racionero (2016) study an optimal non-linear taxation problem in a two-period model when mortality risk (to reach the second period) is imperfectly correlated with occupation. They show that tagging the tax policy to occupation improves welfare and reduces second-period labor supply of workers in the occupation of higher mortality risk while making workers in the other occupation group worse off. These welfare effects are in line with our results on a fair pension system reforms. However, our results are derived in a comprehensive general equilibrium model with endogenous, health-dependent labor supply at the intensive and extensive margin (retirement age), endogenous age-earning profiles, and biologically founded aging calibrated with real world data. To the best of our knowledge, the research question of how a pension system can be optimally designed that ensures the equivalence of the relationship between lifetime contributions and lifetime benefits has not yet been addressed.

Methodologically, we contribute to the discussed strand of literature by capturing health status, aging, and mortality risk in one unifying variable, the frailty index. This allows us to consider new channels of interaction between health and labor supply: (i) labor productivity declines and the disutility of work increases because of deteriorating health (rather than due to advancing chronological age), (ii) the type of work affects individual health and aging, and (iii) retirement has an important feedback mechanism on health since it eliminates the arrival of new work-related health deficits. The self-productive nature of health-deficit accumulation also implies that a hazardous and/or stressful work leaves a legacy in retirement.

3. THE MODEL

We consider a multi-period OLG model in discrete time with stochastic and health-dependent death, a public pension system, and endogenous labor supply (choice of working hours and retirement decision). Individuals are heterogenous in terms of earnings, occupation-specific aging, and mortality. There is an exogenous world market interest rate $\bar{r} > 0$ (small open economy) at which subjects can freely lend and borrow. We deliberately exclude a feedback channel from savings behavior to the interest rate in order to avoid that the implications of pension reforms in our calibrated model are contaminated by second-order effects.⁸ There exists, however, a perfect private annuity market in addition to the public pension system such that the effective interest for annuity

⁸Such feedback effects appear to be negligible in view of the high degree of financial globalization of advanced economies.

savings is endogenous to mortality rates. Goods and factor markets are perfectly competitive and wage rates are endogenous. For simplicity, there is no labor migration.

3.1. Government Policy Instruments. There is a pay-as-you-go (PAYG) pension system that is financed by proportional contributions on earnings at time-invariant rate $\tau^p \in (0, 1)$ and has a balanced budget each period. Pension benefits are paid from age $R \in \mathcal{R} \equiv \{\underline{R}, \dots, \bar{R}\}$ onwards, where \underline{R} is the earliest retirement age and \bar{R} is the SRA. We assume, for simplicity, that working after the SRA is disincentivized (despite some flexibility in most pension systems). This is justified by the observation that entry into retirement is clustered at the statutory level (Seibold, 2021) and employment rates at older ages are strongly correlated with the SRA (Hairault et al. 2010).

Replacement rates may vary with retirement age. For the case that pension benefits can be conditioned on occupation, we allow the replacement rate to differ across occupational groups to account for the fact that blue-collar workers are prone to higher mortality risk. As an alternative instrument, we allow pension wealth to accumulate with a calculatory interest rate that gives higher weight to contributions early in the career. This benefits blue-collar workers who typically have less steep age-earnings profiles than white-collar workers.

Earnings before declared retirement are taxed according to a co-linear tax schedule with (time-invariant) marginal rate $\tau^w \in (0, 1 - \tau^p)$ and a uniform lump-sum element (“earned income tax credit”), \bar{I} . Earnings taxation is fully redistributive with a balanced budget. This means that the average tax rate increases with income (tax progressivity). Progressive taxation adds to the distortions of labor supply implied by pension contributions, as such distortions are at the center of a meaningful welfare analysis. The marginal tax rate on earnings after declared retirement is $1 - \tau^p$ (full taxation with deduction of pension contributions). For simplicity, we assume that pension income and capital income are not taxed.

3.2. Individuals. Each period a unit mass of newly born individuals with stochastic lifetime enters the labor market, choosing their consumption path, the time path of working hours before retirement, and the retirement age. Individuals are either white-collar (high-skilled) workers or blue-collar (low-skilled) workers, indexed by $j \in \{H, L\}$.⁹ Blue-collar workers account for a share $\theta \in (0, 1)$ of the workforce.¹⁰ For simplicity, all individuals start economic life at the same age. The fact that (for educational reasons) the “representative” white-collar worker may fully enter the labor market at a later age than the “representative” blue-collar worker is captured in the calibrated model by a low early-in-life productivity of white-collar workers. Workers born into

⁹Empirical evidence suggests that the gap between white-collar and blue-collar workers in the frailty index among the elderly in Germany is almost identical to the gap between high-skilled and low-skilled workers (Abeliansky and Strulik, 2022). We aim to capture the feature that retirement slows down the accumulation of job-related health deficits and therefore prefer to distinguish groups by occupation.

¹⁰To facilitate the interpretations, we refrain from modeling the choice of occupation. In this way we avoid that results are contaminated by effects of pension reform on occupational choice, which can be of negligible magnitude anyway.

economic life in period v are said to belong to cohort v . Thus, an individual from cohort v and occupational group j choosing retirement age R_v^j is in retirement from period $v + R_v^j$ onwards.

3.2.1. *Health Deficits, Mortality, and Survival.* As explained in the Introduction, modern gerontology suggests to conceptualize (biological) aging as accumulation of health deficits. Health deficit accumulation is measured by the frailty index as the fraction of health deficits present in an individual out of a long list of potential health deficits. The empirical evidence suggests that the frailty index grows, on average, exponentially with age at a rate between 3 and 5 percent (e.g., Mitnitski et al., 2002a; Mitnitski and Rockwood, 2016; Abeliansky and Strulik, 2018), consistent with Figure 1. We additionally capture in line with Figure 1 that the speed of health deficit accumulation is occupation-dependent and may depend on the retirement status. These features are consistent with the notion that health deficit accumulation is particularly fast when the job is physically demanding, involves stress or workplace hazards, and provides little autonomy in decision-making (Marmot and Wilkinson, 2005; Marmot, 2015; Abeliansky and Strulik, 2022). We thus assume faster health deficit accumulation for blue-collar workers (e.g., manufacturers, nurses, construction workers, delivery service workers) especially before retirement.¹¹

Formally, denote the frailty index of an individual from cohort v and occupation-group j in period t by $d_{v,t}^j$, where, initially, $d_{v,v}^j = d_{\min} > 0$, $j \in \{H, L\}$. Health deficits grow at different rates during working life to capture the occupation-health gradient:

$$d_{v,t}^j = \begin{cases} d_{\min}(1 + \bar{\mu}^j)^{t-v} & \text{for } v \leq t \leq v + R_v^j - 1 \\ d_{v+R_v^j-1}^j(1 + \underline{\mu}^j)^{t-v-(R_v^j-1)}, & t \geq v + R_v^j, \end{cases} \quad (1)$$

where the rates of aging fulfill $\underline{\mu}^H \leq \bar{\mu}^H \leq \underline{\mu}^L < \bar{\mu}^L$. The first line on the right-hand side of (1) refers to working life and the second line to the time spent in retirement. For blue-collar individuals, deficit accumulation slows down after retirement, albeit it does not become lower than that of white-collar individuals. According to the evidence in Abeliansky and Strulik (2022), before retirement the aging rate of blue-collar workers is 0.4 percent higher than that of white-collar workers. After retirement, the difference in aging rates is insignificant (the point estimate for the difference is 0.1 percent). The slowdown after retirement for blue-collar workers, as visualized in Figure 1, suggests that the health deficit accumulation process is partly work-related. We allow for the reduction in the rate of aging caused by retirement, $\bar{\mu}^L - \underline{\mu}^L$, to affect early retirement decisions. We also examine the role of the gap in aging rates $\bar{\mu}^L - \bar{\mu}^H$ (measuring the socioeconomic health gradient)

¹¹Two remarks are in order. First, our approach is agnostic about the causes of the socioeconomic health gradient beyond the role of arduous and hazardous jobs supported by Figure 1. In particular, our fairness concept does not rely on the causes for occupational differences in mortality risk. There is a controversy in the empirical literature about the contribution of lifestyle differences (see, e.g., Contoyannis and Jones, 2004; Balia and Jones, 2008). Although we do not explicitly endogenize health-relevant behavior, our formulation indirectly captures occupation-specific unhealthy consumption, such as smoking, as induced by peer-group effects. Second, the assumption that the average white-collar worker experiences less physical and psycho-social job burden does not well represent high-ranked manager or health professionals in the top earning classes who may experience higher stress levels than the average blue-collar worker. However, these high-ranked individuals are typically not covered in the public pension system or are only marginally relevant for it.

and a decline in aging rates (general increase in life expectancy). Noteworthy, $\bar{\mu}^H < \bar{\mu}^L$ not only implies that blue-collar workers enter retirement with a higher level of health deficits, but also that the absolute difference in health deficits between occupation groups increases with age even when the rates of aging have converged in retirement, i.e. even for $\underline{\mu}^H = \underline{\mu}^L$.

The unconditional probability $S_{v,t}^j$ of an individual from occupational group j and cohort v to survive to age $t - v$ decreases in the frailty index $d_{v,t}^j$ of the individual at age $t - v$. Thus, life-expectancy is occupation-specific. Consistent with data from life tables, we assume that the survival probability follows a logistic function of the frailty index (Schuenemann et al., 2017):

$$S_{v,t}^j = S(d_{v,t}^j) = \frac{1 + \omega}{1 + \omega e^{\kappa d_{v,t}^j}}, \quad (2)$$

$\kappa > 0, \omega > 0$. The survival function assumes the value of one at the state of best health (for $d_{v,t}^j = 0$) and is close to zero when the frailty index is high. According to (1) and (2), $\underline{\mu}^H \leq \bar{\mu}^H \leq \underline{\mu}^L < \bar{\mu}^L$ implies that life expectancy of blue-collar workers is lower than that of white-collar workers, in line with Figure 1. Moreover, early retirement raises survival rates of elderly blue-collar workers, consistent with recent evidence by Giesicke (2019).

3.2.2. Preferences and Productive Endowments. An individual from occupational group j and cohort v faces expected lifetime utility

$$U_v^j = \sum_{t=v}^{v+T-1} \beta^{t-v} S(d_{v,t}^j) \left[\frac{(c_{v,t}^j)^{1-\sigma} - 1}{1-\sigma} - \frac{D(d_{v,t}^j)(\ell_{v,t}^j)^{1+1/\eta}}{1+1/\eta} + \bar{u} \right], \quad (3)$$

where $c_{v,t}^j$ denotes the consumption level in period t , $\ell_{v,t}^j$ the number of hours worked, $T > 0$ the maximum length of life, $\beta \in (0, 1]$ the discount rate, $\sigma > 0$ the degree of relative risk aversion, $\eta > 0$ is the inverse of the marginal disutility of labor supply,¹² and $\bar{u} \geq 0$ is a constant base utility level ensuring that instantaneous utility (the term in squared brackets) is positive at all times (e.g. Hall and Jones, 2007). $D(d)$ is an increasing function, capturing that more health deficits are associated with higher disutility from work, which is particularly relevant in the context of early retirement.

We assume occupation-specific fair insurance within a cohort.¹³ The interest factor of a group member j of cohort v in t between date t and $t + 1$ then reads as

$$1 + r_{v,t}^j = \frac{1 + \bar{r}}{1 - m_{v,t-1}^j}, \quad (4)$$

where $m_{v,t-1}^j \equiv -\frac{S_{v,t}^j - S_{v,t-1}^j}{S_{v,t-1}^j}$ is the occupation-specific mortality rate between period $t - 1$ and t . An individual from cohort v and occupation-group j is endowed with productive ability $a_{v,t}^j$ in period

¹²In our context, η equals the Frisch elasticity of labor supply and is calibrated accordingly. See Keane and Rogerson (2012) for a discussion.

¹³This implies an annuity market where zero-profit insurance companies pay a rate of return above \bar{r} and keep the wealth of the deceased (Yaari, 1965).

t and supplies $a_{v,t}^j \ell_{v,t}^j$ efficiency units of labor. With wage rate w_t^j per efficiency unit of labor of type j in t , the hourly wage rate is given by $W_{v,t}^j = w_t^j a_{v,t}^j$ and earnings are $W_{v,t}^j \ell_{v,t}^j$.

3.2.3. Wealth Accumulation and Pension Benefits. Pension benefits of an individual from cohort v and occupation-group j are given by a cohort-specific fraction (“replacement rate”) of “pension wealth”, $P_{v,t}^j$. Assuming that contributions are not possible from age R_v^j onwards, pension wealth accumulates (with contribution rate τ^p) according to

$$P_{v,t+1}^j - P_{v,t}^j = \mathbf{1}_t(\underline{R}) \cdot \tilde{r} P_{v,t}^j + \mathbf{1}_t(R_v^j) \tau^p W_{v,t}^j \ell_{v,t}^j, \quad (5)$$

with initial value $P_{v,v}^j = 0$. $\mathbf{1}_t(\underline{R})$ denotes an indicator function that takes the value of one for $v \leq t \leq v + R - 1$ (prior to age R) and zero otherwise. $\tilde{r} \geq 0$ is a calculatory interest rate set by the policy maker that applies before the earliest possible retirement age, \underline{R} . In a standard PAYG system, $\tilde{r} = 0$ holds. If $\tilde{r} > 0$, pension contributions in the distant past have a larger impact on pension benefits than those close to the retirement age. This may be an advantage for blue-collar workers who typically have less steep age-earnings profiles than white-collar workers (Richter and Werding, 2021). For the advantage to materialize, we assume that $\tilde{r} > 0$ applies only up to some age. For concreteness, we set the threshold age at the earliest possible retirement age, \underline{R} .

We denote the marginal tax rate on earnings of an individual from cohort v in period t by $\tau_{v,t}$. It depends on the age of declared retirement and is given by

$$\tau_{v,t} = \begin{cases} \tau^w & \text{for } v \leq t \leq v + R_v^j - 1, \\ 1 - \tau^p & \text{otherwise.} \end{cases} \quad (6)$$

Individual asset holdings (non-pension wealth), $k_{v,t}^j$, then accumulate according to

$$k_{v,t+1}^j - k_{v,t}^j = r_{v,t}^j k_{v,t}^j + (1 - \tau_{v,t} - \tau^p) W_{v,t}^j \ell_{v,t}^j + I_{v,t}^j - c_{v,t}^j, \quad (7)$$

where $I_{v,t}^j$ is an income component that is equal to the earned income tax credit during working life and to instantaneous pension benefits after declared retirement. It thus depends on the retirement decision. We write

$$I_{v,t}^j = I_{v,t}^j(P_{v,t}^j, R_v^j) \equiv \begin{cases} \bar{I}_t & \text{for } v \leq t \leq v + R_v^j - 1, \\ b_v^j(R_v^j) P_{v,t}^j & \text{for } v + R_v^j \leq t \leq v + T - 1, \end{cases} \quad (8)$$

where $b_v^j(R)$ is the replacement rate of a member of cohort v in occupation group j as a function of retirement age R .¹⁴ Our formulation features that, first, younger cohorts may have to accept lower pension benefits for given pension wealth because of demographic change and, second, early retirement may come with a penalty on the replacement rate. While, in principle, replacement rates could also depend on time for given cohorts, there is strong indication of a political constraint that prevents lowering pension benefits of retirees despite rising dependency ratios.

¹⁴In other words, the replacement rate is the ratio between instantaneous pension benefits and calculatory pension wealth. The term replacement rate is sometimes used differently as the ratio between benefits and earnings in the final year before retirement starts. However, to employ the replacement rate as a policy instrument, its definition has to capture that instantaneous pension benefits depend on the full history of earnings and/or contributions.

