The Determinants of Income in a Malthusian Equilibrium*

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Abstract. This study constructs a simple, two-sector Malthusian model with agriculture and industry, and uses it to identify the determinants of income in a Malthusian equilibrium. We make standard assumptions about preferences and technologies, but in contrast to existing studies we assume that children and other consumption goods are gross substitutes. Consistent with the conventional Malthusian model, the present theory shows that productivity growth in agriculture has no effect on equilibrium income. More importantly, we also show that equilibrium income varies, not just with the death rate as has recently been demonstrated in the literature, but also with the level of productivity in the industrial sector. An empirical analysis using data for pre-industrial England lends support to both hypotheses.

Keywords: Malthusian Model, Subsistence Income.

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1. Introduction

Subsistence economies are often characterized by Malthusian population dynamics. In a Malthusian economy, higher income causes more births and fewer deaths. This temporarily raises the level of population. But because of diminishing returns to labor in production, more people gradually 'eat up' any improvement of income, forcing it back to the level of subsistence.

The terminology *subsistence economy* can, however, lead to the confused notion that in a Malthusian economy people live on the verge of starvation. Even in the mid-seventeenth century—a time when England's population was constant, and income, therefore, at subsistence by construction—the wage of the poorest workers (unskilled agricultural laborers) was well above the biological minimum of about 1,500 calories a day. This leads Clark (2007) to conclude that: 'preindustrial societies, while they were subsistence economies, were not typically starvation economies' (*ibid.*, p. 23).[1]

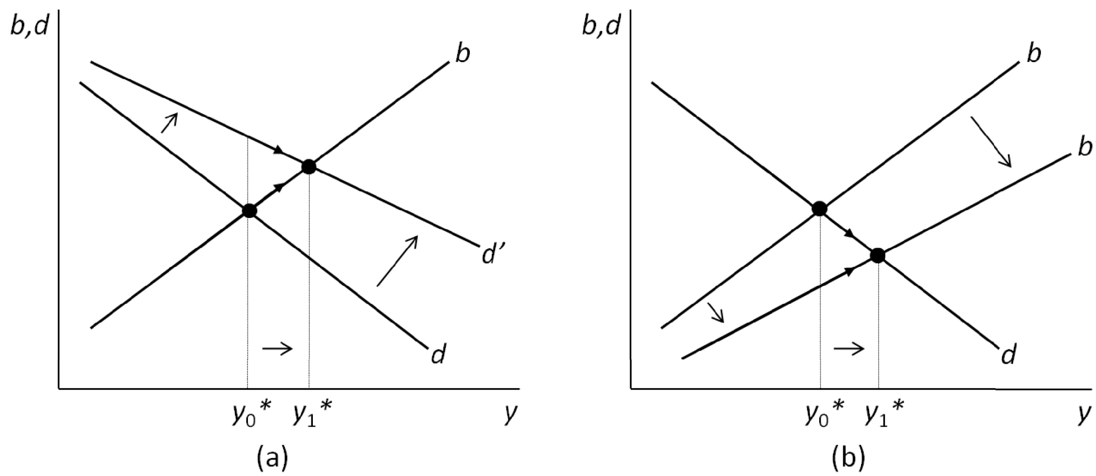
In this note, we construct a simple, two-sector Malthusian economy with agriculture and industry, and use it to identify the determinants of income in a Malthusian equilibrium (or simply *equilibrium income*). We arrive at the conventional conclusion that productivity growth in agriculture has no effect on equilibrium income. However, we show that equilibrium income varies, not only with the death rate as recently emphasized by Voigtländer and Voth (2010), but also with productivity in industry.

Recent attempts to predict the determinants of income in a Malthusian equilibrium are captured by Figure 1. Following Malthus (1798), changes in income have a dual effect on population growth. On the one hand, lower income reduces the marriage rate, leading therefore to fewer births. This is the 'preventive checks' hypothesis, which explains the upward-sloping birth schedule in Figure 1. On the other hand, lower income raises the death rate, as captured by the 'positive checks' hypothesis, which is reflected in the downward-sloping death schedule in Figure 1. Both types of checks have been observed in pre-industrial England (Nicolini 2007).

