The Voracity Effect Revisited

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In an influential article Tornell and Lane (1999) considered an economy, populated by multiple powerful groups, in which property rights in the formal sector are not protected. They argued that then investment in an informal sector may be optimal and set up conditions for "voracity" such that a permanent positive shock in the formal sector leads to lower economic growth. Here I show that whenever investing in the informal sector is feasible, not investing in the informal sector is a Pareto-superior Nash equilibrium under the mild condition of an elasticity of intertemporal substitution in consumption smaller than unity. As a corollary, voracity disappears.

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1. Introduction

In an influential article Tornell and Lane (1999) discussed the impact of powerful groups and weak institutions on economic growth. They considered an economy where powerful groups, the players of the game, appropriate output from a formal sector of high productivity in order to consume it or to invest it in an informal sector in which productivity is low but private property is protected. Both sectors operate with linear technologies.

Given that the groups use linear feedback-Nash (or MPE) strategies in symmetric play in order to maximize their life-time utility, Tornell and Lane, henceforth TL, obtain the following results: 1) There exists a range of parameter values for which groups optimally decide to extract resources from the efficient sector in order to invest them in the inefficient sector. 2) An increase in the number of competing groups increases the long-run growth rate of the economy. 3) The voracity effect: a permanent increase in productivity of the efficient sector leads to a reduction in the economy's long-run growth rate. 4) There exists a range of parameter values for which investing in the informal is not feasible (in linear strategies) whereas not investing and consuming the appropriated output from the formal sector is feasible.

What TL did not investigate is whether groups actually prefer to invest in the informal sector when it is feasible. I perform this exercise below and show that if the elasticity of intertemporal substitution in consumption is smaller than unity, then not investing in the informal sector is the Pareto-superior Nash equilibrium in linear, symmetric play. The intuition for the result is as follows. As long as sufficient curvature of the utility function ensures high enough incentive to smooth consumption over time, that is as long as the elasticity of intertemporal substitution is smaller than one, it is better to "save" in terms of output of the formal sector of high productivity (high return) than in terms of output of the informal sector where productivity is low. In that case the best response of any player in symmetric equilibrium is to increase investment (by reducing consumption in terms of the formal sector) if the other players reduce their investment (increase consumption). This "conservative" strategy of players makes investing in the formal sector a Pareto-superior Nash equilibrium.

In principle, mere existence of a superior equilibrium does not mean that it is attained. But TL assume also that at the beginning of the observation period, at time $t = 0$, there was no informal sector. Given this reasonable assumption it is plausible to conclude that at times $t < 0$ the groups did not invest in the informal sector, i.e. that they attained the superior equilibrium initially. Since payoffs are higher at the superior equilibrium there exists no incentive to deviate by investing in the low productivity sector.

A different question, however, is whether the inferior equilibrium is stable if it "somehow" has been attained. Coordination (a conference of the groups) would of course be sufficient to establish the superior equilibrium.[1] But the superior equilibrium may also be reached in an uncoordinated fashion. In Section 4 I investigate asymmetric play and show that an elasticity of intertemporal substitution below $x/(n-1)$ is a sufficient condition for not investing in the informal sector to be the dominant strategy, where n denotes the number of groups, and x denotes the number of groups that are initially not investing in the informal sector. In particular if the number of groups is small, as suggested by TL, this means that the equilibrium with investment in the informal sector is unstable under a relatively mild condition.

As a corollary, I conclude that there exists no voracity effect when the elasticity of intertemporal substitution is smaller than unity.

2. Recap of the Tornell–Lane Model

Consider an economy populated by $n \geq 2$ groups, indexed by $i = 1, \ldots, n$, each of them maximizing intertemporal utility from consumption (i.e. life-time utility) V.

$$V = \int_0^\infty \frac{\sigma}{\sigma - 1} \cdot c_i^{\frac{\sigma-1}{\sigma}} \mathrm{e}^{-\delta t} \mathrm{d}t \ . \tag{1}$$