As members of cohort v from group j fully retire after R_v^j periods,¹⁵ their corresponding calculatory pension wealth is given by

$$C_v^j(R_v^j, \tilde{r}) \equiv P_{v, v+R_v^j-1}^j. \quad (9)$$

According to (5), it is a function of the retirement age and the calculatory rate of return to annual contributions, \tilde{r} . $C_v^j(R_v^j, 0)$ is the unweighted sum of contributions until retirement age R_v^j . According to (8) and (9), each period during retirement the pension benefit is given by

$$\hat{I}_v^j(R_v^j, \tilde{r}) \equiv b_v^j(R_v^j) C_v^j(R_v^j, \tilde{r}). \quad (10)$$

3.3. Production. There is a single consumption and investment good, chosen as numeraire. Aggregate output in period t is given by

$$Y_t = (K_t)^\alpha (X_t)^{1-\alpha}, \text{ with } X_t = [\chi(H_t)^\rho + (1-\chi)(L_t)^\rho]^{\frac{1}{\rho}}, \quad (11)$$

$\alpha \in (0, 1)$, $\chi \in (0, 1)$, $\rho < 1$, where H and L are efficiency units of white-collar and blue-collar labor, respectively, and K denotes the input of physical capital. Physical capital depreciates at rate $\delta \geq 0$. Variable X may be interpreted as aggregate labor services. The elasticity of substitution between white-collar and blue-collar workers is $\frac{1}{1-\rho}$.

3.4. Equilibrium. The equilibrium is defined as follows.

Definition 1. (Equilibrium) *An equilibrium is a sequence of aggregate quantities $\{Y_t, K_t, H_t, L_t\}$, wage rates $\{w_t^j\}$, individual choices $\{c_{v,t}^j, \ell_{v,t}^j, R_v^j\}$, and individual state variables $\{k_{v,t}^j, P_{v,t}^j, d_{v,t}^j, S_{v,t}^j\}$, $j \in \{H, L\}$, for initial conditions $k_{v,v}^j = P_{v,v}^j = 0$, $d_{v,v}^j = d_{\min} > 0$, $j \in \{H, L\}$, and given the sequences of policy instruments, such that*

- (1) *The representative firm in the numeraire good (Y) sector maximizes profits under technology (11), taking factor prices as given;*
- (2) *Individuals maximize lifetime utility, i.e., the representative member of cohort v in occupation group j chooses $\{c_{v,t}^j, \ell_{v,t}^j, R_v^j\}$ to maximize (3) subject to (1), (2), (5), (6), (7), (8), and terminal condition $k_{v, v+T}^j \geq 0$, taking the time paths of $W_{v,t}^j = w_t^j a_{v,t}^j$ and $r_{v,t}^j = \frac{\bar{r} + m_{v,t-1}^j}{1 - m_{v,t-1}^j}$ as given, $t \in \{v, \dots, v + T - 1\}$;¹⁶*

¹⁵Recall that, *ex ante*, individuals are homogenous within an occupation group. We thus focus on equilibria where all individuals from the same cohort and occupational group make the same early retirement decision.

¹⁶Wage rates w_t^H and w_t^L follow from equilibrium condition 1 (see Appendix A.1); the expression for $r_{v,t}^j$ follows from (4).

(3) *Labor markets clear in any period t , i.e.,*¹⁷

$$H_t = \theta \cdot \sum_{v=t-\bar{R}+1}^t \mathbf{1}_t(R_v^H) S_{v,t}^H a_{v,t}^H \ell_{v,t}^H, \quad (12)$$

$$L_t = (1 - \theta) \cdot \sum_{v=t-\bar{R}+1}^t \mathbf{1}_t(R_v^L) S_{v,t}^L a_{v,t}^L \ell_{v,t}^L. \quad (13)$$

(4) *Health deficit levels of members of cohort v in occupation group j evolve according to (1), survival rates according to (2), pension wealth according to (5), and asset holdings according to (7).*

As individuals optimize subject to constraint (5), they understand that higher labor supply will raise their pension benefits. Individuals also take into account the occupation-specific processes of health deficit accumulation (as given by (1)) and their effect on survival rates (2). In particular, blue-collar workers understand that early retirement slows down their health deficit accumulation rates during retirement (as $\underline{\mu}^L < \bar{\mu}^L$). We solve the individual optimization problem in two steps. In the first step, individuals control consumption and labor supply at the intensive margin for given retirement ages. In the second step, they choose the retirement age (extensive margin) which gives the highest indirect lifetime utility resulting from the first step, given the retirement decisions of all others. As an individual has mass zero, all aggregate quantities and thus factor prices remain unchanged when an individual changes its retirement decision. In equilibrium, no individual gains lifetime utility by deviating from its retirement decision given the aggregates and factor prices. Also note that the financial market and the numeraire good market in the considered small open economy (with exogenously given interest rate \bar{r}) clear due to perfect international mobility of capital.

4. STATUS QUO VS. FAIR PENSION SYSTEM

We characterize the status quo pension system by pension benefits that are independent of the timing of contributions and where replacement rates conditional on retirement ages are equal for all groups. This captures the “conventional equivalence principle” on which the statutory German pension system is based (e.g. Bönke et al., 2019; Eggert, 2021). It is formally defined as follows.

Definition 2. (Conventional equivalence principle) *A pension system fulfills the conventional equivalence principle for a cohort v when the following two conditions hold: (i) $b_v^L(R) = b_v^H(R) = b_v(R)$ for any possible retirement period $R \in \mathcal{R}$, and (ii) $\bar{r} = 0$.*

¹⁷To understand the right-hand side of (12), note that H_t is the sum of the individual supply of white-collar labor in period t , as indicator function $\mathbf{1}_t(R_v^j)$ takes the value of one in period t when surviving members of cohort v from occupation group j with retirement age R_v^j supply labor (and zero during retirement), $a_{v,t}^j \ell_{v,t}^j$ are their efficiency units of labor in period t , and $\theta S_{v,t}^H$ is the mass of surviving white-collar workers. Condition (13) for blue-collar labor supply can be understood analogously.

The conditions in Definition 2 imply that *instantaneous* rather than *lifetime* pension benefits are proportional to the unweighted sum of contributions until declared retirement (Eggert, 2021). The term “equivalence principle” is thus misleading, since it ignores differences in life expectancy between occupational groups and thus in the duration of benefit payments. The German pension system is (in a stylized form) consistent with Definition 2 with pension benefits $\hat{I}_v^j(R_v^j, 0)$ and a replacement rate

$$b_v(R^j) = \bar{b}_v - \psi \cdot (\bar{R} - R_v^j), \quad j \in \{H, L\}, \quad (14)$$

for retirement age R_v^j , where $\bar{b}_v \in (0, 1)$ is the replacement rate when retiring at the SRA, \bar{R} , and $\psi \in (0, 1)$ denotes the reduction in the replacement rate per period of early retirement (“early retirement penalty”).¹⁸

We next formalize the notion of a fair pension system. As motivated in the Introduction, a fair pension system is defined by proportionality between lifetime contributions and the expected PDV of pension benefits at retirement age. The expected PDV of the pension benefits for a member of cohort v in occupational group j at chosen retirement age R_v^j (“lifetime pension benefits”) is calculated as

$$B_v^j(R_v^j, \tilde{r}) \equiv \sum_{t=v+R_v^j}^{v+T-1} S_{v,t}^j \frac{\hat{I}_v^j(R_v^j, \tilde{r})}{\prod_{z=v+R_v^j}^t (1 + r_{v,z}^j)}. \quad (15)$$

Fair pension systems relate lifetime pension benefits and contributions as follows.¹⁹

Definition 3. (Fair pension system) *A fair pension system is non-redistributive from an ante point of view, i.e. behind the veil of ignorance with respect to group-specific survival risk. For members of cohort v with full retirement at age R_v^j for group j it equalizes relative lifetime pension benefits and relative (unweighted) lifetime contributions across groups; i.e. it fulfills:*

$$\frac{B_v^H(R_v^H, \tilde{r})}{B_v^L(R_v^L, \tilde{r})} = \frac{C_v^H(R_v^H, 0)}{C_v^L(R_v^L, 0)} \iff \frac{B_v^H(R_v^H, \tilde{r})}{C_v^H(R_v^H, 0)} = \frac{B_v^L(R_v^L, \tilde{r})}{C_v^L(R_v^L, 0)}. \quad (16)$$

A fair pension system thus equalizes the benefit-contribution ratio across occupational groups for a given cohort (strict equivalence principle).²⁰ We will distinguish two possible implementations of fair pensions that capture different political (or informational) constraints. First, we abandon the requirement of equal replacement rates (i.e. condition (i) of Definition 2). Second, we abandon

¹⁸Eq. (14) formalizes the pension scheme depicted in OECD (2019) for Germany. We calibrate parameter ψ accordingly. \bar{b}_v is endogenous in the calibrated model (balanced pension budget condition).

¹⁹A condition similar to that in Definition 3 has been stated by Breyer and Hupfeld (2009) except that they ignore time discounting of benefits and implicitly assume that lifetime benefits are based on $\tilde{r} = 0$. Breyer and Hupfeld (2009) provide empirical estimates and predictions on the relationship between annual earnings and life expectancy of German retirees with and without fair pensions by abstracting from behavioral responses whereas we compare outcomes in a calibrated overlapping generations model.

²⁰A pension system that is fair for one cohort is not necessarily fair for other cohorts under cohort-heterogeneity (i.e., if there is demographic change). Designing a fair system for all cohorts would require replacement rates which are cohort- and time-variant. As mentioned, our simpler focus reflects the political constraint that prevents lowering replacement rates for given cohorts over time.

the requirement of $\tilde{r} = 0$ (i.e. condition (ii) of Definition 2) while preserving equal replacement rates for both occupation groups. The second implementation accounts for the possibility that policy makers could face difficulties to set distinct replacement rates according to occupation. For instance, conditioning pension policy explicitly on occupation could create incentives for employers or employees to reclassify occupations.

We are now ready to state the implications of Definition 3 for these two possible implementations. For this, we define for cohort v the replacement rate ratio (RRR) for blue-collar to white-collar workers as $\xi_v(R_v^H, R_v^L) \equiv b_v^L(R_v^L)/b_v^H(R_v^H)$.

Proposition 1. *In a pension system that is fair for cohort v in the sense of Definition 3 the following holds for given retirement ages (R_v^H, R_v^L) .*

(i) *When replacement rates can be conditioned on occupation and $\tilde{r} = 0$, the fair RRR, $\xi_v^{fair}(R_v^H, R_v^L)$, reads as*

$$\xi_v^{fair}(R_v^H, R_v^L) = \frac{\sum_{t=v+R_v^H}^{v+T-1} \frac{S_{v,t}^H}{\prod_{z=v+R_v^H}^t (1+r_{v,z}^H)}}{\sum_{t=v+R_v^L}^{v+T-1} \frac{S_{v,t}^L}{\prod_{z=v+R_v^L}^t (1+r_{v,z}^L)}}. \quad (17)$$

(ii) *When replacement rates are not conditioned on occupation, the fair calculatory interest rate $\tilde{r}_v^{fair} = \tilde{r}_v^{fair}(R_v^H, R_v^L)$ is implicitly determined by*

$$\frac{C_v^L(R_v^L, \tilde{r}_v^{fair})}{C_v^H(R_v^H, \tilde{r}_v^{fair})} = \frac{b_v(R_v^H)C_v^L(R_v^H, 0)}{b_v(R_v^L)C_v^H(R_v^L, 0)} \xi_v^{fair}(R_v^H, R_v^L). \quad (18)$$

Proof. Using (15) with (10) in (16) and setting $\tilde{r} = 0$ implies

$$\frac{b_v^H(R_v^H)C_v^H(R_v^H, 0) \sum_{t=v+R_v^H}^{v+T-1} \frac{S_{v,t}^H}{\prod_{z=v+R_v^H}^t (1+r_{v,z}^H)}}{b_v^L(R_v^L)C_v^L(R_v^L, 0) \sum_{t=v+R_v^L}^{v+T-1} \frac{S_{v,t}^L}{\prod_{z=v+R_v^L}^t (1+r_{v,z}^L)}} = \frac{C_v^H(R_v^H, 0)}{C_v^L(R_v^L, 0)} \quad (19)$$

Using $\xi_v(R_v^H, R_v^L) = b_v^L(R_v^L)/b_v^H(R_v^H)$ in (19) confirms part (i) of Proposition 1. Part (ii) also follows from using (15) with (10) in (16) when setting $b_v^H(R) = b_v^L(R) = b_v(R)$ for $R \in \{R_v^H, R_v^L\}$ and using the definition of $\xi_v^{fair}(R_v^H, R_v^L)$ from (17). This concludes the proof. \square

According to part (i) of Proposition 1, the fair RRR for a given cohort v , $\xi_v^{fair}(R_v^H, R_v^L)$, is independent of pension contributions. It only depends on the occupation-specific sequences of mortality rates for cohort v (and the world market interest rate, \bar{r}), in addition to retirement ages. As health deficits are higher for blue-collar workers than for white-collar workers at any age, mortality rates are higher as well. Thus, when workers from different occupations retire at the same age, a fair pension system assigns a higher replacement rate for blue-collar workers, $\xi_v^{fair}(R, R) > 1$, whereas $\xi_v(R, R) = 1$ (same replacement rates) in the status quo system characterized in Definition 2. A larger gap in mortality rates between blue-collar and white-collar workers raises the fair RRR, $\xi_v^{fair}(R_v^H, R_v^L)$, for any given (R_v^H, R_v^L) .

For the case where policy makers are restricted to occupation-invariant replacement rates (part (ii) of Proposition 1), a steeper age-earnings profile of white-collar workers (with lower initial earnings) implies that the left-hand side of (18) is increasing in \tilde{r} , if the calculatory interest rate only applies for pension wealth accumulated sufficiently early in life. In our calibrated model, this is the case for pension wealth accumulated up to period \underline{R} , as assumed in (5).

5. CALIBRATION

In order to quantitatively analyze the effects of establishing a fair pension system, we calibrate the model to the case of Germany under the status quo pension system that can be approximated as the ‘conventional equivalence principle’ of Definition 2, with specification (14). Germany has long served as a prime example for a PAYG pension system that is regressive due to its negligence of heterogeneity in mortality risk (e.g., Haan et al., 2020).

We calibrate a representative male white-collar worker and a representative male blue-collar worker (representing within-group averages) in terms of their rates of health deficit accumulation, survival rates, and earnings. Both workers start their working life at the age of 20, but with different age-earnings profiles. White-collar workers earn initially less (capturing in a stylized way a longer education period) and face a considerably steeper age-earnings profile than blue-collar workers. For computational reasons and since most earnings data are presented in 5-year intervals, a model period comprises five years. We will nevertheless report calibrated parameters as if one period is one year to facilitate the interpretation.

In our calibration strategy, we distinguish between externally set parameters and estimated parameters. Externally set parameter values are taken from other (comparable) studies or are directly observed in the data. Estimated parameters are simultaneously calibrated to fit the response of endogenous variables to moments observed in the data. We numerically solve the model with the relaxation algorithm by Trimborn et al. (2008).

5.1. Parametrization. We start by imposing functional forms on expressions that we kept general so far. For the deficit function, we assume $D(d_{v,t}^j) = \exp(\varrho d_{v,t}^j)$ such that ϱ captures how health deficits affect utility through the disutility of labor supply. We further assume that the labor units per hour $a_{v,t}^j$ evolve over age according to $a_{v,t}^j = \vartheta_0^j \exp[\vartheta_1^j(t - v) + \vartheta_2^j d_{v,t}^j]$. With this formulation, we capture the notion that individual labor productivity exhibits positive returns to age (i.e., experience), $\vartheta_1^j > 0$, and declines due to health deficit accumulation in the course of aging, $\vartheta_2^j < 0$. Confronting our formulation with the Mincerian wage equation, we thus generate the observed diminishing labor productivity at the end of the working life through the deterioration of health rather than the mere advancement of chronological age. In contrast to the original Mincer equation but in line with intuition and recent empirical findings, we thus assume that chronological age (more experience) contributes positively to productivity at all ages, but there is a productivity-reducing impact of declining health for any given age (Dalgaard et al., 2022). Importantly, we allow for

$\vartheta_1^H > \vartheta_1^L$ to capture that the age-earnings profile is steeper for white-collar workers which plays a particular role for establishing fairness by using a calculatory interest rate for contributions, $\tilde{r} > 0$.

5.2. Externally Set Parameters. The preference parameter related to labor supply, η , equals the Frisch elasticity and is set to 0.5 in accordance with the estimates by Chetty et al. (2011). Moreover, we set $\sigma = 2$ which reflects a mean value of 0.5 for the intertemporal elasticity of substitution from the literature (Chetty, 2006; Havranek, 2015).