As is evident from the illustration, the intersection of the birth and the death schedules determines equilibrium income (y^*), defined as the level of income at which the population level remains constant over time. Hence, shifts in the position of the birth and death schedules are

[1] Recent work by Ashraf and Galor (2010) provides empirical support of the idea that pre-industrial economies displayed Malthusian population dynamics. See also Dalgaard and Strulik (2010) for an investigation of subsistence consumption understood as metabolic needs, and Strulik (2010) for an investigation of the implied long-run adjustment dynamics in conventional growth theory, when subsistence needs are introduced into the utility function as the level of consumption ($\bar{c}$) below which utility is minus infinity.

Figure 1: The Effect of Subsistence Income of Shifts in Deaths (a) and Births (b)



responsible for variations in equilibrium income. While Clark (2007) highlights the benign effect of higher death rates on living standards in a diagram similar to Figure 1, Voigtländer and Voth (2010) use this to draw a link between European wars and diseases and the sharp rise in European urbanization, as well as its permanently higher per capita incomes.

In order to understand the mechanics of the Voigtländer-Voth hypothesis, suppose we start off at y_0^* in Figure 1(a). At y_0^*, births equal deaths, so that the population level remains constant, and income per capita remains in equilibrium. An upward shift in the death schedule (higher death rate at any given income level) means that deaths momentarily exceed births. The population thus starts to shrink, and with diminishing returns to labor in production, this gradually raises the level of income. In turn, births rise and deaths fall, until the two meet again—this time at a higher equilibrium income (y_1^*).

Complementary to the hypothesis forwarded in Voigtländer and Voth (2010), the point we make in this paper is that changes in the *birth* schedule have similar effects on income per capita as those of changes in the death schedule. Our main argument is that a shift in the costs of foods, and therefore children, relative to the costs of other goods, affects the position (or more specifically the slope) of the birth schedule. This, too, impacts on the intersection point between the birth and death schedule, and thus on equilibrium income, as reflected in Figure 1(b).

More specifically, we demonstrate theoretically how advances in industrial productivity increase the relative price of food, and hence the costs of raising children. If children are ordinary goods, and if children and other consumption goods are gross substitutes, then parents respond to a price increase by reducing births. In turn, this flattens out the birth schedule for any given level of income, leading ultimately to a higher equilibrium income.

Certain features of our theory are shared with those of Hansen and Prescott (2002). We do differ, however, on two important points. First, the Hansen-Prescott framework is a one-good model, while ours has two goods (three, in fact, since children are considered as consumption goods). The two-goods construction of the present model allows the relative price between the two goods (food and manufactured goods) to play a role in development—a role which is absent in the Hansen-Prescott framework. Second, the Hansen-Prescott model is concerned with the take-off from stagnation to growth. In this regard, it does not address the factors that determine equilibrium income in a subsistence economy, which is the main purpose of the present work.

In the following, we first describe our model, after which we point to the determinants of income in a Malthusian equilibrium. Then we examine the hypothesis relative to historical evidence for England, using regression analysis to show that death and industrial productivity both correlate positively with pre-industrial real wages.

2. The Model

Let b_t denote the number of births per adult, and let d denote a risk of dying before adulthood.[2] The number of surviving children per adult can thus be written as $n_t = b_t(1-d)$. Parents derive utility from the number of surviving children and from consumption of manufactured goods m_t. Each child born costs one unit of food, and the price of food is denoted p_t. Parents divide their income between children and manufactured goods, so that the budget constraint of a parent reads

$$w_t = p_t b_t + m_t \tag{1}$$

where w_t is parental income, measured in units of the manufactured goods. We have normalized the price of the manufactured goods to one, so that p_t now denotes the relative price of food, also known as agricultural terms of trade.

2.1. Preferences.

The results obtained below rely on the crucial assumption that parents consider children and other consumption goods to be gross substitutes. Most related studies assume a Cobb-Douglas type utility function. However, such preferences carry the implicit assumption

[2] As in Voigtländer and Voth (2010), the death risk could be inversely related to income per capita. However, for the point we wish to make, this is not a necessary assumption.

that the cross-price elasticity between children and consumption goods is zero. While a CES (constant elasticity of substitution) utility function would permit any sort of substitutability between children and other consumption goods, such preferences would seriously complicate matters, and prevent us from reaching the tractable closed-form solutions we obtain below.