The time preference rate is δ and σ denotes the elasticity of intertemporal substitution. Capital in the formal sector is denoted by K and evolves according to

$$\dot{K} = \alpha p K - \sum_{i=1}^{n} r_i \ , \quad K(0) = K_0. \tag{2}$$

where αp denotes factor productivity. Property rights in the formal sector are not protected and r_i is the extraction of group i from this sector. Capital in the informal sector is a closed access asset to each group and evolves according to

$$\dot{b}_i = \beta b_i + r_i - c_i, \ \ i = 1, \ldots, n \ . \tag{3}$$

Productivity in the formal sector is higher:

$$\alpha p > \beta \ . \tag{4}$$

The initial stock in the informal sector is zero:

$$b_i(0) = 0 \ . \tag{5}$$

All parameters are positive and all variables are restricted to be non-negative. TL make also additional assumptions about maximum extraction rates (extreme strategies) which I neglect in the following because they are irrelevant for the argument. Also, for linguistical

[1] This may lead to the following question: if the groups can coordinate on the superior Nash-equilibrium, why could they not coordinate to arrive at even better outcomes? The answer is straightforward: in an A-equilibrium playing B is not a credible threat since the defector is worse off. Playing A is a stable equilibrium of assurance (see, for example, Dixit and Skeath, 2004). An even better outcome than playing A requires a credible threat and a trigger strategy and is much more fragile. This is shown by Lindner and Strulik (2008) who investigate trigger strategies in a common pool game within the Ak-growth framework.

convenience, if groups are investing in the informal sector, I say they "play B". Thus, if $r_i > c_i$, group i plays B. If groups are not investing in the informal sector, they "play A". If $r_i = c_i$, group i plays A. These codes a mnemonically derived from productivity in the formal (α) and informal sector (β) . TL show the following.

If all groups play B:

$$r_i = s_R \cdot K, \quad s_R \equiv \frac{\alpha p - \beta}{n-1} \tag{6}$$

$$c_i = s_B \cdot (K + b_i), \quad s_B \equiv \beta(1-\sigma) + \delta\sigma. \tag{7}$$

$$\frac{\dot{c}_i}{c_i} = g_B \equiv \sigma(\beta - \sigma). \tag{8}$$

If all groups play A:

$$c_i = r_i = s_A \cdot K, \quad s_A \equiv \frac{\alpha p(1-\sigma) + \delta\sigma}{n - \sigma(n-1)}. \tag{9}$$

$$\frac{\dot{c}_i}{c_i} = \frac{\dot{K}}{K} = g_A \equiv \frac{\sigma(\alpha p - \delta n)}{n - \sigma(n-1)}. \tag{10}$$

Note that, if σ were larger than unity, there exists a range of parameter values for which playing B is not an option since s_B and thus c_i would be negative. In these instances playing A may still be an option because both the numerator and denominator of s_A are negative such that c_i is still positive. Moreover a solution with investment in the informal sector does not exists if parameters would imply $s_B \geq s_R$ according to (6) and (7), that is negative investment. These cases are investigated by TL and are not re-addressed in the following. Here, the analysis is confined to the case where playing A and playing B are both feasible. The whole point is to show that playing A is preferred in symmetric equilibrium. The actual range of parameter values for which the informal sector is inactive is thus larger than obtained from the subsequent analysis. It consists not only of the (discussed) case where playing A is preferred over B but also of the (neglected) case where playing B is anyway not an option.

In order to understand the result in (9), take a step back and consider linear consumption strategies $s_{A,j}$, $c_j = s_{A,j}K$. Let $\tilde{s}_A \equiv \sum_{j\neq i}^{n} s_{A,j}$. Solving problem (1)-(2) for group i then leads to the response function $s_{A,i} = \sigma\delta + (1-\sigma)(\alpha p - \tilde{s}_A)$. Inspection shows that $s_{A,j}$ and $\tilde{s}_A$ are inversely related for $\sigma < 1$. They are strategic substitutes. Group i consumes less if the other groups consume more. This observation verifies the claim from the Introduction that groups follow a "conservative" strategy for $\sigma < 1$. Assuming symmetry, inserting $\tilde{s}_A = (n-1)s_{A,i}$ into the response function, and solving for $s_A = s_{A,i}$ provides the result in (9).[2]

3. Playing A Pareto-dominates Playing B

The following proposition summarizes the main result.

[2]The reasoning in this paragraph follows Long (2010, Chapter 4.6.2.).