For the initial frailty index of individuals we set $d_0 = 0.0273$. This is in line with the estimate in Mitnitski et al. (2002a) who find by exploiting Canadian data that, out of a potential list of 38 health deficits, males at age 20 roughly have, on average, one deficit.

At the production side, we set the capital share to $\alpha = 0.37$ (Feenstra et al., 2015) and $\rho = 1/3$ such that the elasticity of substitution between high-skilled and low-skilled labor amounts to $\frac{1}{1-\rho} = 1.5$ (Johnson, 1997). For the depreciation rate we assume a value of $\delta = 0.058$ (Davis and Heathcote, 2005) and for the real interest rate a value of $\bar{r} = 0.03$ which is in between the (longer run) rate of return of bonds and equity reported by Jorda et al. (2019).²¹

In the German pension system, pension contributions are proportional to labor income and amount to 18.6% of gross earnings (DRV, 2020). Therefore, we set $\tau^p = 0.186$. The German pension law also requires that pension income is reduced by 0.3% per month (or 3.6% per year) that the individual retires before the SRA. This suggests $\psi = 0.036$ for the early retirement penalty parameter in (14). Thus, if only blue-collar workers decided to retire five years earlier, the effective replacement rate would be only 82% of that of white-collar workers. This reduction in pension benefits comes on top of that implied by reduced pension wealth.

Regarding the income tax, we follow Grossmann and Strulik (2019) and set $\tau^w = 0.25$. Further, we set $\theta = 0.5$ implying that blue- and white-collar workers represent an equal share in the total labor force (Haipeter and Slomka, 2015, Figure 1). We analyze two different statutory retirement ages, $\bar{R} = 45$ and $\bar{R} = 50$. For the status quo analysis, we focus on $\bar{R} = 45$. Since economic life in our model starts at the age of 20, this reflects an SRA of 65 years (OECD, 2019). For computational reasons (i.e. limiting the number of possible retirement age combinations of blue- and white-collar workers), we assume that the earliest possible retirement age is $\underline{R} = 40$. Finally, we set $T = 80$ such that the maximum life span amounts to 100 years. Table 1 summarizes the externally set parameter values along with the associated source.

5.3. Calibrated Parameters. We set the occupation-specific aging parameters in line with the data employed for Figure 1. For the aging parameter for white collar-workers, the evidence suggests no difference before and after retirement, $\bar{\mu}^H = \underline{\mu}^H$. When retirement sets in and the work-related health burden comes to an end, blue-collar workers enjoy a drop in the rate of health deficit accumulation such that the rate of aging between blue- and white-collar workers equalizes after retirement (Abeliansky and Strulik, 2022), $\underline{\mu}^L = \underline{\mu}^H < \bar{\mu}^L$. We fit $\bar{\mu}^H = \underline{\mu}^H = \underline{\mu}^L$ to replicate the deficit accumulation process of white-collar workers as shown in Figure 1 and set $\bar{\mu}^L$ together

²¹In sensitivity analysis (Section 9), we report results for alternative interest rates and consumption profiles.

TABLE 1. Externally Set Parameters

Parameter	Value	Explanation	Source
η	0.5	Frisch elasticity of labor supply	Chetty et al. (2011)
$1/\sigma$	0.5	intertemporal elasticity of substitution	Chetty (2006), Havranek (2015)
d_0	0.0273	initial deficit level	Mitnitski et al. (2002)
α	0.37	capital share	Feenstra et al. (2015)
$\frac{1}{1-\rho}$	1.5	elast. of subst. blue- and white-collar	Johnson (1997)
δ	0.058	depreciation rate	Davis and Heathcote (2005)
\bar{r}	0.03	(real) interest rate	Jorda et al. (2019)
τ^p	0.186	pension contribution rate	DRV (2020)
τ^w	0.25	income tax rate	Grossmann and Strulik (2019)
θ	0.5	share of white-collar workers	Haipeter and Slomka (2015)
\bar{R}	45; 50	length of working life until statutory retirement	OECD (2019); alternative scenario
\underline{R}	40	minimum length of working life until retirement	simplifying assumption
T	80	maximum length of economically active life	convention

with the survival function parameters ω and κ to match three observations in the data. First, the weighted average of the occupation-specific life expectancies equals male life expectancy at age 20 of 59.2 years and at age 65 of 18.1 years – as reported in life tables of the World Health Organization (WHO, 2020) for the year 2019. As life expectancies depend on early retirement decisions in the model, we assume that there is no early retirement and show that this is an equilibrium outcome under the existing early retirement penalties in Germany. Second, we match a life expectancy differential at 65 between these two groups of 2.6 years as reported in Kibele et al. (2013). This procedure yields $\bar{\mu}^L = 0.028$ and $\bar{\mu}^H = \underline{\mu}^H = \underline{\mu}^L = 0.026$. The difference between the two values lies in the confidence interval of a study by Abeliansky and Strulik (2022) which reports a difference in conceptionally equivalent aging parameters of 0.04. In other words, blue-collar workers’ health deficits grow by 2.8% per year before retirement while health deficits of white-collar workers grow by 2.6% per year.

With regard to the utility weight of health deficits ϱ , we rely on a study by Cai et al. (2014). The study presents evidence on the effect of health shocks and health status on labor supply at the intensive margin. According to their estimates, a one-standard-deviation decrease in health status leads to a reduction in working hours of 6.86 %.²² In Appendix A.3, we show that labor supply can be written in the form $\ell_{v,t}^j = \left(\frac{\Xi_{v,t}^j W_{v,t}^j}{D(d_{v,t}^j)} \right)^\eta$. Relative labor supply $\ell/\tilde{\ell}$ of two individuals

²²Cai et al. (2014) find that a unit change in health status changes labor supply by around 3.5 hours. Given that the standard deviation of health status is 0.77, a one-standard-deviation change in health status is associated with a 2.7 hours change in labor supply. Given a mean of 39.27 hours worked in the data set, this amounts to a relative change of 6.86 %.

from the same occupation group with different levels of health deficits d and \tilde{d} thus is $\ell/\tilde{\ell} = D(\tilde{d})^\eta/D(d)^\eta = \exp[\varrho\eta(\tilde{d} - d)]$. In our model, one standard deviation of health deficits in the working age population equals 0.0118. Given the parameter value $\eta = 0.5$, setting $\ell/\tilde{\ell} = 1.068$ and $\tilde{d} - d = 0.0118$ yields a value of $\varrho = 11.25$.

Moreover, we normalize $\vartheta_0^L = 1$ (determining initial productive ability of blue-collar workers) and simultaneously calibrate the remaining parameters β , \bar{u} , γ , ϑ_0^H , ϑ_1^H , ϑ_1^L , ϑ_2^H , ϑ_2^L , and χ under the status quo pension system to match the following cross-sectional data moments for males: (i) earnings differential between blue- and white-collar workers, (ii) the age profile of aggregate earnings, (iii) relative labor supply of blue- to white-collar workers at age 35-39, as well as the following longitudinal data moments: (iv) consumption profile over the life cycle, (v) occupation-specific evolution of earnings over the life cycle, and (vi) the ratio between the empirical value of a statistical life (VSL) and earnings at age 50.

Data for calibration target (i) comes from Haipeter and Slomka (2015) who find an earnings ratio of white- to blue-collar workers in the German manufacturing sector of 1.6. For calibration target (ii) we use data from Dossche and Hartwig (2019) on the cross-sectional age profile of earnings. For calibration target (iii), we assume that blue- and white-collar workers supply on average the same amount of labor at age 35-39. This is plausible as the vast majority of workers is healthy at that age and the fraction of part-time workers among males is low. For calibration target (iv), we match a stable consumption profile according to Browning and Ejrnaes (2009). The authors find that once family size is controlled for, the consumption trajectory over the life cycle becomes flat. With $\bar{r} = 0.03$, this implies $\beta = (1 + \bar{r})^{-1} = 0.97$. Calibration target (v) is included to capture occupation-specific differences in life-cycle earnings. According to Ruzik-Sierdzinska et al. (2013), earnings of white-collar workers increase at a higher pace at the beginning of life than those of blue-collar workers. Complementing this observation, Bönke et al. (2015) find that earnings of high-skilled workers start off lower than those of low-skilled workers, coincide between age 25-29, and then continue to grow at higher rate thereafter. Further, the findings by Ruzik-Sierdzinska et al. (2013) suggest that the earnings of both blue- and white-collar workers peak between 45-49. We aim to capture these stylized facts by matching occupation-specific earnings to those moments of the data. Finally, for calibration target (vi), we use the observation from Murphy and Topel (2006) that the ratio between the VSL and earnings at age 50 is equal to 100. Table 2 shows the calibration results.

Our estimates for the productivity parameters suggest that there are positive returns to chronological age (capturing experience) and negative returns to (physiological) aging due to health deficit accumulation. This pattern is reflected in positive estimates for ϑ_1^H and ϑ_1^L and negative estimates for ϑ_2^H and ϑ_2^L . The feature $\vartheta_1^L < \vartheta_1^H$ implies that earnings of white-collar workers grow at a higher rate over age than those of blue-collar workers. Finally, the earnings ratio between blue- and white-collar workers of 1.6 together with a similar population share implies an earnings share of white-collar workers in total earnings of 0.62 and a white-collar weight in production (χ) of 0.68.

TABLE 2. Calibration Results

Parameter	Value	Explanation
$\bar{\mu}^L$	0.028	rate of aging of blue-collar workers before retirement
$\underline{\mu}^L$	0.026	rate of aging of blue-collar workers after retirement
$\bar{\mu}^H = \underline{\mu}^H$	0.026	rate of aging of white-collar workers
β	0.97	time preference
ϱ	11.25	preference parameter health deficits
\bar{u}	65.7	utility constant
ϑ_0^L	1	level parameter blue-collar ability (normalized)
ϑ_0^H	1.3	level parameter white-collar ability
ϑ_1^L	0.25	experience coefficient blue-collar ability
ϑ_1^H	0.64	experience coefficient white-collar ability
ϑ_2^L	-31.4	health deficit coefficient blue-collar ability
ϑ_2^H	-88.7	health deficit coefficient white-collar ability
χ	0.68	white-collar weight in production
κ	37.42	survival function parameter
ω	0.0055	survival function parameter

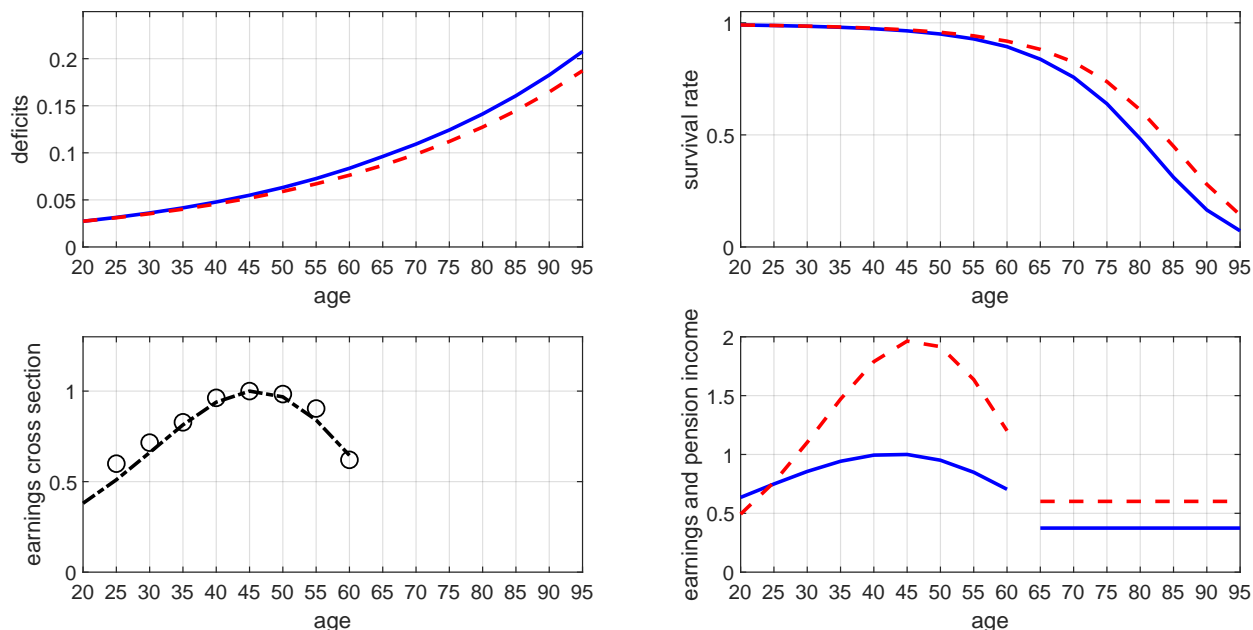
6. IMPLICATIONS OF THE CONVENTIONAL EQUIVALENCE PRINCIPLE

6.1. Age Profiles and Benefit-Contribution Ratio. Figure 2 illustrates implications for important observables under the status quo policy (i.e., the ‘conventional equivalence principle’ in Definition 2) with the current early retirement penalty $\psi = 0.036$ in schedule (14) and an SRA of 65. In equilibrium, all workers retire at age 65 (no early retirement). Blue (solid) lines represent blue-collar workers, red (dashed) lines represent white-collar workers, and black (dash-dotted) lines represent the model response for the weighted average between these two.

The upper left panel shows the evolution of health deficits over age. Due to the higher work-related health burden, deficits of blue-collar workers accumulate at a higher rate than those of white-collar workers. At retirement age 65, the health deficit trajectory of blue-collar workers exhibits a kink due to the drop in the rate of aging once those individuals retire (i.e., reflecting $\underline{\mu}^L < \bar{\mu}^L$). Although blue- and white-collar workers face the same rate of aging after retirement, the deficit paths continue to diverge. The reason behind this observation is that the evolution of health deficits over age is path-dependent, meaning that the speed of deficit accumulation is positively affected by the amount of health deficits an individual has already accumulated. Since blue-collar workers have accumulated a higher amount of health deficits at the time of retirement, health deficit accumulation remains faster than that of white-collar workers.

The upper right panel shows the associated survival functions which for both groups decline logistically with age. The higher health deficit levels of blue-collar workers result in a lower survival

FIGURE 2. Model Results: Benchmark Run



Blue (solid lines): blue-collar workers. Red (dashed) lines: white-collar workers. Black (dash-dotted) lines: weighted average between blue- and white-collar workers. Dots indicate data points. Data for cross-sectional earnings are from Dossche and Hartwig (2019).

probability at any age, according to (2). The model predicts that life expectancy at age 20 is 3.4 years higher for white-collar workers than for blue-collar workers. Luy et al. (2015) report that in Germany male life expectancy at age 40 of employees and public servants exceeds that of manual workers by 3.2 to 3.7 years. The non-targeted prediction of the model accords well with these empirical observations.

The lower left panel illustrates the age profile of earnings in the cross section, i.e., across cohorts for a given point in time, relative to earnings at age 45-49. We calculate a weighted average between the earnings of blue- and white-collar workers according to their share in the population and their survival rate $S_{v,t}^j$. Dots indicate data points from Dossche and Hartwig (2019). As can be seen in the figure, the model predicts the typical hump-shaped pattern of the cross sectional earnings trajectory reasonably well.

The lower right panel shows occupation-specific earnings over the life-cycle. The reported numbers are computed relative to earnings of a blue-collar worker at age 45-49. As intended by our calibration strategy and consistent with empirical evidence, earnings of white-collar workers start off at a lower level, but grow at a higher rate and quickly surpass earnings of blue-collar workers. After a peak around age 45-49, earnings start to decrease as a result of growing health deficits that impair labor productivity and induce lower labor supply. Starting at age 65, individuals receive a per-period pension benefit proportional to their pension contributions as implied by the status

quo pension system. Occupational differences in per-period pension benefits combined with mortality rate differences imply a lifetime benefit ratio between the two groups $B^L/B^H = 0.52$ under the status quo system (with equilibrium retirement ages of 65), whereas the contribution ratio is $C^L/C^H = 0.62$ and thus 10 percentage points lower than required by a fair pension system. The implied benefit-contribution ratio is $0.52/0.62 = 0.84$, indicating that blue-collar workers receive 16% less benefits from contributions than white-collar workers.