A simple way in which to allow children and other consumption goods to be gross substitutes, and yet arrive at closed-form results, is by assuming that parents maximize a quasi-linear utility function. This could be given by

$$u_t = m_t + \gamma \ln n_t, \ \gamma > 0, \tag{2}$$

where γ denotes the relative weight of children in utility. By maximizing (2) subject to the budget constraint given by (1), the first-order condition tells us that the number of births and surviving children per adult are given by

$$b_t = \gamma / p_t \qquad \Rightarrow \qquad n_t = (1-d)\gamma / p_t. \tag{3}$$

Note that gross substitutability between children and manufactured goods is represented by the negative effect of prices on the demand for children.[3]

2.2. **Production.** Consistent with the existing literature, suppose that the agricultural sector's output is subject to constant returns to land and labor, and that land is fixed and its amount set to unity. Furthermore, industrial output is subject to constant returns to labor, so that total output of the two sectors is given by

$$Y_{A,t} = \Omega_A L_{A,t}^{\alpha}, \quad \alpha \in (0,1), \tag{4}$$

$$Y_{M,t} = \Omega_M L_{M,t}. \tag{5}$$

where Ω_i is total factor productivity in sector $i \in \{A, M\}$ (subscript A refers to agricultural and M to manufacturing), and where L_i is the number of workers employed in sector $i \in \{A, M\}$. The fraction of labor allocated to agriculture and industry, respectively, is determined endogenously below.

[3]Although it is possible to relax this assumption about quasi-linear preferences, allowing for a more general utility function, such as the CES, will not affect the qualitative nature of the results presented shortly, but will severely complicate matters (notably at a cost to the closed-form solutions obtained in the following).

2.3. **Labor Market Equilibrium.** Suppose that land rents are zero, that there is free labor mobility, and that each sector is characterized by perfect competition. This means that the workers of each sector are paid according to their average product, i.e.

$$w_t = p_t \frac{Y_t^A}{L_t^A} = \frac{Y_t^M}{L_t^M}. \tag{6}$$

Full employment implies that

$$L_t = L_t^A + L_t^M. \tag{7}$$

2.4. **Food Market Equilibrium.** Suppose that, over the course of a lifetime, each individual consumes a fixed quantity of foods (or calories) measured by $\eta \equiv 1$.[4] For tractability reasons, food is demanded only during childhood and some of it stored for adulthood.[5] The fact that each individual demands a fixed amount of calories implies that, as income increases, people allocate a growing share of their income to manufactured goods (and vice versa). This is a main implication of Engel's Law.

By equating total food demand to total food supply, given by (4), the food market equilibrium condition implies that

$$b_t L_t = \Omega_A L_{A,t}^{\alpha}. \tag{8}$$

2.5. **Population Dynamics.** Finally, it follows from the demographic components described above that change in the size of the labor force between two consecutive periods is given by

$$L_{t+1} = n_t L_t = b_t \left(1 - d_t\right) L_t. \tag{9}$$

Equation (9) completes the model.

3. Analysis

In the following, we derive the closed-form solutions for a number of variables relevant for analyzing a Malthusian equilibrium. These include fertility, agricultural terms of trade, the share of labor employed in agriculture, and equilibrium income. First, we compute the variables in the static equilibrium, then we turn to the Malthusian (i.e. constant population) equilibrium.

[4] It will not affect the qualitative nature of the results, if we allow children to consume more food goods as their parents receive more income. For such a construction, see Strulik and Weisdorf (2008).

[5] It will not affect the qualitative nature of the results, if, instead, individual food demand is divided over two periods. Such a construction, however, severely complicates matters.

We begin by rewriting (8) to obtain the share of labor employed in agriculture, which we denote $l_A \equiv L_A/L$. This is given by

$$l_A = \left(\frac{\gamma L_t^{1-\alpha}}{\Omega_A p_t}\right)^{1/\alpha}. \tag{10}$$

It shows the fraction of workers in agriculture increases with the size of the labor force, but decreases with agricultural productivity, as well as agricultural terms of trade.