Proposition 1. *Consider the TL setup and assume that it is feasible that the informal sector exists ($r_i > c_i$ if all groups play B). Then, if $\sigma < 1$, not investing in the informal sector (all groups play A) yields higher life-time utility for (the representative member of) each group.*

The proposition is most conveniently proven by establishing a series of lemmata. We begin by calculating life-time utility.

Lemma 1. *If all groups play A life-time utility $V = V_A$. If all groups play B life-time utility $V = V_B$.*

$$V_x = \frac{\sigma}{\sigma - 1} \cdot (K_0)^{(\sigma-1)/\sigma} \cdot s_x^{-1/\sigma}, \qquad x \in \{A, B\}. \tag{11}$$

For the proof, consider first the case when all groups play A and insert the consumption strategy (9) into the utility function (1).

$$V = V_A = \frac{\sigma}{\sigma - 1} \cdot (s_A \cdot K_0)^{\frac{\sigma-1}{\sigma}} \int_0^\infty \exp\left[\left(g_A \cdot \frac{\sigma - 1}{\sigma} - \delta\right) t\right] \mathrm{d}t.$$

After inserting consumption growth (10) the exponent of the above expression simplifies to

$$g_A \cdot (\sigma - 1)/\sigma - \delta = -\frac{\alpha p(1 - \sigma) + \delta\sigma}{n - \sigma(n - 1)} = -s_A. \tag{12}$$

Thus life-time utility is given by (13).

$$V_A = \frac{\sigma}{\sigma - 1} \cdot (s_A \cdot K_0)^{\frac{\sigma-1}{\sigma}} \int_0^\infty \exp(-s_A t) \mathrm{d}t = \frac{\sigma}{\sigma - 1} \cdot (K_0)^{(\sigma-1)/\sigma} \cdot s_A^{-1/\sigma}. \tag{13}$$

This is (11) for $x = A$. Next consider the case when all groups play B. Noting that $b_i(0) = 0$ and thus $c_i(0) = s_B K_0$ provides the following life-time utility.

$$V = V_B = \frac{\sigma}{\sigma - 1} \cdot (s_B \cdot K_0)^{\frac{\sigma-1}{\sigma}} \int_0^\infty \exp\left[\left(g_B \cdot \frac{\sigma - 1}{\sigma} - \delta\right) t\right] \mathrm{d}t.$$

Inserting consumption growth (8) the exponent of the above expression simplifies to

$$g_B \cdot (\sigma - 1)/\sigma - \delta = -s_B. \tag{14}$$

Thus life-time utility is given by (15).

$$V_B = \frac{\sigma}{\sigma - 1} \cdot (s_B \cdot K_0)^{\frac{\sigma-1}{\sigma}} \int_0^\infty \exp(-s_B) \mathrm{d}t = \frac{\sigma}{\sigma - 1} \cdot (K_0)^{(\sigma-1)/\sigma} \cdot s_B^{-1/\sigma}. \tag{15}$$

This is (11) for $x = B$. Compare V_A and V_B to arrive immediately at the following result.

Lemma 2. *For $\sigma < 1$, $\quad V_A > V_B \quad \Leftrightarrow \quad s_A > s_B$.*

Next inspect (12) and (14) and recall that $\sigma < 1$ and $\delta > 0$ to arrive immediately at the following conclusion about the relation between consumption shares and growth rates.

Lemma 3. *For $\sigma < 1$, $\quad s_A > s_B \quad \Leftrightarrow \quad g_A > g_B$.*

The next Lemma establishes the relation between s_R and s_B.

Lemma 4. *For the informal sector (the B sector) to exist, $s_R > s_B$.*

For the proof insert $b_i(0) = 0$ and (6) and (7) into (3) to get $\dot{b}_i = (s_R - s_B)K$. Thus $s_R > s_B$ is needed for the B-sector to get started. For that the groups have to extract more from the A-sector than they consume. The constraint is also acknowledged by TL.

The final Lemma provides the closing link to prove Proposition 1.