While we focus on within-group averages for both types of workers to derive optimal fair policies behind the veil of ignorance, it is interesting to consider deviations in behavior in response to an individual health shock to gain further insights in the mechanics of the model. In Appendix B.1, we analyze the effect of a sudden increase in the health deficit index of a blue-collar worker. We document the differential evolution of the health outcomes relative to the average blue-collar worker and the result of individual re-optimization. Because working becomes more painful and productivity of the individual declines after the health shock, the individual reduces both labor supply and consumption on impact.

6.2. Increasing the Statutory Retirement Age. A popular theme in the discussion of pension system reforms is an extension of the retirement age. If the SRA is universally increased to age 70 while maintaining the other characteristics of the pension system, we find that all workers still work until the SRA in equilibrium. Because of the detrimental health effects of working longer (up to age 70 rather than age 65), according to (1) with $\underline{\mu}^L < \bar{\mu}^L$, the life expectancy gap increases from 3.44 to 3.77 years at age 20. Remaining life expectancy of blue-collar workers at age 65 decreases from 16.8 to 16.4 years, compared to 19.4 years for white-collar workers (i.e., the gap at age 65 increases to 3 years). The replacement rate \bar{b} (the ratio of annual benefits to total contributions in the case where individuals retire at the SRA) increases from 4.77% to 6.09% due to the later retirement. The lifetime benefit ratio between the two groups declines to $B^L/B^H = 0.49$ though, whereas the contribution ratio increases to $C^L/C^H = 0.63$. The implied benefit-contribution ratio thus declines to $0.49/0.63 = 0.78$. That is, blue-collar workers now receive 22% instead of 16% less benefits from contributions than white-collar workers.

The higher SRA has opposing effects on occupation-specific welfare which we measure as consumption equivalent changes (see Appendix A.4 for details). Without an accompanying pension reform, the higher SRA reduces the welfare of blue-collar workers equivalent to a permanent consumption drop of 0.78% whereas white-collar workers gain 0.79%. Because of the calibrated strongly concave utility function ($\sigma = 2$), these relative changes are associated with a reduction of aggregate social welfare.

7. SOCIAL PLANNING PROBLEMS

The social planner has the objective of achieving fairness across occupation groups from an ante point of view (i.e., at the beginning of working life) by controlling retirement ages along with

replacement rates.²³ The social planner also needs to take into account the behavioral responses of workers and firms and a balanced government budget requirements. The fairness constraint may be viewed as an institutional device ensuring that lifetime returns to contributions are equal across occupational groups. We derive socially optimal and fair pension policies under both a utilitarian welfare criterion (i.e., the social planner assigns equal weights to all individuals) and a Rawlsian welfare criterion. In our calibrated model, a Rawlsian welfare criterion effectively means that the social planner maximizes welfare of blue-collar workers because they are the least advantaged social group.

7.1. Social Welfare and Constraints. Indirect lifetime utility of an individual from cohort v and occupation group j for a given retirement age R_v^j is a function of time sequences of the earned income tax credit, $\{\bar{I}_t\}_{t=v}^{v+R_v^j-1}$, pension benefits $\{\hat{I}_{v,t}^j\}_{t=v+R_v^j}^{v+T-1}$, the rate of return to asset holdings, $\{r_{v,t}^j\}_{t=v}^{v+T-1}$, and the group-specific wage rate $\{w_t^j\}_{t=v}^{v+R_v^j-1}$. For the sake of tractability, we focus on steady states at which cohorts are identical. Accordingly, we drop the time index and the cohort index.²⁴

Wage rates as implied by equilibrium condition 1 in Definition 1 are functions of aggregate labor supply H and L .²⁵ According to (12) and (13), they depend on the retirement ages in both occupation groups, R^H and R^L . As those are controls of the social planner, we write equilibrium wage rates as $w^H = w^H(R^H, R^L)$ and $w^L = w^L(R^H, R^L)$. Similarly, the earned income tax credit, that is related to aggregate labor income, can be written as $\bar{I} = \bar{I}(R^H, R^L)$.²⁶ As the streams of both labor-related income and pension benefits depend on individual retirement decisions, indirect utility is explicitly a function of the individually declared retirement age as well, according to (5), (6), and (8). The sequence of the interest rate of a member of occupation group j over the course of the lifetime may also depend on individual retirement age R^j . To see this, recall from (4) that interest rates depend on mortality rates, which are determined by the process of health deficit accumulation, according to (2). In turn, according to (1), health deficits during retirement depend on the retirement decision (at least for blue-collar workers, as $\underline{\mu}^L < \bar{\mu}^L$). Finally, recall that pension benefits depend on the replacement rate and the calculatory interest rate, \tilde{r} . Summing up, we can denote the indirect lifetime utility function of each individual from occupation group j by $\tilde{U}^j(w^j(R^H, R^L), \bar{I}(R^H, R^L), R^j, b^j(R^j), \tilde{r})$.

We next derive the constraints of the social planner. For the pension budget constraint, recall from (10) that the pension benefit of a member of group j is given by $b^j(R^j)C^j(R^j, \tilde{r})$. Aggregate pension contributions in the economy are given by $\tau^p(w^H H + w^L L)$. A balanced budget thus

²³The policies do not depend on decision time of the social planner and are thus time-consistent (i.e. do not require commitment).

²⁴We numerically confirmed that the dynamic system converges very fast (within a few periods) to the steady state in response to shocks. In order to focus on the heterogeneity of aging processes within cohorts, we abstract from transitional dynamics caused by demographic trends. Cohort heterogeneity is important for analyzing fiscal sustainability problems of pension systems but of secondary importance for designing fairness within cohorts.

²⁵See expressions (28) and (29) in Appendix A.1.

²⁶The full expression is derived in Appendix A.2., see (30).

requires

$$\begin{aligned} & \tau^p \cdot (w^L(R^H, R^L)L + w^H(R^H, R^L)H) \\ = & \theta \cdot b^H(R^H) \cdot C^H(R^H, \tilde{r}) \sum_{i=R^H}^{T-1} \tilde{S}_i^H + (1 - \theta) \cdot b^L(R^L) \cdot C^L(R^L, \tilde{r}) \cdot \sum_{i=R^L}^{T-1} \tilde{S}_i^L \end{aligned} \quad (20)$$

to hold, where \tilde{S}_i^j is the survival rate at age i of a member from occupation group j .

Moreover, the social planner regards incentive compatibility constraints that ensure consistency with individual choices of retirement ages. Formally, these are given by

$$\begin{aligned} & \tilde{U}^H(w^H(R^H, R^L), \bar{I}(R^H, R^L), R^H, b^H(R^H), \tilde{r}) \geq \\ & \tilde{U}^H(w^H(R^H, R^L), \bar{I}(R^H, R^L), R, b^H(R), \tilde{r}) \forall R \in \mathcal{R}, \end{aligned} \quad (21)$$

$$\begin{aligned} & \tilde{U}^L(w^L(R^H, R^L), \bar{I}(R^H, R^L), R^L, b^L(R^L), \tilde{r}) \geq \\ & \tilde{U}^L(w^L(R^H, R^L), \bar{I}(R^H, R^L), R, b^L(R), \tilde{r}) \forall R \in \mathcal{R}. \end{aligned} \quad (22)$$

for white-collar and blue-collar workers, respectively. Conditions (21) and (22) mean that an individual deviation from planned retirement ages does not pay off, when taking the equilibrium paths of wage rates and the earned income tax credit resulting from planned retirement ages in the economy as given (since an individual deviant has mass zero).

7.2. Optimization Problems. The fairness constraint for the social planner problem in the scenario analyzed in part (i) of Proposition 1 reads as

$$b^L(R^L) = b^H(R^H) \cdot \xi^{fair}(R^H, R^L). \quad (23)$$

That is, the replacement rates can be conditioned on occupation to fulfill the fairness condition (16) in Definition 3 with $\tilde{r} = 0$.

The corresponding optimization problem, labeled SPP1, can then be written as

$$\begin{aligned} & \max_{b^H(R^H), b^L(R^L), R^H, R^L} \left\{ \tilde{\theta} \tilde{U}^L(w^L(R^H, R^L), \bar{I}(R^H, R^L), R^L, b^L(R^L), \tilde{r}) + \right. \\ & \left. (1 - \tilde{\theta}) \tilde{U}^H(w^H(R^H, R^L), \bar{I}(R^H, R^L), R^H, b^H(R^H), \tilde{r}) \right\} \text{ s.t. (20), (21), (22), (23), } \tilde{r} = 0, \end{aligned} \quad (24)$$

We consider welfare weights $\tilde{\theta} = \theta$ (utilitarian case) and $\tilde{\theta} = 1$ (Rawlsian case). To solve SPP1, we first calculate for all possible (R^H, R^L) -combinations the replacement rates $b^H(R^H)$ and $b^L(R^L)$ that jointly fulfill pension budget constraint (20) and the fairness constraint (23). We then identify the combination that gives the highest welfare, denoted by (R^{H*}, R^{L*}) . Notably, the social planner can always make the desired policy schedule (R^{H*}, R^{L*}) incentive compatible by appropriately designing replacement rate schedules $b^H(R)$ and $b^L(R)$ on the right-hand side of (21) and (22), $R \in \mathcal{R}$, respectively, when a white-collar or blue-collar individual deviates from R^{H*} or R^{L*} , respectively.²⁷

²⁷For instance, this could be done by implementing linear occupation-specific schedules, $b^j(R) = \bar{b} - \varrho^j \cdot (\bar{R} - R)$, that fulfill fairness constraint (23) at the planned retirement ages (R^{H*}, R^{L*}) , where $\varrho^j > 0$ denotes the early

Moreover, we solve the social planner problem in the scenario analyzed in part (ii) of Proposition 1, labeled SPP2, with fairness constraint

$$\tilde{r} = \tilde{r}^{fair}(R^H, R^L). \quad (25)$$

That is, we derive the fair calculatory interest rate on pension contributions that incentivizes welfare-maximizing retirement decisions. Recall that this policy option does not require the social planner to observe individual occupations, i.e., we now assume that the social planner applies the same replacement rate schedule for both occupation groups. For concreteness, replacement rates follow the linear status quo policy schedule (14) in Germany. SPP2 can then be written as

$$\max_{\tilde{r}, R^H, R^L} \left\{ \tilde{\theta} \tilde{U}^L + (1 - \tilde{\theta}) \tilde{U}^H \right\} \text{ s.t. (20), (21), (22), (25),} \quad (26)$$

$$b^H(R) = b^L(R) = \bar{b} - \psi \cdot (\bar{R} - R), R \in \mathcal{R}. \quad (27)$$

To solve SPP2, we first find for all possible (R^H, R^L) -combinations the fair calculatory interest rate $\tilde{r}^{fair}(R^H, R^L)$ that fulfills (18). Among those we identify, in a second step, the combination of retirement ages that gives the highest welfare when ignoring incentive compatibility constraints (21) and (22). In a third step, we check if anybody has an incentive to deviate from those retirement ages for given income paths during working life and given the replacement rate schedule (27).²⁸ If neither blue-collar nor white-collar workers have an incentive to deviate, we have found a solution. If an individual gains higher utility from deviating, however, we repeat the procedure for the combination $(R^H, R^L, \tilde{r}^{fair}(R^H, R^L))$ found in the first step that gives the second-highest welfare, etc. We report the solutions to (SPP1) and (SPP2) for various levels of the SRA, \bar{R} .

8. FAIR AND SOCIALLY OPTIMAL PENSION POLICIES

Results for the case of occupation-specific replacement rates (part (i) of Proposition 1 and solution to SPP1) are presented in Section 8.1 and results for fair and socially optimal calculatory rates of return on pension contributions (part (ii) of Proposition 1 and solution to SPP2) are presented in Section 8.2. Table 3 shows the fair policies for the baseline calibration for different combinations of the retirement age. The socially optimal pension policies are displayed in bold under utilitarian welfare ($\tilde{\theta} = 0.5$) and underlined with a Rawlsian welfare function ($\tilde{\theta} = 1$). We distinguish the cases where the set of retirement ages is $\mathcal{R} = \{40, 45\}$ (i.e., a real life SRA of 65 years and earliest retirement at age 60, as economic life starts at age 20 in the calibrated model) and $\mathcal{R} = \{40, 45, 50\}$ (SRA of 70 years), displayed in the upper and lower part of Table 3, respectively.

8.1. Fair and Socially Optimal Replacement Rates.

retirement penalty parameter for occupation group j . The replacement rate that applies when retiring at the SRA, \bar{b} , endogenously adjusts to fulfill the pension budget constraint (20).

²⁸Note that ϱ and \bar{R} are calibrated in the numerical analysis, whereas, again, \bar{b} endogenously adjusts to the pension budget constraint (20).

TABLE 3. Fair Pension Policies

case	R^L	R^H	ξ^{fair}	\tilde{r}^{fair}
SRA 65				
1)	65	65	1.19	<u>0.060</u>
2)	60	65	<u>0.95</u>	0.036
3)	65	60	1.42	0.077
4)	60	60	1.13	0.042
SRA 70				
5)	70	70	1.29	<u>0.091</u>
6)	65	70	0.96	0.044
7)	70	65	1.61	0.125
8)	60	70	<u>0.76</u>	0.037
9)	70	60	1.91	0.137
10)	65	65	1.19	0.060
11)	60	65	0.95	0.049
12)	65	60	1.42	0.055
13)	60	60	1.13	0.042

R^L : retirement age blue-collar workers. R^H : retirement age white-collar workers. ξ^{fair} : fair replacement rate ratio between blue- and white-collar workers. \tilde{r}^{fair} : fair calculatory interest rate. Optimal policies are indicated by bold (for utilitarian welfare criterion) and underlined (for Rawlsian welfare criterion).

8.1.1. *The Fair Replacement Rate Ratio.* We start with a discussion of the fair RRR, $\xi^{fair}(R^H, R^L)$, according to part (i) of Proposition 1, and the solution to SPP1.

With a utilitarian welfare function and a SRA of 65, it is socially optimal to incentivize both blue- and white-collar workers to retire at age 65. In this case, the fair RRR is $\xi^{fair} = 1.19$ (line 1 of Table 3). That is, accounting for occupation-specific mortality in a fair pension system implies that the replacement rate of blue-collar workers should exceed the one of white-collar workers by 19%. This closes the 16% gap in the benefit-contribution ratio in the status quo case where the RRR is one ($\xi = 1$). As long as both groups retire at the same age, the fair RRR increases in the retirement age from 1.13 for age 60 to 1.29 for age 70, because blue-collar workers suffer a longer time from a higher health-related job burden. This adversely affects old-age survival rates and thereby reduces life expectancy.

In general, once an occupational group decides to retire early, the fair RRR rate tends to move in favor of the other group to account for the shorter retirement period. A fair pension system requires to increase the RRR from 0.82 to $\xi^{fair} = 0.95$ for a SRA of 65 when blue-collar workers

decide to retire five years early and white-collar workers work until the SRA (line 2 of Table 3). This is indeed the optimal fair policy under Rawlsian welfare, that maximizes welfare for the more disadvantaged group. The result implies that the monthly penalty of early retirement should lie around 0.1% rather than the current 0.3%. Blue-collar workers can expect to live 0.3 years longer when retiring at age 60 rather than 65 due to the slowdown in health deficit accumulation.

When the SRA is 70, early retirement of blue-collar workers (and no early retirement of white-collar workers) is optimal under both welfare criteria. Under the utilitarian welfare criterion, blue-collar workers optimally retire five years earlier with a small penalty ($\xi^{fair} = 0.96$) while under the Rawlsian welfare criterion, they optimally retire ten years earlier with a considerably higher penalty ($\xi^{fair} = 0.76$).

8.1.2. *Pension Reform Implications.* Table 4 reports the behavioral responses (lifetime labor supply of someone who survives until declared retirement, average hourly wages over the lifetime, the PDV of expected earnings, the PDV of expected pension income) as well as changes in occupation-specific welfare (in consumption equivalents) and changes in aggregate lifetime utility from deviating from the status quo by implementing the fair and optimal replacement rate policy (solution to SPP1).