Next, inserting (3), (4), (5) and (10) into (6), the market-clearing agricultural terms of trade is given by

$$p_t = \frac{\Omega_M^{\alpha}\left(\gamma L_t\right)^{1-\alpha}}{\Omega_A}, \tag{11}$$

This increases with industrial productivity and labor, but decreases with agricultural productivity.

Further, inserting (11) into (3), we obtain the fertility rate which is

$$b_t = \left(\frac{\gamma}{\Omega_M}\right)^{\alpha} \frac{\Omega_A}{L_t^{1-\alpha}} = \frac{\gamma}{\Omega_M}\frac{w_t}{p_t} \equiv \frac{\gamma}{\Omega_M} y_t. \tag{12}$$

It follows that births—opposite to agricultural terms of trade—decrease with industrial productivity and labor, but increase with agricultural productivity.

Note for the purpose of understanding how this model relates to Figure 1 how births, by the use of (6) and (8), can be expressed as a function of income per capita, measured in units of food. It follows from (12) that the slope of the birth schedule is inversely related to the level of productivity in industry.

Finally, income per capita, measured in units of food, is obtained by dividing w_t by (11), and is given by

$$\frac{w_t}{p_t} = \Omega_A \left(\frac{\Omega_M}{\gamma L_t}\right)^{1-\alpha}. \tag{13}$$

While (10)-(13) are all static equilibrium variables, our main interest is to identify the determinants of income per capita in a dynamic Malthusian equilibrium, i.e. in a situation where the population level remains constant over time. We know that a constant population implies that $L_{t+1} = L_t$, and hence from (9) that $b_t = 1/\left(1-d\right)$. Inserting (12) into (9) we find that the law of motion of population is

$$L_{t+1} = (1-d)\gamma^{\alpha}\Omega_A L_t^{\alpha} \equiv f(L_t). \tag{14}$$

Solving for the population level in a Malthusian equilibrium (denoted by an asterisk), we find that

$$L^* = \left((1-d)\,\Omega_A \left(\frac{\gamma}{\Omega_M} \right)^{\alpha} \right)^{\frac{1}{1-\alpha}}. \tag{15}$$

This leads us to conclude the following.

Proposition 1. *The two-sector Malthusian model has a unique, globally stable, dynamic equilibrium (a steady-state) at which population size is given by (15). The steady-state population size is a positive function of agricultural productivity (Ω_A), but a negative function of the death rate (d) and of industrial productivity (Ω_M).*

Proof. Stability of the steady state follows from the fact that $f(L_t)$, defined in (14), is a concave function that intersects the $L_{t+1} = L_t$ identity-line in the positive quadrant of a phase diagram exactly at L^*, with $f(L_t) > L_t$ for $L_t < L^*$, and vice versa for $L_t > L^*$. □

We can now compute the steady state equilibrium income, using (13) and (15), so as to get

$$\left(\frac{w}{p} \right)^* = \frac{\Omega_M}{\gamma(1-d)}. \tag{16}$$

Based on (16), the following can be observed.

Proposition 2. *The two-sector Malthusian model predicts that: (i) higher agricultural productivity leads to a higher steady state population level, but has no effect on equilibrium income; (ii) higher death rates lead to a lower steady state population level and a higher equilibrium income; and (iii) higher manufacturing productivity leads to a lower steady state population level and a higher equilibrium income.*

Proof. The Proof follows directly from observing equation (16). □

Starting with part (i) of Proposition 2, this is the conventional result of the standardized Malthusian model. Namely that productivity growth in agriculture is eventually 'eaten up' by a larger population, and so, in the long run, has no effect on equilibrium income in steady state. Accordingly, variations in agricultural productivity cannot account for variations in equilibrium income across time and space. Turning to part (ii) of Proposition 2, this captures the benign effect of higher death rates on living standards, as highlighted by Clark (2007) and discussed at length by Voigtländer and Voth (2010).