Lemma 5. *For $\sigma < 1$, $\quad s_R > s_B \quad \Leftrightarrow \quad g_A > g_B$.*

For the proof compare s_R and s_B:

$$\begin{aligned} s_R = \frac{\alpha p - \beta}{n-1} &> \beta(1-\sigma) + \delta\sigma = s_B \qquad (16) \\ \alpha p - \beta &> \beta(n-1)(1-\sigma) + \delta\sigma(n-1) \\ \alpha p - \delta n &> (\beta - \delta)\left[n - \sigma(n-1)\right] \\ \frac{\sigma(\alpha p - \delta n)}{n - \sigma(n-1)} &> \sigma(\beta - \delta) \end{aligned}$$

that is $g_A > g_B$.

The proof of Proposition 1 is now straightforward. For investment in the B sector, $s_R > s_B$ from Lemma 4. Then $g_A > g_B$ from Lemma 5. Then $s_A > s_B$ from Lemma 3. Then $V_A > V_B$ from Lemma 2. q.e.d.

It is interesting to assess quantitatively whether the informal sector does not exist or whether investing in it is Pareto-dominated. For a back of the envelope "calibration" I assume that the economy could grow at a rate of 2 % annually if property rights were secure. The growth rate under secure property rights (the market solution, or the "first best" in TL) is computed as $g_{fb} = \sigma(\alpha p - \delta)$. I set $\alpha p = 0.06$ according to a frequent calibration of the real rate of return under secure property rights (e.g., Barro et al., 1995). We consider alternative σ and recalibrate δ such that $\delta = \alpha p - g_{fb}/\sigma$.

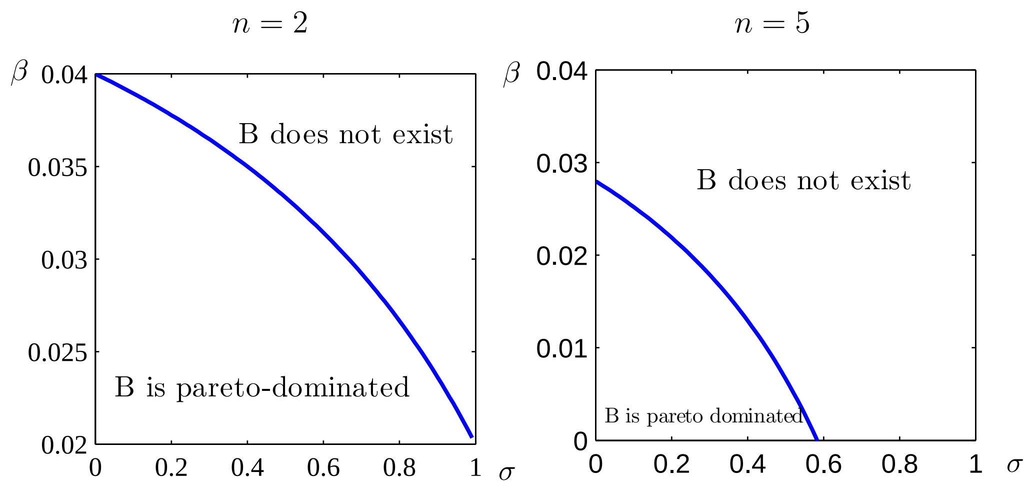
Inspection of (16) shows that investment in the informal sector is most likely if n is small, since the investment strategy s_R is decreasing in n. Generally, after substitution of the calibrated δ, condition (16) requires that

$$\frac{\alpha p\left[1/(n-1) - \sigma\right] + g_{fb}}{1/(n-1) + (1-\sigma)} > \beta.$$

Perhaps surprisingly, existence of the informal sector is more likely if productivity of the sector is low. This result (also obtained by TL) is explained by no-arbitrage, which requires equality of the net return of investment (foregone consumption in the A sector, calculated as $\alpha p - (n-1)r'(K)$, and in the B sector, which is simply β. If β is too large compared to αp, extraction from the informal sector is too small compared to consumption needs s_B, which are increasing in β.

Figure 1 shows the threshold between non-existence and inferiority of the informal sector for 2 and 5 groups. Recalling that returns in the formal sector $\alpha p = 0.06$ the impression is that productivity in the informal sector must be quite low for the sector exist. Moreover, the size of the set of parameters $\{\sigma, \beta\}$ for which the informal sector exists contracts quickly as the number of powerful gets larger.