Utilitarian Welfare. The first part of Table 4 shows the results when switching to the fair replacement rate policy for utilitarian welfare ($\tilde{\theta} = 0.5$) and an SRA of 65. Anticipating the higher replacement rate in old age, blue-collar workers slightly reduce average labor supply by 0.18% (line 1). Since blue-collar labor becomes more scarce, average hourly wages go up by 0.13%. Both effects combined result in an overall decline of the PDV of expected lifetime earnings of 0.10%. The PDV of expected lifetime pension income increases by 11.8%. With respect to welfare, the negative effect from (slightly) lower wage income is dominated by the positive effects of lower lifetime labor supply and higher pension income, leading to a consumption equivalent welfare gain of 0.81% compared to the status quo.²⁹

The pension reform affects behavior of white-collar workers in the opposite direction. The lower replacement rate induces white-collar workers to work slightly more (0.15%), resulting in a small reduction in hourly wages of 0.08% (line 2). The net effect on earnings, however, is still positive (0.09%). Despite slightly higher earnings during working age, the drop in lifetime pension income of 6.08% leads to a welfare loss for white-collar workers of 0.57%. Recalling that blue- and white-collar workers contribute equal shares to the total population, we assign the same weight to both groups in the computation of aggregate welfare and conclude that the fair pension system improves aggregate lifetime utility compared to the status quo system by 0.03%³⁰.

²⁹In order to put the welfare gain in perspective, it may be worthwhile to recall that a one percent welfare gain (from abolishing capital taxation) has been appraised as the largest gain that quantitative welfare economics can provide (Lucas, 1990) and that the welfare gained from eliminating all business cycles has been estimated to be somewhere between 0.008 and 0.1 percent (Krusell and Smith, 1999).

³⁰ $\Delta Welfare$ is measured as the relative change in aggregate lifetime utility while occupation-specific welfare changes are measured in consumption equivalents. Therefore, these figures are not directly comparable. Also note that an increase in relative consumption of blue-collar workers in conjunction with an equiproportionate decline in consumption of white-collar workers would leave aggregate welfare unchanged if the utility functions were logarithmic. For the

TABLE 4. Effects of Optimal Fair Replacement Rates

Utilitarian Welfare	$\Delta\ell$	ΔW	$\Delta W\ell$	ΔI	ΔU	$\Delta Welfare$
<u>SRA of 65, $R^L = 65$, $R^H = 65$</u>						
<i>optimal fair replacement rate ratio: $\xi^{fair} = 1.19$</i>						
1) blue-collar workers	-0.18	0.13	-0.10	11.8	0.81	0.03
2) white-collar workers	0.15	-0.08	0.09	-6.08	-0.57	
<u>SRA of 70, $R^L = 65$, $R^H = 70$</u>						
<i>optimal fair replacement rate ratio: $\xi^{fair} = 0.96$</i>						
3) blue-collar workers	-6.46	1.90	0.51	39.4	1.93	0.12
4) white-collar workers	-0.05	-1.16	-1.17	-10.3	-0.90	
Rawlsian Welfare	$\Delta\ell$	ΔW	$\Delta W\ell$	ΔI	ΔU	$\Delta Welfare$
<u>SRA of 65, $R^L = 60$, $R^H = 65$</u>						
<i>optimal fair replacement rate ratio: $\xi^{fair} = 0.95$</i>						
5) blue-collar workers	-7.38	2.77	0.55	27.4	1.04	0.02
6) white-collar workers	-0.03	-1.66	-1.68	-8.59	-1.09	
<u>SRA of 70, $R^L = 60$, $R^H = 70$</u>						
<i>optimal fair replacement rate ratio: $\xi^{fair} = 0.76$</i>						
7) blue-collar workers	-13.2	4.60	1.19	58.9	2.14	0.09
8) white-collar workers	-0.21	-2.71	-2.88	-12.7	-1.45	

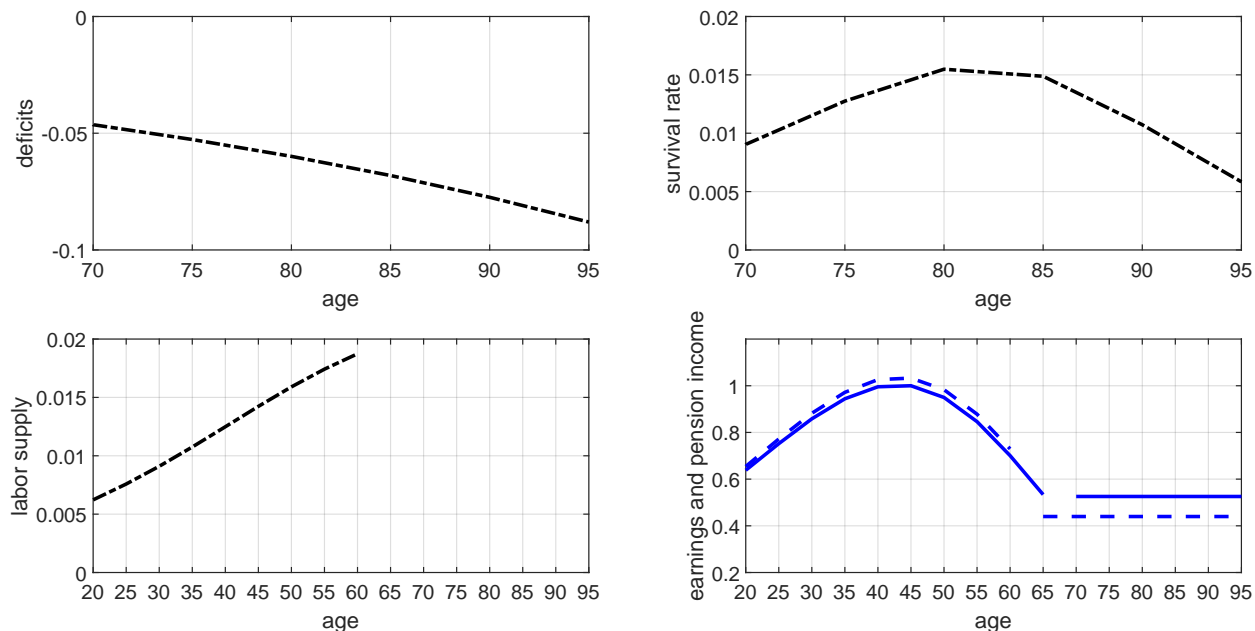
We report changes in percent when deviating from the status quo to the optimal system for lifetime labor supply conditional on survival until declared retirement (ℓ), average hourly wages over the lifetime (W), the PDV of expected earnings ($W\ell$), the PDV of expected pension benefits (I), occupation-specific welfare measured in consumption equivalents (U), and aggregate lifetime utility (Welfare). R^L and R^H refer to retirement ages of blue- and white-collar workers, respectively. ξ^{fair} refers to the fair RRR between blue-collar and white-collar workers.

Recalling the results in Table 3, solving SPP1 for utilitarian welfare and an SRA of 70 involves early retirement of blue-collar workers at age 65 and almost no early retirement penalty. The retirement decision involves the following trade-off. On the one hand, early retirement shortens the length of the working period when supplying labor is particularly painful because of a relatively high number of health deficits (captured by function $D(d)$ in utility function (3)). Further, early retirement slows down the work-related accumulation of health deficits for blue-collar workers, according to (1) with $\underline{\mu}^L < \bar{\mu}^L$. This results in higher survival rates after retirement (and thus higher life expectancy). Both effects imply higher expected utility. On the other hand, early

calibrated $\sigma = 2$, however, the relative increase in blue collar consumption receives a greater weight than the relative decline in white collar consumption. Thus, a higher welfare loss of white-collar workers compared to the welfare gain of blue-collar workers may still result in an aggregate welfare gain due to the curvature of the utility function.

retirement reduces the periods in which the individual contributes to the pension system and thus decreases annual pension benefits.

FIGURE 3. Blue-Collar Life-Cycle Outcomes: Status Quo vs. Fair Replacement Rates with Early Retirement for an SRA of 70



Blue (solid) lines: per-period earnings and per-period pension benefits under the status quo pension system. Blue (dashed) lines: per-period earnings and per-period pension benefits with a fair RRR and early retirement. Black (dash-dotted) lines: Difference in outcomes between status quo pension system and fair pensions with early retirement, normalized by the respective initial value under the status quo pension system.

The effects of switching to the fair and optimal replacement rate policy for utilitarian welfare and an SRA of 70 are shown in Figure 3 and lines 3 and 4 of Table 4. Black (dash-dotted) lines in the first three panels of Figure 3 show the difference across the two policy scenarios for health deficits, survival, and labor supply of blue-collar workers (normalized by the respective initial value under status quo policy). Until age 65, health deficits coincide under both pension policies. Since blue-collar workers retire at age 65 instead of 70 under the fair policy, they enjoy five more years of leisure without accumulating work-related deficits. As a result, deficits accumulate at lower pace, which is reflected in the figure by a negative difference of deficits by age. Reflecting the self-productive nature of deficit accumulation, the health gain from early retirement increases with advancing age. The gain in health translates into improved survival rates in old age, which are depicted in the upper right panel of Figure 3. Consistent with the gain in health deficits, the gain in survival first increases with age; it eventually declines because in the end everybody dies. Recall that the associated gain in remaining life expectancy at age 65 of blue-collar workers when retiring at age 65 rather than 70 is 0.4 years.

Anticipating that working life is shorter and thus life time earnings and pension income will be lower, blue-collar workers increase labor supply at any age to partly compensate for this loss of income, as visible in the lower left panel of Figure 3. The lower right panel of Figure 3 shows that increased labor supply translates into higher annual earnings. The blue solid lines represent the life-cycle trajectory of a blue-collar worker in the status quo pension system, while the blue dashed lines represent the outcome with a fair RRR. The combined effect of increased labor supply when working and early retirement with respect to lifetime labor supply is negative. According to line 3 of Table 4, it decreases by 6.46%, which leads to an increase in hourly wages by 1.90%. The PDV of expected earnings still increases by 0.51%, since the relatively large drop in earnings occurs at the end of the working life.

Lifetime pension income of blue-collar workers increases by 39.4%. On the one hand, both pension contributions and the replacement rate decreases because contributions are paid five years less, lowering annual pension income. On the other hand, blue-collar workers collect pension income already five years earlier and, through better health and thus higher survival rates, exhibit a higher probability of collecting pension for a longer time in retirement. This, taken for itself, has a positive impact on lifetime pension income and is the dominating force in this experiment. Due to the higher pension income combined with better health and higher life expectancy and the reduction of labor supply (especially at the end of the working life when working is especially painful), there is a consumption equivalent welfare gain for blue-collar workers of 1.93%.

The considerably lower labor supply of blue-collar workers causes a decline in the marginal product of white-collar labor, which results in a drop of their hourly wages by 1.16%. This leads to a reduction in their lifetime earnings by 1.32%. Since both blue-collar and white-collar workers contribute less to the pension system and white-collar workers have to cross-subsidize early retirement of blue-collar workers, the replacement rate for white-collar workers drops from 6.09% to 5.59%. This leads to a decline in the PDV of expected pension income by 10.3%. As both lifetime earnings and lifetime pension income decrease, welfare of white-collar workers declines by a consumption equivalent of 0.90%. Aggregate lifetime utility, however, again increases when switching to the fair policy regime. In fact, it is higher than for an SRA of 65.

Rawlsian Welfare. Recall that, with Rawlsian welfare ($\tilde{\theta} = 1$), early retirement of blue-collar workers at age 60 is optimal for both an SRA of 65 and 70. Results from switching to the policy that solves the planner problem SPP1 for $\tilde{\theta} = 1$ are shown in lines 5-8 of Table 4. In case the SRA is 65 (70) years, the consumption equivalent welfare gain of blue-collar workers is 1.04% (2.14%) while white-collars lose 1.09% (1.45%). The gain for the blue-collar workers are primarily determined by a longer retirement duration and increased life expectancy, while white-collar workers suffer from cross-subsidizing early retirement of blue-collar workers. Moreover, white-collar workers face lower wages compared to the utilitarian optimal policy since the reduced labor force of blue-collar workers decreases the marginal product of white collar workers.

When comparing the optimal fair policies for different SRAs, we have seen that early retirement becomes socially more rewarding for a higher SRA. The reason is twofold. First, the disutility

of labor increases in the number of health deficits such that working becomes more painful at higher ages. Therefore, retiring early for a SRA of 70 entails larger welfare gains for blue-collar workers than for a SRA of 65. Second, the reduction in hours worked caused by early retirement is smaller for ages 65-69 than for ages 60-64, reflecting labor supply decreases for older ages because of deteriorating health. Because of their complementarity to blue-collar workers in production, earnings of white-collar workers thus deteriorate by less than for statutory retirement at age 65.

8.2. Fair and Socially Optimal Rate of Return on Contributions. Recall that we account in the calibrated model for the less steep age-earnings profiles of blue-collar workers compared to white-collar workers. Relative earnings in younger age are less unequal. Attributing a calculatory interest rate to past contributions in (5) can thus lead to less old-age inequality without requiring observability of occupations.

8.2.1. *Fair Calculatory Interest Rate.* The last column of Table 3 shows for different combinations of retirement ages the calculatory interest rate $\tilde{r}^{fair}(R^H, R^L)$ that applies to pension wealth accumulation until age 60 and equalizes the benefit-contribution ratio as implied by part (ii) of Proposition 1. We find that the socially optimal solution is the same for utilitarian and Rawlsian welfare. For an SRA of 65, it involves that both occupational groups work until 65 and \tilde{r}^{fair} equals 6% (line 1). For an SRA of 70, it is still socially optimal that both groups work until the SRA but the fair calculatory interest that compensates blue-collar workers for the implied lower life expectancy when retiring at 70 rather than 65 rises to 9.1% (line 5).³¹

The fair calculatory interest rate moves along with the fair RRR. If both occupational groups retire at the same age, \tilde{r}^{fair} is increasing in the age of retirement in order to compensate for the associated decrease in life expectancy of blue-collar workers. If blue-collar workers retired early, \tilde{r}^{fair} would decline to account for the longer retirement period. If white-collar workers retired early, it would be fair to compensate blue-collar workers by a higher calculatory interest due to the adverse effects on the pension budget.

8.2.2. *Policy Reform Implications.* Table 5 displays the implications of switching the pension system towards the optimal solution.

Again, blue-collar workers gain welfare and white-collar workers lose welfare compared to the status quo. For an SRA of 65 (70), the gain in terms of the consumption equivalent is 0.71% (0.87%) for blue-collar workers and the loss is 0.70% (1.03%) for white-collar workers. Aggregate lifetime utility (Welfare) increases.

The welfare results reflect that a fair policy reform drastically raises pension income of blue-collar workers along with a reduction in labor supply and an increase in earnings (due to equilibrium repercussions on the hourly wage rate) while the opposite holds for white-collar workers. As the

³¹Welfare would be higher if the social planner were able to force blue-collar workers into early retirement along with setting \tilde{r}^{fair} according to line 6 and 8 under utilitarian and Rawlsian welfare, respectively. But for these combinations of retirement ages and calculatory interest rates blue-collar workers would have an incentive to work longer, violating the incentive compatibility constraint (22).

TABLE 5. Effects of Optimal Fair Return on Contributions

Utilitarian/Rawlsian Welfare	$\Delta\ell$	ΔW	$\Delta W\ell$	ΔI	ΔU	ΔW_{elfare}
SRA of 65, $R^L = 65$, $R^H = 65$						
<i>optimal fair calculatory interest rate: $\tilde{r}^{fair} = 0.060$</i>						
1) blue-collar workers	-0.24	0.15	0.74	11.64	0.71	0.02
2) white-collar workers	0.56	-0.10	0.89	-6.18	-0.70	
SRA of 70, $R^L = 65$, $R^H = 65$						
<i>optimal fair calculatory interest rate: $\tilde{r}^{fair} = 0.091$</i>						
3) blue-collar workers	-0.54	0.20	0.97	17.2	0.87	0.01
4) white-collar workers	0.52	-0.12	1.23	-8.73	-1.03	

We report changes in percent when deviating from the status quo to the optimal system for lifetime labor supply conditional on survival until declared retirement (ℓ), average hourly wages over the lifetime (W), the PDV of expected earnings ($W\ell$), the PDV of expected pension benefits (I), occupation-specific welfare measured in consumption equivalents (U), and aggregate lifetime utility (Welfare). R^L and R^H refer to retirement ages of blue- and white-collar workers, respectively. \tilde{r}^{fair} refers to the fair calculatory interest rate on pension contributions.

equilibrium retirement age after the policy switch does not change (all individuals still work until the SRA), there are no welfare gains from longer benefit duration. Overall, welfare gains are lower under the “second-best” policy scenario than with distinct replacement rates.