Finally, part (iii) of Proposition 2 points to yet another reason why equilibrium incomes may differ across time and space, and captures the main contribution of the current work. Namely that advances in industrial productivity have a permanent and benevolent impact on standards of living. The mechanics are the following. Productivity growth in industry increases agricultural terms of trade, which escalates the costs of raising children relative to the costs of other consumption goods. If children are ordinary goods and gross substitutes to other consumption goods (as captured by the quasi-linear utility function in (2)), then parents respond to this by lowering births. This corresponds to a reduction in the slope of the birth schedule—see Figure 1(b)—causing its intersection with the death schedule to take place at a higher equilibrium income.

4. Empirical Evidence

The following section sets out to examine some empirical evidence relative to the theory presented above. As captured by equation (16), the main implication of the model is that equilibrium income is determined by two factors: the death rate and the level of productivity in industry, both of which are positively correlated with income.

Taking logarithms to equation (16), we get

$$\log(\frac{w}{p}) = \log \Omega_M - \log \gamma - \log (1-d)\,.$$

We define $\log(w/p) \equiv$ wage, $\log \Omega_M \equiv$ industry, and $-\log(1-d) \equiv$ death, so the expected long run empirical relationship is of the following form:

$$wage = \sigma + \delta \cdot death + \lambda \cdot industry,$$

where the expectation is that $\delta = 1$ and $\lambda = 1$.

As a proxy for income we use the wage rate of the poorest workers: unskilled agricultural laborers. Annual nominal wages are provided by Allen (1992) and Beveridge (1936) for southern England for the period 1300-1830. These we deflate by an annual consumer price index offered by Allen (2001), so as to get a time series of real wages. Real wages are thus measured in units of consumption goods rather than in agricultural or industrial goods, which is of course crucial for us to be able to draw conclusions from the data.

Crude death rates (i.e. number of deaths per 1,000 individuals per year) are taken from Wrigley and Schofield (1989), and are available for the period 1541-1871. Of course this is an imperfect proxy for child mortality (the variable used in the model above), but life tables are only available by decade (Wrigley et al. 1997) and so cannot be used in the analysis, since we then would have very few observations to work with. The fact that we use crude death rates instead of child mortality may potentially bias the results.[6] Since it is likely that child mortality is more sensitive to economic fluctuations than is adult mortality, it is probable that our point estimates are underestimating the actual effect. Hence, although δ should be equal to 1 in the model, it would not be surprising if the effect of mortality on wages fell between zero and one. Additionally, if crude death rates are an imperfect proxy for child mortality, then our analysis will be subject to more noise, leaving us with larger standard errors, and hence less significant point estimates, than had we used child mortality.