Figure 1: The Informal Sector: Threshold between Non-Existence and Inferiority



Calibration: $A = 0.06$, alternative σ and δ recalibrated such that the economy would grow at rate 2% at the first best equilibrium (under protected property rights); see main text for details.

Note that s_B and g_B are independent from how many other players there are and what they play. If the initial symmetric equilibrium strategy was playing A there exists thus no incentive to deviate and play B since the obtained payoff V_B falls short of V_A. The equilibrium "all play A" is stable. Since the assumption is that initially there was no capital stock in the B sector (condition (4)), it is plausible to conclude that initially the groups played A. As long as preferences are stable and σ stays below unity there is thus no reason for an informal sector to occur.

Finally, taking the derivative $\partial g_A / \partial(\alpha p)$ of (10) verifies the following conclusion.

Corollary 1. *For $\sigma < 1$ there exists no voracity effect.*

An improvement of factor productivity (or the terms of trade) leads, as usual in the Ak growth model, to higher growth.

4. Asymmetric Play

Although it is plausible to conclude that the groups played A initially from the assumption that there was no capital in the informal sector initially, it is interesting to assume that "somehow" the economy is nevertheless in a situation in which all groups play B. In principle they should then coordinate and assume the symmetric equilibrium of playing A. But suppose that there is no coordination device available. The following analysis considers deviation of one or several groups and shows that under an additional requirement on the elasticity of intertemporal substitution playing A is preferred, i.e. playing B is not stable. If there are only a few powerful groups the additional requirement is rather mild.

Consider the TL setup with the following modification. Suppose x groups are initially playing A, $0 \leq x \leq n$, while $n-x$ groups are initially playing B. Suppose wolog that the first $i = 1, \ldots, x$ players are playing A. As TL we confine the analysis to linear strategies. The following proposition, proven in the Appendix, summarizes the results from asymmetric play.

Proposition 2. *In the TL setup in asymmetric equilibrium at which x groups are playing A (are not investing in the informal sector) and $n-x$ groups are playing B (are investing in the informal sector), extraction- and consumption strategies and associated growth rates are as follows.*

For those playing B:

$$r_i = s_R \cdot K, \quad s_R \equiv \frac{\sigma\left[\alpha p - \beta(1-x)\right] - bx - \delta x \sigma}{\sigma(n-1) - x} \tag{17}$$

$$c_i = s_B \cdot (K + b_i), \quad s_B \equiv \beta(1-\sigma) + \delta\sigma. \tag{18}$$

$$\frac{\dot{c}_i}{c_i} = g_B \equiv \sigma(\beta - \delta). \tag{19}$$

For those playing A:

$$c_i = r_i = s_A \cdot K, \quad s_A \equiv \frac{(1-\sigma)\left[\alpha p - \beta(n-x)\right] - (n-x-1)\delta\sigma}{x - \sigma(n-1)}. \tag{20}$$

$$\frac{\dot{c}_i}{c_i} = \frac{\dot{K}}{K} = g_A \equiv \frac{\sigma\left[\alpha p - (n-x)\beta - \delta x\right]}{x - \sigma(n-1)}. \tag{21}$$

Note that (17)–(19) collapse to (6)–(8) if all groups play B ($x = 0$) and that (20)–(21) collapse to (9)–(10) if all groups play A ($x = n$). Symmetric play of A or B is thus included as border cases. Note also that the consumption strategy s_B and the associated growth rate are, as claimed above, invariant to what the other players do. This result follows naturally from the fact that property rights in the B sector are respected. Moreover inspection of (17) and comparison with (6) shows that the fact that some groups are playing A further reduces the parameter range for which playing B is an option since either the numerator or the denominator of (17) gets negative. In this case we assume, similar as TL did for symmetric play, that all groups play A, i.e. return to strategies (9).

It is straightforward to check that Lemmata 1 to 4 hold for symmetric strategies as well as for asymmetric strategies. It is thus only Lemma 5 that gets replaced by the following result.