9. SENSITIVITY ANALYSIS

9.1. Increasing Life Expectancy. To address demographic change, we investigate in a comparative dynamics analysis the effect of higher life expectancy. To this end, we consider a lower rate of aging for both blue- and white-collar workers and redo the policy experiments from the last section. We first consider an increase in life expectancy by three years which corresponds to the increase in male life expectancy observed over the last 20 years in Germany (World Bank, 2021). In a further analysis, we increase life expectancy by six years. We assume that the increase in life expectancy occurs equally for both occupational groups. We thus follow evidence for Germany that differences in life expectancy between subpopulations, including different education strata, have remained stable in recent decades (Kibele et al., 2013). In model terms, we reduce the rates of aging before retirement from $\bar{\mu}^L = 0.28$ to $\bar{\mu}^L = 0.27$ for blue-collar workers and from $\bar{\mu}^H = \underline{\mu}^H = 0.26$ to $\bar{\mu}^H = 0.24$ for white-collar workers to simulate a universal life expectancy increase (ΔLE) of three years. If it is six years, aging rates decrease to $\bar{\mu}^L = 0.25$ and $\bar{\mu}^H = 0.23$. We maintain assumption $\underline{\mu}^H = \underline{\mu}^L = \bar{\mu}^H$ for post-retirement aging rates.

Unlike in Section 6.2, the higher life expectancy now implies that under the status quo policy, a uniform increase in the SRA from 65 to 70 years does not anymore lead to an aggregate welfare gain. However, blue-collar workers still lose (i.e. Rawlsian welfare still declines).

TABLE 6. First-Best Fair Pension Policy and Welfare Effects with Demographic Change

	Utilitarian Welfare				Rawlsian Welfare			
	R^L, R^H	ξ^{fair}	ΔU^L	ΔU^H	R^L, R^H	ξ^{fair}	ΔU^L	ΔU^H
SRA of 65								
1) benchmark	65, 65	1.19	0.81	-0.57	60, 65	0.95	1.04	-1.09
2) $\Delta LE^L = \Delta LE^H = 3$	65, 65	1.17	0.73	-0.51	60, 65	0.97	0.82	-1.27
3) $\Delta LE^L = \Delta LE^H = 6$	65, 65	1.17	0.69	-0.47	60, 65	0.99	0.78	-1.45
4) $\Delta LE^H = 2.1$	60, 65	1.04	2.20	-1.22	60, 65	1.04	2.20	-1.22
5) $\Delta LE^L = -2.1$	60, 65	1.03	2.37	-1.12	60, 65	1.03	2.37	-1.12
SRA of 70								
6) benchmark	65, 70	0.96	1.93	-0.90	60, 70	0.76	2.14	-1.45
7) $\Delta LE^L = \Delta LE^H = 3$	65, 70	0.98	1.74	-1.06	60, 70	0.81	1.84	-1.83
8) $\Delta LE^L = \Delta LE^H = 6$	65, 70	1.01	1.76	-1.23	60, 70	0.85	1.90	-2.17
9) $\Delta LE^H = 2.1$	60, 70	0.86	4.21	-1.54	60, 70	0.86	4.21	-1.54
10) $\Delta LE^L = -2.1$	60, 70	0.82	2.28	-1.02	60, 70	0.82	2.28	-1.02

R^L and R^H refer to the retirement ages of blue- and white-collar workers under the optimally fair policy. ξ^{fair} refers to the optimal fair replacement rate ratio. ΔU^L and ΔU^H show the percentage change in welfare for blue- and white-collar workers, respectively, when deviating from the status quo to the optimal system. The effect on welfare is measured in consumption equivalents. ΔLE^L and ΔLE^H refer to an increases in life expectancy for blue- and white-collar workers, respectively.

Table 6 shows the results for the first-best policies (solution to SPP1). The first set of columns shows the optimally fair policies for the utilitarian welfare criterion and the second set for the Rawlsian one. Each set of columns reports under the optimal fair policy the retirement ages, fair replacement ratio, and the welfare implications for blue- and white-collar workers. The upper and the lower part of the table include results for a statutory retirement age of 65 and 70, respectively. The first line in each part reiterates the effects of a switch to the optimal replacement rates for the benchmark calibration. Lines 2 and 7 show results for a three years higher life expectancy and lines 3 and 8 for the six year increase. The retirement ages under the optimally fair policies remain unchanged despite the increase in life expectancy for both welfare criteria. The social planner still incentivizes white-collar workers to work until age 70 and blue-collar workers to retire at age 65. Since we keep the occupational life expectancy gap constant, the fair RRR, ξ^{fair} changes only moderately.

Table 7 shows the same experiment for the second-best policies (solution to SPP2). Compared to the benchmark, a lower calculatory interest rate on pension contributions is needed in order to

TABLE 7. Second-Best Fair Pension Policy and Welfare Effects with Demographic Change

	Utilitarian/Rawlsian Welfare			
	R^L, R^H	\tilde{r}^{fair}	ΔU^L	ΔU^H
SRA of 65				
1) benchmark	65, 65	0.060	0.71	-0.70
2) $\Delta LE^L = \Delta LE^H = 3$	65, 65	0.041	0.67	-0.70
3) $\Delta LE^L = \Delta LE^H = 6$	65, 65	0.030	0.67	-0.52
4) $\Delta LE^H = 2.1$	65, 65	0.058	1.11	-0.95
5) $\Delta LE^L = -2.1$	65, 65	0.084	1.15	-1.14
SRA 70				
6) benchmark	70, 70	0.091	0.87	-1.03
7) $\Delta LE^L = \Delta LE^H = 3$	70, 70	0.057	0.84	-0.82
8) $\Delta LE^L = \Delta LE^H = 6$	70, 70	0.040	0.84	-0.72
9) $\Delta LE^H = 2.1$	65, 70	0.052	3.04	-1.23
10) $\Delta LE^L = -2.1$	65, 70	0.066	1.13	-0.93

R^L and R^H refer to the retirement ages of blue- and white-collar workers under the optimally fair policy. interest refers to the optimal fair calculatory interest rate. ΔU^L and ΔU^H show the percentage change in welfare for blue- and white-collar workers, respectively, when deviating from the status quo to the optimal system. The effect on welfare is measured in consumption equivalents. ΔLE^L and ΔLE^H refer to an increases in life expectancy for blue- and white-collar workers, respectively.

establish a fair pension system while retirement ages remain unchanged. Also welfare effects from the fair policy reform are similar.

9.2. Higher Life Expectancy Gap. We next analyze how the results are affected when the occupational gap in life expectancy increases. To this end, we consider two experiments. In the first experiment, white-collar workers become more healthy, while in the second experiment, blue-collar workers become less healthy. In the benchmark scenario, the life expectancy gap at the age of 20 amounts to 3.4 years. For the first experiment, we reduce the rate of aging of white-collar workers from $\bar{\mu}^H = 0.26$ to $\bar{\mu}^H = 0.25$ and for the second experiment, we increase the pre-retirement rate of aging of the blue-collar worker from $\bar{\mu}^L = 0.28$ to $\bar{\mu}^L = 0.30$. Both of these changes result in an increase in the life expectancy gap between blue- and white-collar workers by around two years to 5.5 years.

The results are shown in lines 4 and 9 (when health of white-collar workers improves by $\Delta LE^H = 2.1$) and lines 5 and 10 (when health of the blue-collar workers worsens by ($\Delta LE^L = -2.1$)) of Tables 6 and 7. Focusing on the first-best policy, the higher life expectancy gap results in an optimal retirement age of 60 for blue-collar workers for both a SRA of 65 and 70. If white-collar workers become healthier and thus have higher earnings (wage rate and labor supply go up), blue-collars benefit from higher pension contributions of white-collar workers. These effects raise the incentives of blue-collar worker to retire early under the fair policy. If blue-collar workers become less healthy, incentivizing early retirement again is the optimal fair policy because working becomes more painful. In both cases, the higher life expectancy gap results in a higher fair RRR, ξ^{fair} , for a given combination of retirement ages. Inspecting Table 3, ξ^{fair} increases in the first experiment from 0.95 to 1.04 for an SRA of 65 and from 0.76 to 0.86 for an SRA of 70; in the second experiment, ξ^{fair} increases to 1.03 and 0.82, respectively. The optimal fair policies under the utilitarian approach coincide with those in the Rawlsian case.

One noteworthy result is that blue-collar workers gain significantly more welfare under an SRA of 70 from the fair policy in the first experiment, i.e. when white-collar workers become more healthy as compared to when health of blue-collars declines. The reason behind this result is that the implications of the status quo policy (14), which the fair policy is compared to, differs between the two experiments. When white-collar workers become more healthy, the equilibrium outcome in the status quo is that both groups retire at the age of 70. When blue-collar workers are less healthy, the equilibrium outcome in the status quo features early retirement of blue-collar workers at age 65 because of their poorer health. Therefore, the effect of the optimal fair policy featuring early retirement is greater in the first experiment where blue-collar workers work until the age of 70 in the status quo.

For an SRA of 70, the second-best policy now implies early retirement of blue-collar workers at age 65 while it was 70 in the benchmark scenario. For an SRA of 65, results are qualitatively the same as in the benchmark case while the welfare results are more pronounced in magnitude.

9.3. Alternative Interest and Time Preference Rate. We also checked sensitivity of the results for different interest rates and time preference rates. These results, which are shown in detail in Appendix B, are summarized as follows.

First, a lower interest rate implies that the forgone labor income from early retirement is discounted less heavily. Therefore, we find that the optimal policy does not feature early retirement anymore. By contrast, a lower time preference rate means that the increase in life expectancy associated with early retirement is discounted less heavily when evaluating lifetime utility. Thus, the optimal policy tends to be associated with stronger early retirement incentives. Reducing both the interest rate and time preference rate by the same amount and thus keeping the calibration target of a flat consumption profile basically counterbalances both effects and thus does not alter the qualitative results.

Finally, we check sensitivity of the fair RRR when the interest rate follows a declining time trend (Del Negro et al., 2019). We find that a time-varying interest rate affects the fair RRR only mildly.

10. CONCLUSION

Fairness concerns call for pension reforms that raise the rate of return to pension contributions from individuals facing higher mortality risk. We analyzed fair and welfare-maximizing pension policies in a multi-period OLG model with stochastic survival, endogenous retirement ages, and an occupation-specific health gradient that captures two sets of empirical evidence. First, blue-collar workers have higher mortality risk than white-collar workers, associated with faster health deficit accumulation. Second, the health deficit accumulation process of blue-collar workers is associated with a job-related health burden and thus slows down after retirement. Consequently, early retirement incentives potentially reduce life expectancy gaps between blue-collar and white-collar workers.

Calibrating the model to Germany showed that the (lifetime) benefit-contribution ratio of blue-collar workers is considerably lower than that of white-collar workers. Universally increasing the SRA harms blue-collar workers further by raising the life expectancy gap between occupational groups and the difference in lifetime contributions. It also reduces aggregate welfare.

We considered two sets of policy instruments to achieve fair pensions. First, we derived occupation-specific replacement rates that equalize the ratio of the PDV of expected pension benefits and lifetime pension contributions across occupations and analyzed both allocative and welfare effects of implementing the optimal fair policy. Fairness considerations call for a significant increase in the replacement rate ratio between blue-collar and white-collar workers for given retirement age choices. If the weight of blue-collar workers in the social welfare function or the SRA are sufficiently high, the optimal fair replacement rate policy implies early retirement of blue-collar workers by lowering their early retirement penalty compared to the current system. Switching to the optimal fair replacement rate policy implies that pension income and welfare of blue-collar workers considerably increase while labor supply declines. The opposite holds for white-collar workers. With optimal fair replacement rates, welfare is higher for an SRA of 70 (involving early retirement of blue-collar workers) than for an SRA of 65. Overall, our analysis suggests that raising the SRA in Germany can be welfare-enhancing only if accompanied with early retirement incentives for workers in arduous jobs.

Second, as an alternative “second-best” policy we derived the optimal fair calculatory interest rate to earlier contributions to account for possible informational or political constraints to design distinct replacement rates. This policy option is easily implementable and exploits the less steep age-earnings profiles of blue-collar workers. The optimal fair policy involves a sizable interest rate but, given the early retirement penalty in the current system, no early retirement. In all considered scenarios, irrespective of the specific policy instrument considered, switching from the status quo to the optimal fair policy raises aggregate welfare.

Finally, we considered how the optimal fair policy schedules respond to longevity changes. A larger difference of health deficit accumulation rates between the occupational groups, leading to more pronounced longevity gaps, calls for higher replacement rate ratios to the favor of blue-collar workers or a higher calculatory interest rate on contributions. The policy schedules include more favorable early retirement incentives for blue-collar workers. In contrast, a universal increase in life expectancy associated with a slowdown in health deficit accumulation for both groups has little effect on the fair pension system.

We focussed on social planning problems to inform policy makers and researchers how to reform Bismarckian pension systems such that benefit-contribution ratios are equalized. Such systems are intended to be non-distributive. It would be interesting to use our life-cycle model with heterogeneity of health status, aging, and mortality risk to co-design income taxation and pension policies in a unifying framework. While challenging, this is an important task for future research as welfare states are currently designed very differently to achieve redistributive goals and to provide longevity insurance.

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APPENDIX A. MATHEMATICAL APPENDIX

A.1. Wage rates. Equilibrium condition 1 implies that the user cost of capital, $\bar{r} + \delta$, is equal to its marginal product. According to (11), $\bar{r} + \delta = \alpha(X_t/K_t)^{1-\alpha}$, i.e., $K_t = \left(\frac{\alpha}{\bar{r} + \delta}\right)^{\frac{1}{1-\alpha}} X_t$. Consequently, the wage rates, that equal the marginal products of labor, read as

$$w_t^H = (1 - \alpha) \left(\frac{\alpha}{\bar{r} + \delta}\right)^{\frac{\alpha}{1-\alpha}} \chi \left(\chi + (1 - \chi) \left(\frac{L_t}{H_t}\right)^\rho\right)^{\frac{1-\rho}{\rho}}, \quad (28)$$

$$w_t^L = (1 - \alpha) \left(\frac{\alpha}{\bar{r} + \delta}\right)^{\frac{\alpha}{1-\alpha}} (1 - \chi) \left(\chi \left(\frac{H_t}{L_t}\right)^\rho + 1 - \chi\right)^{\frac{1-\rho}{\rho}}. \quad (29)$$

Thus, the wage rate of occupation group j is decreasing in the employment of group j relative to the other group. Recall that employment levels, H and L , are given by (12) and (13), respectively. Thus, their steady state values depend on retirement decisions, R^H and R^L .

A.2. Government Budget Constraints. Since the government budgets are balanced at any t , it must hold that the earned income tax credit is given by

$$\bar{I}_t = \frac{\tau^w \cdot (w_t^L L_t + w_t^H H_t)}{\theta \cdot \sum_{v=t-\bar{R}+1}^t \mathbf{1}_t(R_v^H) S_{v,t}^H + (1 - \theta) \cdot \sum_{v=t-\bar{R}+1}^t \mathbf{1}_t(R_v^L) S_{v,t}^L}. \quad (30)$$

To see this, recall that the tax on earnings (with marginal tax rate τ^w) is used solely for redistributive purposes, such that the earned income tax credit is a lump-sum payment. The tax base (total labor income) in period t is $w_t^L L_t + w_t^H H_t$ and the denominator of (30) is the total number of surviving individuals that are employed in t .

Moreover, for the pension budget of the PAYG system, the contemporaneous revenue must equal total benefits of all surviving retirees in the same period, i.e.,

$$\tau^p \cdot (w_t^L L_t + w_t^H H_t) = \theta \cdot \sum_{v=t-T+1}^{t-R} S_{v,t}^H \hat{I}_v^j(R_v^j, \tilde{r}) + (1 - \theta) \cdot \sum_{v=t-T+1}^{t-R} S_{v,t}^L \hat{I}_v^j(R_v^j, \tilde{r}). \quad (31)$$

Using (10) confirms (20) in steady state.