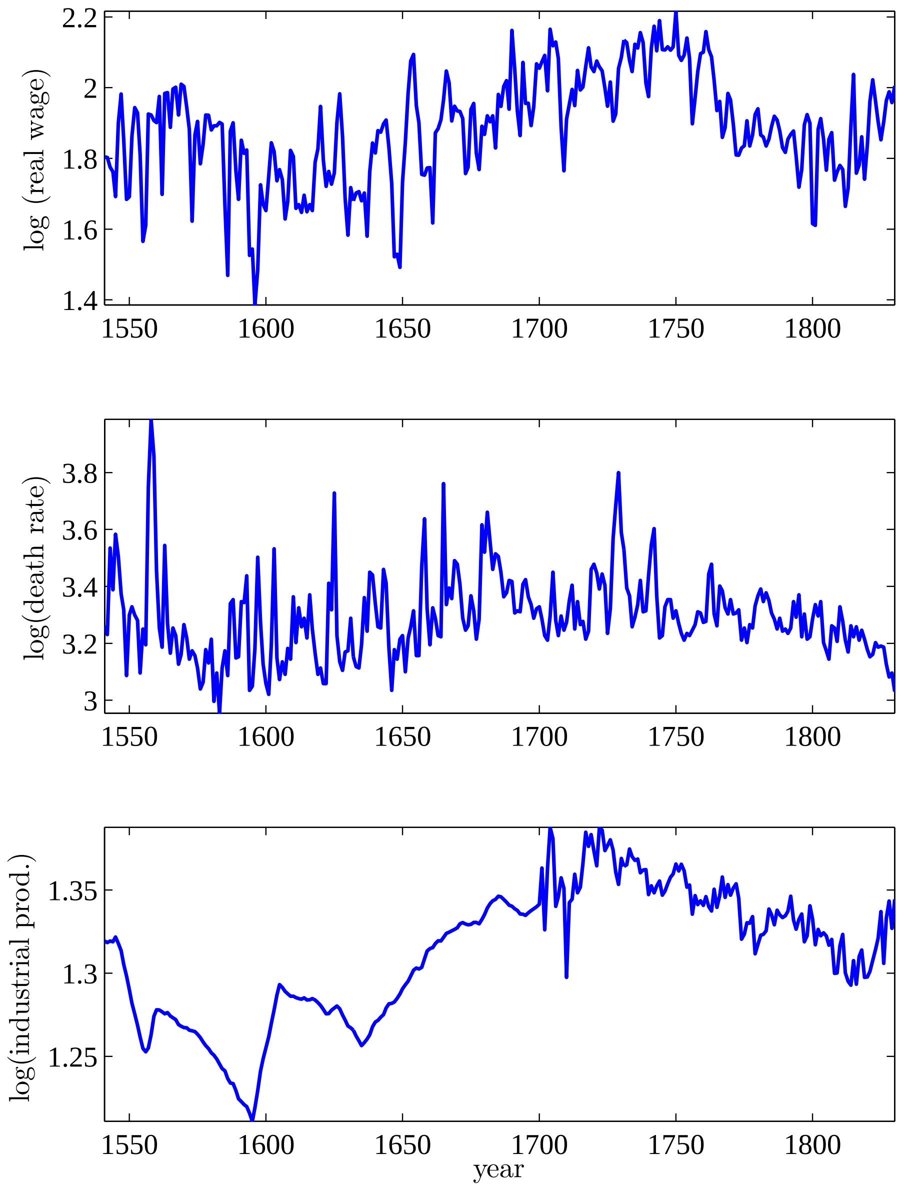
Furthermore, digging up historical evidence concerning industrial productivity is a challenging task, even for England. The most ambitious attempt to generate a time series useful for the purpose at hand is found in O'Rourke and Williamson (2005). This series is constructed based on data from five sources: Broadberry (1997), Crafts and Harley (1992), Crafts (1985), and Clark (2001, 2007). Further details on the origins of the data are available from their appendix (*ibid.*, p. 31). The series is indexed (1900=100), and runs from 1500 to 1936.

The intersection of all three time series—real wages, death rates and industrial productivity—comprises the period 1541 to 1830, spanning nearly three centuries and illustrated in Figure 2. As is clear from the graph, the time series for industrial productivity is a composite of two series meeting in 1700, so we divide the data into two samples: 1541-1700 and 1700-1830. Moreover, it is also clear from the graphs that the series are non-stationary, so we perform cointegration analyses using Dynamic OLS (DOLS). This estimation technique has the advantage, compared to the usual static Engle-Granger approach, that the model is well-specified (since it includes dynamic effects) so, given cointegration (which we test for below), t-ratios constructed from the standard errors follow standard normal distributions under the null.

For time series analysis, it is essential to define the length of a time-period. In the framework presented above the length of a period is one generation. On the face of it, this makes it seem that a household in the model takes decisions only every 20-25 years. In reality, of course, time

[6] We thank an anonymous referee for pointing this out.

Figure 2: Wages, Death Rates, and Industrial Productivity 1541-1830



is continuous, and our notion thus simply reflects the fact that a cohort of households takes decisions at any point in time. Against this background, the length of a time-period should simply be the shortest interval for which data is available—in our case, a year.