Lemma 6. *If the informal sector exists when groups use symmetric strategies then, if $\sigma < x/(n-1)$, $g_A > g_B$ when groups use asymmetric strategies.*

For the proof recall that for the informal sector to exist, $s_R > s_B$, which means that when all groups use symmetric strategies

$$s_R = \frac{\alpha p - \beta}{n-1} > \beta(1-\sigma) + \delta\sigma = s_B.$$

$$\alpha p > \beta n - \sigma\beta(n-1) + \delta\sigma(n-1) \quad \Rightarrow \quad \alpha p - (n-x)\beta - \delta x > \beta x - \sigma\beta(n-1) - \delta x + \delta\sigma(n-1)$$

Noting that $x - \sigma(n-1) > 0$ for $\sigma < x/(n-1)$ and dividing the above expression by $x - \sigma(n-1)$ provides

$$\frac{\alpha p - (n-x)\beta - \delta x}{x - \sigma(n-1)} > \beta - \delta.$$

Multiplying both sides by σ we get $g_A > g_B$. The growth rate of consumption is higher for the groups that play A.

As for symmetric strategies it is now straightforward to prove Proposition 3. From $g_A > g_B$ we conclude $s_A > s_B$ from Lemma 3 and $V_A > V_B$ from Lemma 2.

Proposition 3. *If the informal sector exists when groups use symmetric strategies then, if $\sigma < x/(n-1)$, the groups playing A (not investing in the informal sector) experience higher life-time utility than those playing B (investing in the formal sector).*

If there are only two powerful groups and one group plays A ($n = 2, x = 1$), the A-player gets higher utility if $\sigma < 1$, implying that there is no further restriction of the initial assumption that the elasticity of intertemporal substitution is below unity. If there are more powerful groups, unilateral deviation from a hypothetical initial equilibrium where all are playing B is worthwhile only for a restricted set of parameter values for σ. But then, multilateral deviation becomes interesting. For example, if there are three groups and two groups play A, the A-players experience higher utility for all $\sigma < 1$. They form an internally stable coalition in the sense that no A player wants to join the B-player. On the other hand – recalling that V_B is invariant to the numbers of players and the payoff of A-players – the coalition is not externally stable: the B-player could actually improve life-time utility by joining the A players ($V_A > V_B$). If there are a lot of groups initially playing B, deviation of one or a few playing A becomes less worthwhile. But we have also seen that the parameter space for which which a symmetric B-equilibrium exists contracts quickly as the number of groups gets larger. Summarizing, the B-equilibrium is not stable if there are only a few powerful groups whereas the A-equilibrium is stable.

5. Conclusion

This paper has shown that the explanatory power of the TL model with respect to resource-shifting from a formal sector to an informal sector and voracity depends crucially on the size of the elasticity of intertemporal substitution in consumption σ. If $\sigma < 1$, the B-equilibrium is unlikely to exist and – if it exists – it is Pareto-dominated, eliminated by coordination, and not stable against uni- or multi-lateral deviation.

While the empirical literature does not always agree with Hall (1988) that σ is close zero, it is probably fair to say that there is consensus between quantitative macro- and empirical microeconomists that the elasticity of intertemporal substitution is rarely larger than unity.[3]

This means that under the assumption that groups maximize utility of their representative member the theory has little power to explain the existence of an informal sector and the phenomenon of voracity. One possibility to immunize the theory against this critique could be to argue that groups maximize a "group utility function" and that the elasticity of intertemporal substitution of consumption of a group is larger than that for individuals and, in particular, larger than unity. However, since the "elasticity of intertemporal substitution of consumption of a group" is unobservable, the theory becomes unfalsifiable and loses power for a different, methodological reason (Popper, 1963).

Finally, I would like to emphasize that this paper has not tried to argue against the voracity effect as an empirical phenomenon. It has just suggested that there may exist better rationalizations of the voracity effect than the TL model. An alternative view of the voracity effect is offered in Strulik (2012) by endogenizing the elasticity of intertemporal substitution in consumption. It is shown that the voracity effect then becomes a situation-specific phenomenon. In particular an economy is prone to voracity when aggregate productivity is low, when the society is largely fractionalized, and when the economy is in decline. With σ being endogenous the income effect means that voracity occurs when the elasticity of substitution in consumption is particularly low.

[3]See, for example, Lucas (1990), Campbell and Mankiw (1989), Patterson and Peseran (1992), Attansio and Browning (1995), Ogaki et. al (1996), Atkeson and Ogaki (1996), and Guvenen (2006).