A.3. Individual Optimization. According to equilibrium condition 1 in Definition 1, the intertemporal optimization problem of an individual from cohort v and occupation-group j conditional on the retirement age R_v^j is given by

$$\max_{\{c_{v,t}, \ell_{v,t}, P_{v,t+1}, k_{v,t+1}\}_{t=v}^{v+T-1}} \sum_{t=v}^{v+T-1} S_{v,t}^j \beta^{t-v} \left[\frac{(c_{v,t})^{1-\sigma} - 1}{1 - \sigma} - D(d_{v,t}^j) \frac{(\ell_{v,t})^{1+1/\eta}}{1 + 1/\eta} + \bar{u} \right] \text{ s.t.} \quad (32)$$

$$P_{v,t+1}^j - P_{v,t}^j = \mathbf{1}_t(\underline{R}) \tilde{r} P_{v,t}^j + \mathbf{1}_t(R_v^j) \tau^p W_{v,t}^j \ell_{v,t}^j, \quad (33)$$

$$k_{v,t+1} - k_{v,t} = r_{v,t}^j k_{v,t} + (1 - \tau_{v,t} - \tau^p) W_{v,t}^j \ell_{v,t} + I_{v,t}^j (P_{v,t}^j, R_v^j) - c_{v,t}, \quad (34)$$

(6), $\ell_{v,t} \geq 0$ for all $t \in \{v, \dots, v+T-1\}$,³² and boundary conditions $k_{v,v} = 0$ and $k_{v,v+T} \geq 0$. Using $S_{v,v}^j = 1$, survival rates and mortality rates are related according to

$$S_{v,t}^j = \prod_{u=v}^{t-1} (1 - m_{v,u}^j). \quad (35)$$

The hourly wage rate net of taxes and contributions reads as

$$\tilde{W}_{v,t}^j = \begin{cases} (1 - \tau^w - \tau^p)W_{v,t}^j, & v \leq t \leq v + R_v^j - 1, \\ 0 & \text{otherwise,} \end{cases} \quad (36)$$

according to (6). Using (35) and (36) in (32), the Lagrangian \mathcal{L}_v^j for the optimization problem of an individual from cohort v and occupation-group j can be written as

$$\begin{aligned} \mathcal{L}_v^j &= \dots + \beta^{t-v} S_{v,t}^j \left[\frac{(c_{v,t})^{1-\sigma} - 1}{1-\sigma} - D(d_{v,t}^j) \frac{(\ell_{v,t})^{1+1/\eta}}{1+1/\eta} \right] + \\ &\quad \beta^{t+1-v} S_{v,t+1}^j \left[\frac{(c_{v,t+1})^{1-\sigma} - 1}{1-\sigma} - D(d_{v,t+1}^j) \frac{(\ell_{v,t+1})^{1+1/\eta}}{1+1/\eta} \right] + \dots + \\ &\quad \lambda_{v,t}^j \left[(1 + r_{v,t}^j)k_{v,t} + \tilde{W}_{v,t}^j \ell_{v,t} + I_{v,t}^j(P_{v,t}, R_v^j) - c_{v,t} - k_{v,t+1} \right] + \\ &\quad \lambda_{v,t+1}^j \left[(1 + r_{v,t+1}^j)k_{v,t+1} + \tilde{W}_{v,t+1}^j \ell_{v,t+1} + I_{v,t+1}^j(P_{v,t+1}, R_v^j) - c_{v,t+1} - k_{v,t+2} \right] + \dots + \\ &\quad \varepsilon_{v,t}^j \left[P_{v,t} + \mathbf{1}_t(\underline{R}) \cdot \tilde{r} P_{v,t}^j + \mathbf{1}_t(R_v^j) \tau^p W_{v,t}^j \ell_{v,t}^j - P_{v,t+1} \right] + \\ &\quad \varepsilon_{v,t+1}^j \left[P_{v,t+1} + \mathbf{1}_{t+1}(\underline{R}) \tilde{r} P_{v,t+1}^j + \mathbf{1}_{t+1}(R_v^j) \tau^p W_{v,t+1}^j \ell_{v,t+1} - P_{v,t+2} \right] + \dots, \end{aligned} \quad (37)$$

where $\{\lambda_{v,t}^j\}_{t=v}^{v+T-1}$ denotes the sequence of shadow prices for capital holdings and $\{\varepsilon_{v,t}^j\}$ is the sequence of shadow prices for the stock of pension contributions. At the end of life, either the pension stock or the shadow price of the pension stock must be equal to zero. Since the pension stock is always greater than zero, transversality condition $\varepsilon_{v,v+T-1}^j = 0$ must hold. Moreover, as there is no bequest motive, capital holdings after death must be zero, $k_{v,v+T}^j = 0$.

The first-order conditions $\partial \mathcal{L}_v^j / \partial c_{v,t} = \partial \mathcal{L}_v^j / \partial c_{v,t+1} = \partial \mathcal{L}_v^j / \partial k_{v,t+1} = \partial \mathcal{L}_v^j / \partial P_{v,t+1} = \partial \mathcal{L}_v^j / \partial \ell_{v,t} = 0$ can be written as

$$\beta^{t-v} (c_{v,t}^j)^{-\sigma} S_{v,t}^j = \lambda_{v,t}^j, \quad (38)$$

$$\beta^{t+1-v} (c_{v,t+1}^j)^{-\sigma} S_{v,t+1}^j = \lambda_{v,t+1}^j, \quad (39)$$

$$\lambda_{v,t}^j = \lambda_{v,t+1}^j (1 + r_{v,t+1}^j), \quad (40)$$

$$(1 + \mathbf{1}_{t+1}(\underline{R}) \cdot \tilde{r}) \varepsilon_{v,t+1}^j - \varepsilon_{v,t}^j + \frac{\partial I_{v,t+1}^j(P_{v,t+1}^j, R_v^j)}{\partial P_{v,t+1}^j} \lambda_{v,t+1}^j = 0, \quad (41)$$

$$\beta^{t-v} D(d_{v,t}^j) (\ell_{v,t}^j)^{1/\eta} S_{v,t}^j \leq \lambda_{v,t}^j \tilde{W}_{v,t}^j + \mathbf{1}_t(R_v^j) \varepsilon_{v,t}^j \tau^p W_{v,t}^j. \quad (42)$$

³²The non-negativity constraints for labor supply are explicitly stated because of the possibility of corner solutions.

Evidently, (42) holds with equality for $v \leq t \leq v + R_v^j - 1$,³³ whereas we get corner solutions $\ell_{v,t}^j = 0$ for $v + R_v^j \leq t \leq v + T - 1$ (after declared retirement), according to (36) and the fact that $\mathbf{1}_t(R_v^j) = 0$ for $v + R_v^j \leq t \leq v + T - 1$.

We can rewrite (41) to the first-order difference equation

$$\varepsilon_{v,t}^j = \mathbf{a}_t(R_v^j)\varepsilon_{v,t-1}^j + \mathbf{b}_{v,t}^j(R_v^j), \quad (43)$$

where³⁴

$$\mathbf{a}_t(R_v^j) \equiv \frac{1}{1 + \mathbf{1}_t(\underline{R})\tilde{r}} = \begin{cases} \frac{1}{1+\tilde{r}} & \text{for } t \in \{v, \dots, v + R_v^j - 1\}, \\ 1 & \text{otherwise,} \end{cases} \quad (44)$$

$$\mathbf{b}_{v,t}^j(R_v^j) \equiv -\frac{\lambda_{v,t}^j}{1 + \mathbf{1}_t(\underline{R})\tilde{r}} \frac{\partial I_{v,i}^j(P_{v,i}^j, R_v^j)}{\partial P_{v,t}^j} = \begin{cases} 0 & \text{for } t \in \{v, \dots, v + R_v^j - 1\}, \\ -\lambda_{v,t}^j \mathbf{b}_v^j(R_v^j) & \text{otherwise,} \end{cases} \quad (45)$$

The solution to (43) is³⁵

$$\begin{aligned} \varepsilon_{v,t}^j &= \varepsilon_{v,v}^j \prod_{\tau=1}^{t-v} \mathbf{a}_\tau(R_v^j) + \sum_{i=v+1}^{t-1} \prod_{\tau=i+1}^t \mathbf{a}_\tau(R_v^j) \mathbf{b}_{v,i}^j(R_v^j) + \mathbf{b}_{v,t}^j(R_v^j) \\ &= \begin{cases} \frac{\varepsilon_{v,v}^j}{(1+\tilde{r})^{t-v}} & \text{for } v \leq t \leq v + R_v^j - 1, \\ \frac{\varepsilon_{v,v}^j}{(1+\tilde{r})^{R_v^j}} - \mathbf{b}_v^j(R_v^j) \sum_{i=v+R_v^j}^t \lambda_{v,i}^j & \text{for } v + R_v^j \leq t \leq v + T - 1, \end{cases} \end{aligned} \quad (47)$$

where we used (44) and (45) for the latter equation. To obtain initial value $\varepsilon_{v,v}^j$, note that (47) implies for the final period $t = v + T - 1$ that

$$\varepsilon_{v,v+T-1}^j = \frac{\varepsilon_{v,v}^j}{(1+\tilde{r})^{R_v^j}} - \mathbf{b}_v^j(R_v^j) \sum_{i=v+R_v^j}^{v+T-1} \lambda_{v,i}^j. \quad (48)$$

Thus, using transversality condition $\varepsilon_{v,v+T-1}^j = 0$, we find

$$\varepsilon_{v,v}^j = (1 + \tilde{r})^{R_v^j} \mathbf{b}_v^j(R_v^j) \sum_{i=v+R_v^j}^{v+T-1} \lambda_{v,i}^j. \quad (49)$$

³³In this case, (42) equates the marginal disutility of labor with the marginal labor income (converted into utils through shadow price λ) plus marginal pension contributions (converted into utils through shadow price ε) from working an additional hour.

³⁴To see the latter equations in (44) and (45), recall that $\mathbf{1}_t(R_v^j) = 1$ for $t \in \{v, \dots, v + R_v^j - 1\}$ and $\mathbf{1}_t(R_v^j) = 0$ otherwise. Moreover, note from (8) that $\partial I_{v,t}^j / \partial P_{v,t}^j = 0$ for $t \in \{v, \dots, v + R_v^j - 1\}$ and $\partial I_{v,t}^j / \partial P_{v,t}^j = s_v^j(R_v^j)$ otherwise.

³⁵It is well-known that the solution to first-order difference equation $x_t = \mathbf{a}_t x_{t-1} + \mathbf{b}_t$ with initial value $x_0 \neq 0$ is

$$x_t = x_0 \prod_{\tau=1}^t \mathbf{a}_\tau + \sum_{i=1}^{t-1} \prod_{\tau=i+1}^t \mathbf{a}_\tau \mathbf{b}_i + \mathbf{b}_t.$$

We adapt the solution to the case where the initial value is given in time period v .

Substituting (49) into (47), we finally obtain

$$\varepsilon_{v,t}^j = \begin{cases} (1 + \bar{r})^{v+R_v^j-t} b_v^j(R_v^j) \sum_{i=v+R_v^j}^{v+T-1} \lambda_{v,i}^j & \text{for } v \leq t \leq v + R_v^j - 1, \\ b_v^j(R_v^j) \sum_{i=t+1}^{v+T-1} \lambda_{v,i}^j & \text{for } v + R_v^j \leq t \leq v + T - 1, \end{cases} \quad (50)$$

where $\lambda_{v,i}^j = \beta^{i-v} (c_{v,i}^j)^{-\sigma} S_{v,i}^j$, according to (38).

Combining (38)–(40) and using (35) leads to

$$\left(\frac{c_{v,t+1}^j}{c_{v,t}^j} \right)^\sigma = \beta(1 - m_{v,t}^j)(1 + r_{v,t+1}^j). \quad (51)$$

Using $1 + r_{v,t+1}^j = \frac{1+\bar{r}}{1-m_{v,t}^j}$ from (4) in (51) implies

$$c_{v,t+1}^j = [\beta(1 + \bar{r})]^{\frac{1}{\sigma}} c_{v,t}^j. \quad (52)$$

By iterating we obtain for $t \geq v$ that

$$c_{v,t}^j = [\beta(1 + \bar{r})]^{\frac{t-v}{\sigma}} c_{v,v}^j. \quad (53)$$

According to (34), $k_{v,v} = 0$ and $k_{v,v+T} = 0$, we find that the intertemporal budget constraint of a member of cohort v is given by

$$c_{v,v}^j + \sum_{t=v+1}^{v+T-1} \left(\frac{c_{v,t}^j}{\prod_{\tau=v+1}^t (1 + r_{v,\tau}^j)} \right) = \tilde{W}_{v,v}^j \ell_{v,v}^j + I_{v,v}^j + \sum_{t=v+1}^{v+T-1} \left(\frac{\tilde{W}_{v,t}^j \ell_{v,t}^j + I_{v,t}^j}{\prod_{\tau=v+1}^t (1 + r_{v,\tau}^j)} \right), \quad (54)$$

where $I_{v,t}^j = I_{v,t}^j(P_{v,t}^j, R_v^j)$. Substituting $1 + r_{v,\tau}^j = \frac{1+\bar{r}}{1-m_{v,\tau-1}^j}$ from (4) and (53) into the left-hand side of (54), we find

$$\begin{aligned} c_{v,v}^j + \sum_{t=v+1}^{v+T-1} \left(\frac{c_{v,t}^j}{\prod_{\tau=v+1}^t (1 + r_{v,\tau}^j)} \right) &= c_{v,v}^j + \sum_{t=v+1}^{v+T-1} \left(\frac{[\beta(1 + \bar{r})]^{\frac{t-v}{\sigma}} c_{v,v}^j \prod_{\tau=v+1}^t (1 - m_{v,\tau-1}^j)}{(1 + \bar{r})^{t-v}} \right) \\ &= c_{v,v}^j \left(1 + \sum_{t=v+1}^{v+T-1} \left(\frac{\beta}{(1 + \bar{r})^{\sigma-1}} \right)^{\frac{t-v}{\sigma}} S_{v,t}^j \right), \end{aligned} \quad (55)$$

where we used (35) for the latter equation. Using (4) and (35), we also obtain

$$\sum_{t=v+1}^{v+T-1} \left(\frac{\tilde{W}_{v,t}^j \ell_{v,t}^j + I_{v,t}^j}{\prod_{\tau=v+1}^t (1 + r_{v,\tau}^j)} \right) = \sum_{t=v+1}^{v+T-1} S_{v,t}^j \frac{\tilde{W}_{v,t}^j \ell_{v,t}^j + I_{v,t}^j}{(1 + \bar{r})^{t-v}}. \quad (56)$$

Recall that $I_{v,t}^j = \bar{I}_t$ for $v \leq t \leq v + R_v^j - 1$, according to (8), and $I_{v,t}^j = b_v^j(R_v^j) P_{v,t}^j$ otherwise, according to (10). Also recall that $\ell_{v,t}^j = 0$ for all $t \geq v + R_v^j$. Substituting (55) and (56) in (54)

and using (36) thus implies that the optimal initial consumption level, $c_{v,v}^j$, is given by

$$c_{v,v}^j = \frac{(1 - \tau^w - \tau^p)W_{v,v}^j \ell_{v,v}^j + \bar{I}_v + \sum_{t=v+1}^{v+R_v^j-1} S_{v,t}^j \frac{(1-\tau^w-\tau^p)W_{v,t}^j \ell_{v,t}^j + \bar{I}_t}{(1+\bar{r})^{t-v}} + b_v^j(R_v^j) \sum_{t=v+R_v^j}^{v+T-1} S_{v,t}^j \frac{P_{v,t}^j}{(1+\bar{r})^{t-v}}}{1 + \sum_{t=v+1}^{v+T-1} \left(\frac{\beta}{(1+\bar{r})^{\sigma-1}} \right)^{\frac{t-v}{\sigma}} S_{v,t}^j}. \quad (57)$$

$c_{v,v}^j$ as given in (57) gives us the consumption path by using (53) for a given labor supply path, a given path of income from public sources, and a given path of wage rates.

We finish the equilibrium analysis with the labor supply path. Using (36) in (42), we find that, for all $t \in \{v, \dots, v + R_v^j - 1\}$,

$$\ell_{v,t}^j = \left(\frac{\lambda_{v,t}^j (1 - \tau^w - \tau^p) + \varepsilon_{v,t}^j \tau^p}{\beta^{t-v} D(d_{v,t}^j) S_{v,t}^j} W_{v,t}^j \right)^\eta, \quad (58)$$

with $\lambda_{v,t}^j = \beta^{t-v} (c_{v,t}^j)^{-\sigma} S_{v,t}^j$ and $\varepsilon_{v,t}^j = b_v^j(R_v^j) \sum_{\tau=v+R_v^j}^{v+T-1} \beta^{\tau-v} (c_{v,\tau}^j)^{-\sigma} S_{v,\tau}^j$ as given by (38) and (50).