Furthermore, as the impulse response analysis in the Appendix demonstrates, the economy adjusts rather quickly when disturbed, reaching its new steady-state only five years after a permanent shock. Although our main interest here concerns the steady-state (i.e. long-run) relationship predicted by the model, we do however take into account the short-run adjustment dynamics by adding lags. We thus proceed to estimate the model as follows:

$$\begin{aligned} wage_t &= \beta_0 + \beta_1 wage_{t-1} + \beta_2 wage_{t-2} + \beta_3 death_t + \beta_4 death_{t-1} \\ &\quad + \beta_5 death_{t-2} + \beta_6 industry_t + \beta_7 industry_{t-1} + \beta_8 industry_{t-2} + \beta_9 t + \varepsilon_t, \end{aligned} \tag{17}$$

where $wage_t$ is the (log of the) daily wage rate at time t, $death_t$ is minus the log of one minus the crude death rate, $industry_t$ is the (log of the) industrial productivity index number, and t is a trend. It is assumed that the error term ε_t is iid normally distributed. We include two lags, although the data suggest only one is necessary. This will increase the standard deviation of the estimates, but we keep this specification throughout to avoid ambiguity and to ease comparison with later results.

Formal tests do not suggest any major problems with non-normality or non-independence of the residuals: the LM test for no autocorrelation and the Doornik and Hansen (2008) test for normality of the residuals cannot be rejected at the 1% level. Moreover, the trend is insignificant. Solving the estimated short-run relationship (17) for the steady-state relationship we compute $\sigma = \beta_0/(1-\beta_2-\beta_2)$, $\delta = (\beta_3+\beta_4+\beta_5)/(1-\beta_2-\beta_2)$, and $\lambda = (\beta_6-\beta_7-\beta_8)/(1-\beta_2-\beta_2)$ and find:

$$wage = -\underset{(-1.45)}{1.08} + \underset{(1.87)}{0.32} \cdot death + \underset{(2.24)}{1.45} \cdot industry.$$

The unit root t-statistic, which gives the PcGive test for the null of no cointegration, is -5.71, which should be compared to a 5% critical value of 3.93—so a clear rejection of the null. As explained above, this means that the test statistics follow the standard distributions. Individually, the variables are clearly then significant, and the Wald test for joint significance is accepted with a p-value of 0.000.

Since all variables are in logarithms, the parameters can be interpreted as elasticities. So a 1 percent increase in death leads to a long-run increase of 0.32 percent in wage. Likewise, a 1 percent increase in industry leads to a 1.45 percent increase in wage in the long-run. The coefficient for industry is positive and insignificantly different from 1, in line with the model.[7] The coefficient for death is also positive, yet significantly different from 1, and hence falls statistically

[7]The t-value for the test of the null that $\lambda = 1$ is 0.70

between 0 and 1. That the latter finding is not entirely consistent with the model could simply reflect the fact that the use of crude death rates instead of child mortality underestimates the effect.[8]

Clearly in a fuller model, the death rate need not be exogenous to wages, which is the assumption here. For instance, the Malthusian positive check suggests that death rates should increase when real wages fall. To control for possible endogeneity between the variables, we also experimented with modelling the data within the framework of the cointegrated VAR model (Johansen 1996, Juselius 2006). Not unexpectedly, we can within this very general framework identify other relations than the one identified above, and we have experimented with many alternatives. In all specifications, by far the most robust relationship was that between industrial productivity and wages. The death rate relationship was rather more sensitive to the exact specification of the model, which however is not surprising, given that we are using an imperfect proxy for child mortality, as described above.[9]

Returning to dynamic OLS and focussing now on the second period, 1700-1830, the results are much less clear. Formal tests for this model do not suggest any problems with non-normality or non-independence of the residuals. Estimating and solving in the same way for steady-state relationship we find:

$$wage = -\underset{(-2.23)}{2.79} + \underset{(0.77)}{0.19} \cdot death + \underset{(2.28)}{3.08} \cdot industry.$$

Again, the Wald test for joint significance of the variables is accepted with a p-value of 0.000. Moreover, the PcGive unit root t-statistic is lower, however, at -4.51, and is thus a rather less clear rejection of the null. The coefficient for industry is positive and, though relatively large, borderline insignificantly different from 1, in line with the model. The fact that the coefficient for death is now insignificant seems consistent with the idea that Malthusian mechanisms were becoming much less important by the turn of the eighteenth century.

[8]As a robustness check, we perform the same analysis using Clark's alternative series of real wages for agricultural workers (Clark 2001). As is well known, this series displays less long-run fluctuations than Allen's wage series used above, which would be expected to impact on our estimates. Moreover, it turns out that a trend is significant. Nevertheless, the cointegration results are qualitatively similar: the long-run coefficient to death has a parameter value of 0.20 and the coefficient for industry is 1.24, although they are not significant at the 5% level. The PcGive test for the null of no cointegration is -5.58 in this case, i.e. a rejection of the null.