Appendix: Asymmetric Play

It is notation-wise easiest to assume that those groups playing A satisfy consumption directly from extraction of the formal sector (rather than to introduce the complementary slackness conditions for the case $c_i = r_i$). Thus aggregate capital K and group-specific capital b_i evolve according to

$$\dot{K} = \alpha p K - \sum_{j=1}^{x} c_j - \sum_{h=x+1}^{n} r_h, \qquad \dot{b}_i + r_i - c_i, \;\; i = x+1, \ldots n. \tag{A.1}$$

The associated Hamiltonian is

$$H_i = \frac{\sigma}{\sigma - 1} c_i^{\frac{\sigma-1}{\sigma}} + \lambda_i \left[\alpha p K - \sum_{j=1}^{x} c_j - \sum_{h=x+1}^{n} r_h \right] + \mu_i \left[\beta b_h - c_h + r_h \right].$$

After applying within-group symmetry the first order conditions for the groups playing A ($i = j$) are

$$c_j^{-1/\sigma} - \lambda_j = 0$$
$$\lambda_j \left[\alpha p - (x-1) c_j' - (n-x) r_h' \right] = \lambda_j \delta - \dot{\lambda}_j.$$

Differentiating the first equation wrt time and substituting λ_j and $\dot{\lambda}_j$ in the second equation provides (A.2).

$$\sigma \left[\alpha p - (x-1) c_j' - (n-x) r_h' - \delta \right] = \dot{c}_j / c_j. \tag{A.2}$$

After applying within-group symmetry the first order conditions for the groups playing B ($i = h$) are

$$c_h^{-1/\sigma} - \mu_h = 0$$
$$-\lambda_h + \mu_h = 0$$
$$\lambda_h \left[\alpha p - x c_h' - (n-x-1) r_h' \right] = \lambda_h \delta - \dot{\lambda}_h$$
$$\mu_h \beta = \mu_h \delta - \dot{\mu}_h.$$

Eliminating λ_h with use of the second condition, differentiating the first equation wrt time and substituting μ_h and $\dot{\mu}_h$ in the the third equation provides (A.3).

$$\frac{1}{\sigma} \frac{\dot{c}_h}{c_h} = \beta - \delta = \alpha p - x c_h' - (n-x-1) r_h' - \delta. \tag{A.3}$$

As TL we confine the analysis to linear strategies, i.e. we begin with guessing that the solution has the form $c_j = s_A K$ for $j = 1, \ldots, x$ and $r_h = s_R K$ and $c_h = s_B(K + b_h)$ for $h = x+1, \ldots, n$. Thus from (A.3):

$$\frac{1}{\sigma} \frac{\dot{c}_h}{c_h} = \beta - \delta = \alpha p - x s_A - (n-x-1) s_R - \delta \quad \Rightarrow \quad x s_A = \alpha p - \beta - (n-x-1) s_R. \tag{A.4}$$

From the equation on the left hand side we obtain (19). Noting that the linear consumption strategy for A-players implies that consumption grows that the rate of K we get from (2) and (A.2)

$$\dot{c}_j/c_j = \sigma\left[\alpha p - (x-1)s_A - (n-x)s_R - \delta\right] = \alpha p - xs_A - (n-x)s_R = \dot{K}/K.$$

Inserting (A.4) and solving for s_R provides (17). Re-inserting (17) into (A.4) provides (20). Inserting (A.4) into

$$\dot{K}/K = xs_A - (n-x)s_R$$

obtained from (A.1) provides (A.5).

$$\dot{K}/K = \beta - s_R. \tag{A.5}$$

Inserting s_R from (17) provides (21). From (A.1) and (A.5)

$$\dot{K} + \dot{b}_h = (\beta - s_R)K - \beta b_h + s_R K - s_B(K + b_i) = (\beta - s_B)(K + b_h).$$

And thus for $\dot{c}_h/c_h = s_B(\dot{K} + \dot{b}_h)/[s_B(K + b_h)]$:

$$\frac{\dot{c}_h}{c_h} = \frac{(\beta - s_B)(K + b_h)}{K + b_h} = \sigma(\beta - \delta).$$

The right hand side originates from (19). Solving for s_B provides (18).

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