A.4. Welfare Comparisons. To enable welfare comparisons, we compute group-specific consumption equivalents with respect to alternative policy scenarios, i.e. compute the factor by which consumption in the benchmark scenario is multiplied each period such that an individual experiences the same utility under the status quo policy in Definition 2 and the considered policy alternative (*equivalent variation*).

Formally let superscripts “0” and “1” endogenous variables indicate the values in the benchmark and alternative policy scenario, respectively, and define

$$\tilde{U}_v^{j,k}(\phi) := \sum_{t=v}^{v+T-1} S_{v,t}^{j,k} \beta^{t-v} \left[\frac{(\phi \cdot c_{v,t}^{j,q})^{1-\sigma} - 1}{1-\sigma} - D(d_{v,t}^{j,i}) \frac{(\ell_{v,t}^{j,q})^{1+1/\eta}}{1+1/\eta} + \bar{u} \right] \quad (59)$$

as the intertemporal utility of any individual from cohort v and occupational group j under pension policy $q \in \{0, 1\}$ when the optimally chosen consumption levels are multiplied by factor ϕ . The equivalent variation measure ϕ_v^j is then implicitly defined as

$$\tilde{U}_v^{j,0}(\phi) = \tilde{U}_v^{j,1}(1). \quad (60)$$

APPENDIX B. SUPPLEMENTARY MATERIAL

This supplement provides details on the behavioral effects of a health shock at the individual level under the status quo policy (Section 9.1) that we discussed at the end of Section 6.1, a graphical representation of the evolution of the occupational life expectancy gap in Germany (Section 9.2), and on the sensitivity analysis with respect to the interest rate \bar{r} and time discount factor β (Section 9.3).

B.1. Health Shock. In order to provide further insights on the mechanics of the model mechanisms, we analyze the impact of health shocks on the behavior of the individual under the status quo pension system. To this end, we consider a blue-collar worker who, at age 50, suffers an unexpected health shock that shortens remaining life expectancy by five years. In model terms, the health deficit level increases by 34 percent of the initial deficit level (d_{min}) at the age of 50. In order to isolate the effect of the health shock, we assume that the individual hit by the shock has mass zero such that the equilibrium paths of wage rates are taken as given. Figure 5 shows the model responses for health deficits, survival rates, labor supply, and consumption for the average blue-collar worker (blue solid lines) and a blue-collar worker that is hit by a health shock (green dashed lines). The figures of labor supply and consumption are normalized by the respective values of a 50-year old average blue-collar worker.

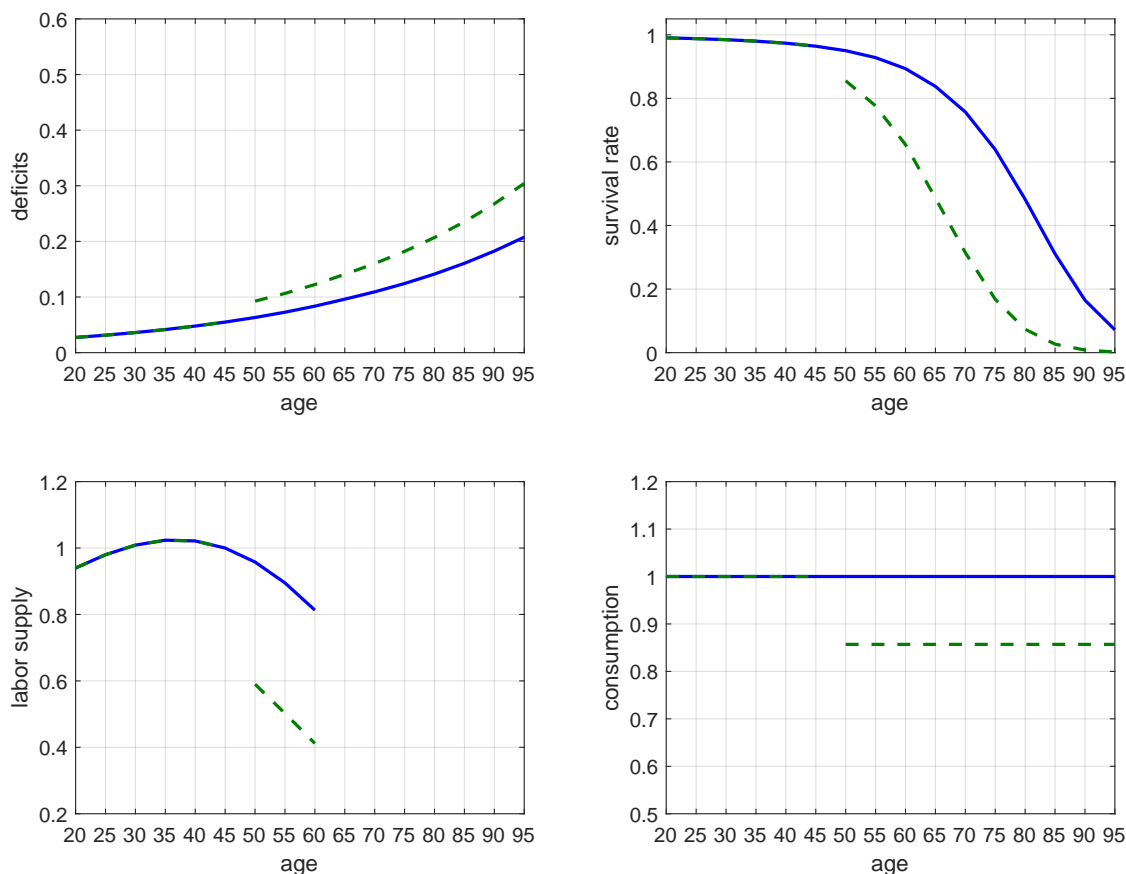
Until age 50, the model responses for both types coincide. After the age of 50, the health shock leads to a sudden increase in health deficits and sudden drop in the survival rate. As a result, the individual re-optimizes and reduces labor supply because working becomes more painful and productivity of the individual declines. Thus, the individual has to reduce consumption on impact as well in response to the health shock.

We note that without a health shock (an unforeseen event) the individual would always choose the same time paths when re-optimizing at different ages. Therefore, the model exhibits time consistent behavior.

B.2. Life Expectancy Gap. Figure 5 shows the evolution of the life expectancy gap between white-collar and blue-collar workers in Germany between 1996-2004, based on data from the Research Data Centre of the German Pension Insurance (FDZ-RV). (The data collection of occupation-specific mortality stopped in 2004.) We see that the gap first increased and remained stable thereafter.

B.3. Alternative Interest and Time Preference Rate. We consider three different sets of experiments. In the first set of experiments, we separately reduce both \bar{r} and $\bar{\beta} \equiv \frac{1-\beta}{\beta}$ by one percentage point from their benchmark values of $r = \bar{\beta} = 0.03$. This allows us to isolate the impact of each parameter on the model outcomes. Note that, according to Euler equation (52), $\bar{r} > (=, <) \bar{\beta}$, which is equivalent to $\beta(1+\bar{r}) > (=, <) 1$, implies that the individual age-consumption profile is increasing (flat, decreasing) over time. In the second experiment, we reduce the parameters \bar{r} and $\bar{\beta}$ simultaneously. This way, we check sensitivity to a lower interest rate while keeping the

FIGURE 4. Health Shock



Blue (solid lines): Average blue-collar worker. Green (dashed) lines: Blue-collar worker hit by a health shock at age 50.

calibration target of an empirically observed consumption path. In the third experiment, we check sensitivity of the fair replacement ratio when the interest rate follows a time trend.

Tables 8 and 9 show the results for alternative discounting for the first-best and second-best fair policies, respectively. In Table 8, the first set of columns shows the optimal fair policies for the utilitarian welfare criterion and the second set for the Rawlsian criterion while Table 9 shows the second-best policies. Each set of columns reports under the optimal fair policy the retirement ages, the fair replacement ratio (Table 8) or the fair calculatory interest rate (Table 9), and the welfare implications for blue- and white-collar workers. The upper and the lower part of the tables include results for a statutory retirement age of 65 and 70, respectively. The first line in each part reiterates the results from our benchmark experiment.

When changing $\bar{r} = 0.03$ to $\bar{r} = 0.02$ (lines 2 and 6) while leaving $\bar{\beta} = 0.03$, we observe that the fair replacement rate (first-best) policies now come with a reduction in the incentive for early retirement. This can be seen when looking at the Rawlsian case for an SRA of 65 and both the

utilitarian and Rawlsian case for an SRA of 70. In the case where $\bar{r} = \bar{\beta} = 0.03$, these featured early retirement for blue-collar workers, while for $\bar{r} = 0.02$ the optimal fair policy incentivizes blue-collar workers to work until the respective SRA. The reason behind this result is that the reduction in earnings through early retirement gets a higher weight when calculating the PDV of lifetime income because the foregone earnings are discounted less heavily. Blue-collar workers are then compensated with a higher RRR. Since second-best policies do not feature early retirement in the benchmark case, there is no change in retirement ages when adjusting the optimal fair calculatory interest rate, \tilde{r}^{fair} , to the lower interest rate. As a result, \tilde{r}^{fair} changes only slightly.

The picture changes when reducing $\bar{\beta}$ from 0.03 to 0.02 while keeping $\bar{r} = 0.03$ (lines 3 and 7). In this case, early retirement is more beneficial for blue-collar workers since the associated increase in life expectancy is discounted less heavily when evaluating lifetime utility. Consequently, regarding the first-best policies, early retirement for blue-collar workers at age 60 becomes optimal fair for both welfare criteria (and not only for the Rawlsian case). Thus, the fair RRR, ξ^{fair} , for the utilitarian welfare function is lower than in the case where $\bar{\beta} = 0.03$ to account for earlier retirement. As far as the second-best policies are concerned, the optimal fair calculatory interest rate, \tilde{r}^{fair} , is now lower and associated with early retirement at the age of 65 (instead of 70) for an SRA of 70.

When reducing both parameters simultaneously to $\bar{r} = \bar{\beta} = 0.02$ and thus keeping the calibration target of a flat consumption profile, retirement ages under the optimal fair policies are not affected. Moreover, ξ^{fair} (first-best policy) and \tilde{r}^{fair} (second-best policy) change only mildly.

Noteworthy, in all experiments considered, not only do blue-collar workers gain welfare from switching to the optimal fair policy regime but also aggregate lifetime utility (Welfare) increases.

In the last experiment, we check sensitivity of the fair replacement rate ratios when the interest rate follows a time trend. For this purpose, we consider an interest rate that linearly drops from 0.07 to 0.02 over the life cycle of an individual. Table 10 shows the results for the fair replacement rate ratios for any given combination of retirement ages. Comparing the results to the benchmark calibration in Table 3, we observe that the fair RRR is almost not affected by a time-varying interest rate.

TABLE 8. First-Best Fair Pension Policy and Welfare Effects with Alternative Discounting

	Utilitarian Welfare				Rawlsian Welfare			
	R^L, R^H	ξ^{fair}	ΔU^L	ΔU^H	R^L, R^H	ξ^{fair}	ΔU^L	ΔU^H
SRA of 65								
1) benchmark	65, 65	1.19	0.81	-0.57	60, 65	0.95	1.04	-1.09
2) $r = 0.02, \bar{\beta} = 0.03$	65, 65	1.21	1.22	-0.85	65, 65	1.21	1.22	-0.85
3) $r = 0.03, \bar{\beta} = 0.02$	60, 65	0.95	1.71	-0.92	60, 65	0.95	1.71	-0.92
4) $r = 0.02, \bar{\beta} = 0.02$	65, 65	1.21	1.22	-0.85	65, 65	1.21	1.22	-0.85
SRA 70								
5) benchmark	65, 70	0.96	1.93	-0.90	60, 70	0.76	2.14	-1.45
6) $r = 0.02, \bar{\beta} = 0.03$	70, 70	1.30	1.68	-1.16	70, 70	1.30	1.68	-1.16
7) $r = 0.03, \bar{\beta} = 0.02$	60, 70	0.76	3.42	-1.15	60, 70	0.76	3.42	-1.15
8) $r = 0.02, \bar{\beta} = 0.02$	65, 70	1.17	1.93	-1.09	65, 70	1.17	1.93	-1.09

R^L and R^H refer to the retirement ages of blue- and white-collar workers under the optimally fair policy. ξ^{fair} refers to the optimal fair replacement rate ratio. ΔU^L and ΔU^H show the percentage change in welfare for blue- and white-collar workers, respectively, when deviating from the status quo to the optimal system. The effect on welfare is measured in consumption equivalents.

TABLE 9. Second-Best Fair Pension Policy and Welfare Effects with Alternative Discounting

Utilitarian/Rawlsian Welfare				
	R^L, R^H	\tilde{r}^{fair}	ΔU^L	ΔU^H
SRA of 65				
1) benchmark	65, 65	0.060	0.71	-0.70
2) $r = 0.02, \bar{\beta} = 0.03$	65, 65	0.065	1.13	-1.01
3) $r = 0.03, \bar{\beta} = 0.02$	65, 65	0.059	0.71	-0.68
4) $r = 0.02, \bar{\beta} = 0.02$	65, 65	0.064	1.13	-0.99
SRA 70				
5) benchmark	70, 70	0.091	0.87	-1.03
6) $r = 0.02, \bar{\beta} = 0.03$	70, 70	0.098	1.36	-1.56
7) $r = 0.03, \bar{\beta} = 0.02$	65, 70	0.044	2.43	-0.83
8) $r = 0.02, \bar{\beta} = 0.02$	70, 70	0.096	1.38	-1.52

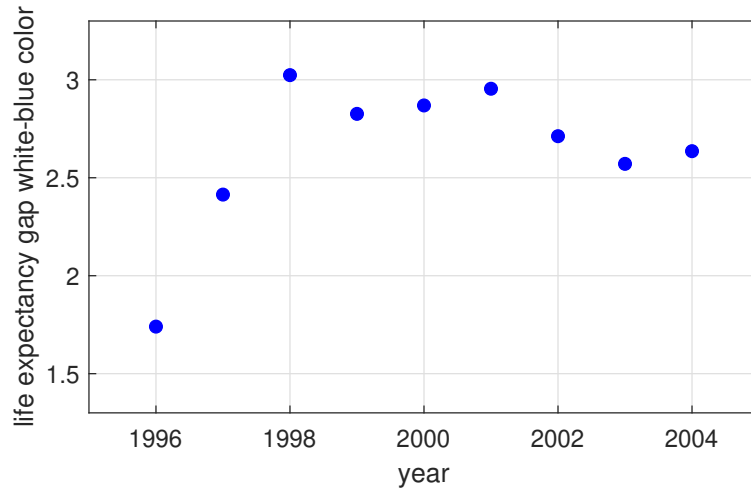
R^L and R^H refer to the retirement ages of blue- and white-collar workers under the optimally fair policy. interest refers to the optimal fair calculatory interest rate. ΔU^L and ΔU^H show the percentage change in welfare for blue- and white-collar workers, respectively, when deviating from the status quo to the optimal system. The effect on welfare is measured in consumption equivalents.

TABLE 10. Fair Pension Policies with Time-Varying Interest Rate

case	R^L	R^H	ξ^{fair}
SRA 65			
1)	65	65	1.18
2)	60	65	0.97
3)	65	60	1.37
4)	60	60	1.12
SRA 70			
5)	70	70	1.28
6)	65	70	0.97
7)	70	65	1.56
8)	60	70	0.80
9)	70	60	1.80
10)	65	65	1.18
11)	60	65	0.97
12)	65	60	1.37
13)	60	60	1.12

R^L : retirement age blue-collar workers. R^H : retirement age white-collar workers. ξ^{fair} : fair replacement rate ratio between blue- and white-collar workers.

FIGURE 5. Difference in Life Expectancy at 65: White vs. Blue Collar Workers in Germany



Data Source: Research Data Centre of the German Pension Insurance (FDZ-RV).