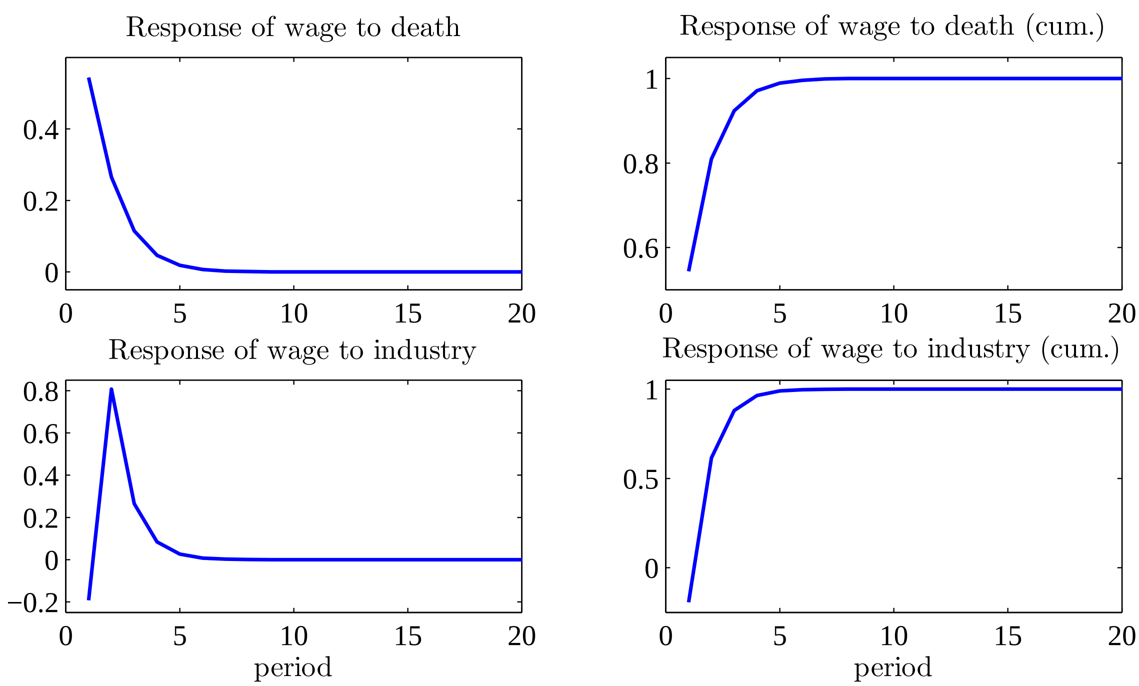
[9]Restricting the cointegrated VAR model to the formulation above, i.e. where we assume weak exogeneity of death and industry and only one cointegrating relation, resulted in a rejection of the restrictions with a p-value of 0.012. This is not entirely surprising, however, given the severity of the restrictions within a general model. The important point is that the main results hold within a more general framework.

Overall, our empirical analysis lends support, not only to the current hypothesis, but also in the pre-industrial period to the work of Voigtländer and Voth (2010), sustaining the idea that an increase in death rates and industrial productivity are jointly responsible for lifting the steady state equilibrium income in a society subject to Malthusian dynamics.

5. Appendix: Impulse Response Functions

The impulse response functions (see Figure 3) illustrate the adjustment of wage after a permanent shock to death or industry. The two upper graphs illustrate the response of wage to a permanent shock to death. The two lower graphs illustrate the response of wage to a permanent shock to industry. The graphs on the left illustrate the proportion of the total adjustment to the new steady state for wage in each period, while the graphs on the right illustrate the accumulated response. Note that the total response of wage (the long-run multiplier, which is the sum of the dynamic multipliers for each period) has been normalized to one in each case. Clearly, a shock to either variable leads to a rapid adjustment (i.e. curves are very steep) which means that the adjustment is more or less complete after five periods.

Figure 3: Impulse Responses



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